MATHEMATICS FOR ELECTRONIC TECHNOLOGY

by

D. P. HOWSON

University of Bradford

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PREFACE

THIS is the second edition of a book originally written as an introduction to the mathematics required for circuit analysis. In the first edition it was necessary to omit several important topics, owing to shortage of space, and this has now been rectified in the present volume. It is hoped that sufficient breadth of material is included to cover most of the needs of undergraduates taking an electronic engineering course, with the exception of the specialised work necessary for computers and digital systems, which is often taught with the technical material, and probability and statistics, often a subject for a particular lecturer from another department.

At the same time as the extra material has been added to the original text, the opportunity has been taken to broaden the application of the work, since the emphasis placed upon circuit analysis in electronic engineering courses has rightly declined, this being hastened by the widespread use of integrated circuits for general systems.

The material in the book is presented as a necessary tool to solve a variety of electronic problems, and to strengthen this approach to the topic some of the examples at the end of each chapter are concerned with electronics. Many of the "proofs" presented are plausible rather than exact, since it is felt that only in this way can an engineering undergraduate cover the wide variety of topics that he is expected to be familiar with, in the time at his disposal. Even so, it has not been possible to cover all necessary points in sufficient detail within the text of the book, and so a number have been left as examples. It is hoped in this way to secure the reader's active co-operation, rather than his passive attention!

Some advanced material has been included in this book, and has been marked with a dagger (†). In this way it is hoped to indicate some of the directions which a more thorough study of the topics

CONTENTS

CHAPTER 1

INTRODUCTORY CONCEPTS

STUDENTS and engineers will come to a textbook such as this with widely differing mathematical training and skills. It therefore seemed useful to devote an introductory chapter to a range of topics, some of which may be familiar to the reader but all of which are indispensable if a proper grounding in the subject is to be obtained. As in the rest of the book, a number of examples of varying degrees of difficulty are included and solutions, at least in outline, are usually provided. The reader should make a point of working through these to improve the grasp of the subject, particularly as the text treatment of each topic is necessarily brief in a little book like this. Nevertheless, it has been thought worth while to commence with a formal grounding in differentiation and integration, partly to refresh the mind as to the precise meanings of these concepts, but also to allow comparison of the definitions with those used as the subject is developed in other sections of the work. The last part of the chapter is devoted to vector theory, commencing with elementary material, but proceeding to a relatively advanced level.

1.1 Differentiation

The idea of the rate of change of a variable is basic to a study of many engineering problems, and it will be assumed that the student is familiar with this. Here only the salient points of the theory will be covered, and a summary provided of some of the most important results.

The differential, or rate of change, of a function of x, $f(x)$, with respect to an infinitesimal change in x, will be denoted by $f'(x)$ or $\mathrm{d}f/\mathrm{d}x$ and defined at a point x_0 by

$$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x\to x_0} = \lim_{x\to x_0}\left\{\frac{f(x_0+\delta x)-f(x_0)}{\delta x}\right\} \qquad (1)$$

$f(x)$ will accordingly be said to be differentiable at x_0 if such a limit exists. This will occur if $f(x)$ is *continuous* at x_0, in other words if

$$\lim_{x\to x_0} f(x) = f(x_0) \qquad (2)$$

independently of the way in which x approaches x_0.

From the basic definition a number of key results follow:

(a) $\mathrm{d}c/\mathrm{d}x = 0$ where c is a constant
(b) $\mathrm{d}x/\mathrm{d}x = 1$
(c) $\mathrm{d}\{x^n\}/\mathrm{d}x = nx^{n-1}$ (3)

This follows since $(x_0 + \delta x)^n = x_0{}^n\left\{1 + \dfrac{\delta x}{x_0}\right\}^n$

$$\simeq x_0{}^n\left\{1 + \frac{n\delta x}{x_0}\right\}$$

by the Binomial theorem (which is given in (36)).

(d) $\{c\,f(x)\}' = c\,f'(x)$, where c is a constant
(e) $\{f(x) \pm g(x)\}' = f'(x) \pm g'(x)$
(f) $\{f(x)\,g(x)\}' = f(x)\,g'(x) + g(x)f'(x)$ (4)

Because

$$\{f(x)g(x)\}'_{x\to x_0} = \lim_{x\to x_0}\left\{\frac{f(x_0+\delta x)\,g(x_0+\delta x) - f(x_0)\,g(x_0)}{\delta x}\right\}$$

If $f(x_0 + \delta x) - f(x_0) = \Delta f$, similarly for g, then

$$\{f(x)\,g(x)\}' \simeq \lim_{x\to x_0}\left\{\frac{f(x_0)\,g(x_0) + f(x_0)\,\Delta g + g(x_0)\,\Delta f - f(x_0)\,g(x_0)}{\delta x}\right\}$$

and the result follows.

(g) $$\{f(x)/g(x)\}' = \frac{g(x)f'(x) - f(x)\,g'(x)}{g^2(x)} \tag{5}$$

Since, following the notation of (f)

$$\begin{aligned}\frac{f(x_0 + \delta x)}{g(x_0 + \delta x)} &= \frac{f(x_0) + \Delta f}{g(x_0) + \Delta g} \\ &\simeq \frac{(f(x_0) + \Delta f)}{g(x_0)}\left(1 - \frac{\Delta g}{g(x_0)}\right) \\ &= \frac{f(x_0)}{g(x_0)} + \frac{g(x_0)\Delta f - f(x_0)\Delta g}{g^2(x_0)}\end{aligned}$$

Substitution of this expression into (1) gives the result.

Repeated differentiation of a function leads to the concept of the nth *derivative*.

$$f^{(n)}(x) \quad \text{or} \quad \frac{\mathrm{d}^n f}{\mathrm{d}x^n} \tag{6}$$

Differentiation emphasises the features of a function, in the sense that a rapid change of shape in the function becomes a step in the derivative, a step in the function becomes a discontinuity in the derivative. Therefore there is only a restricted range of functions that can have nth derivatives at all points in a finite range of x, and the restriction increases as n increases. Two most important functions that can be differentiated n times at all points whatever the value of n are

$$f(x) = \sin x \quad \text{and} \quad f(x) = \cos x \tag{7}$$

and another is the exponential function

$$f(x) = \exp(x) \tag{8}$$

A list of functions and their derivatives is given in Table 1.

A useful generalisation of (4), for the nth derivative is

$$\begin{aligned}\{f(x)\,g(x)\}^{(n)} &= f(x)\,g^{(n)}(x) + nf'(x)\,g^{(n-1)}(x) + \\ &+ \frac{n(n-1)}{1.2} f^{(2)}(x)\,g^{(n-2)}(x) + \ldots + f^{(n)}(x)\,g(x)\end{aligned} \tag{9}$$

This result can be proved by repeated application of the technique used to prove (4).

The *turning points* of a graph $y = f(x)$ can be determined by finding the values of (x, y) for which

$$dy/dx = 0 \tag{10}$$

Such a turning point is called a *maximum* if y reaches its local maximum value at the point, and a *minimum* is defined correspondingly. In Fig. 1.1 points A, B and C are maxima, D, E and F are

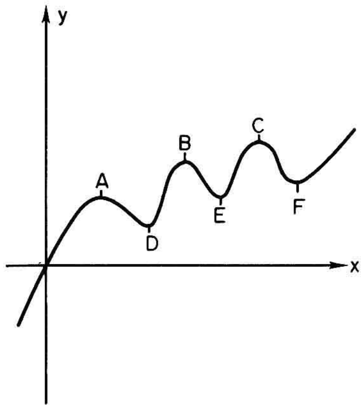

FIG. 1.1. Maxima and minima.

minima. Note that the absolute maximum and minimum values of this function are *not* defined by examination of the maxima and minima—the latter are properly described as turning points of the function. Nevertheless, the determination of turning points for a function is important, and although it can be done analytically for simple functions, in practice numerical methods may have to be employed.

To determine whether a point for which (10) holds is a maximum or minimum is often simplest achieved by consideration of the problem being solved. Often properties or even the general shape

TABLE 1. FUNCTIONS AND THEIR DERIVATIVES

$f(x)$	$f'(x)$
x^n	nx^{n-1}
y^n	$ny^{n-1}(dy/dx)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\exp(ax)$	$a \exp(ax)$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\log x$	$1/x$
$\sin^{-1} x$	$(1 - x^2)^{-\frac{1}{2}}$
$\cos^{-1} x$	$-(1 - x^2)^{-\frac{1}{2}}$
a^x	$a^x \log a$

of the function under investigation are clear before calculation takes place. A mathematical technique for this purpose is to perturb the value of x slightly from that at the turning point and to check whether y increases or decreases. Alternatively the function can be differentiated a second time, since *for a maximum* d^2y/dx^2 *is negative* as x increases through the turning point value, and *for a minimum* d^2y/dx^2 *is positive.*

For example,

$$\begin{aligned} y &= 3x^3 - 9x + 1 \\ y' &= 9x^2 - 9 \\ &= 0 \text{ when } x \text{ is } \pm 1 \\ y^{(2)} &= 18x \end{aligned}$$

When $x = +1$, $y^{(2)} > 0$, when $x = -1$, $y^{(2)} < 0$
Therefore at $x = +1$, y has a minimum and at

$x = -1$, y has a maximum.

Check this result by drawing the graph.

Occasionally (10) will be satisfied, and yet the point determined will be neither maximum nor minimum. In this case it will be a *point of inflection*, as shown in Fig. 1.2, probably best described as a momentary pause in the progress of y, whether increasing or decreasing. At such a point d^2y/dx^2 will also be zero.

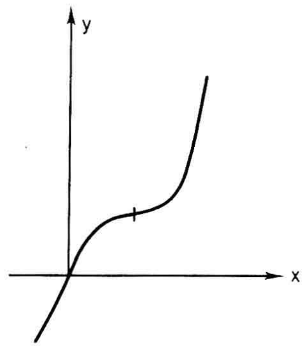

FIG. 1.2. Point of inflection.

There are certain special cases in which the above rules for determining whether a point be a maximum, minimum, or point of inflection do not hold, notably when the second and some higher order differentials are zero, but these will be rarely met in practice.

1.2 Integration

Turning now from differentials, an *indefinite integral* can be defined as a function whose derivative is a given function. The indefinite integral of $f(x)$ is written

$$g(x) = \int f(x)\, dx \tag{11}$$

and from the definition

$$g'(x) = f(x) \tag{12}$$

Since the derivative of a constant is zero, it is clear that $g(x)$ will in general have an added arbitrary constant. Some of the properties of the indefinite integral can be deduced from those of the derivative. Thus

(i) $\int dx = x + C$, C an arbitrary constant

This follows from (b)

(ii) $\int n x^{n-1}\, dx = x^n + C$, from (c)

(iii) $\int C f(x)\, dx = C \int f(x)\, dx$, C is constant

Noting (d) and (iii), (ii) may be rewritten

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

(iv) $\int \{f(x) + g(x)\}\, dx = \int f(x)\, dx + \int g(x)\, dx$

(v) From (f), considering the integral of both sides

$$f(x)\, g(x) = \int f(x)\, g'(x)\, dx + \int f'(x)\, g(x)\, dx$$

which is more usually written

$$\int u\, dv = uv - \int v\, du \tag{13}$$

and is a most useful result.

Table 2 gives some useful indefinite integrals.

Turning now to *definite integrals*, these are defined with reference to the graph of the function $y = f(x)$, see Fig. 1.3. If the function

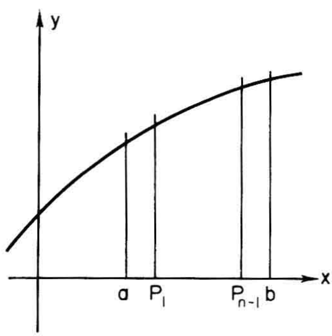

FIG. 1.3. Integration.