Mathematical Thinking in the Measurement of Behavior

Small Groups, Utility, Factor Analysis

EDITED BY *Herbert Solomon*

WITH CONTRIBUTIONS BY

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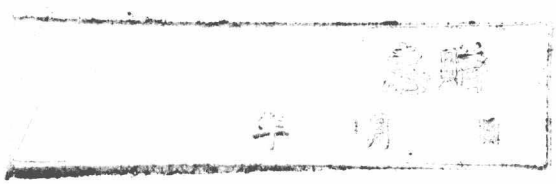

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Mathematical Thinking
in the Measurement of Behavior

A Study of the Behavioral Models Project,
Bureau of Applied Social Research,
Columbia University

To Paul F. Lazarsfeld

Preface

COLUMBIA UNIVERSITY possesses a long tradition in studies which feature the interplay of mathematics and social science. Particularly since the war, there has been much activity in this area in diverse parts of the University involving research in and teaching of mathematical methods for the social sciences, and the resolution of substantive problems in social science settings through mathematical approaches. This activity has been integrated informally at Columbia University by the accepted devices of department courses, faculty seminars and conferences, and sponsored research projects. A more formal organized program, especially in connection with sponsored research projects, has existed in the Bureau of Applied Social Research at Columbia University since 1952.

This volume is a product of the Bureau program and an outgrowth of the Behavioral Models Project; a project in the Bureau program sponsored by the Office of Naval Research from 1952—1956. It is the second of three volumes to appear from the work of this project. The first volume, entitled *Games and Decisions*, was published by John Wiley and Sons, Inc., New York in 1957 under the joint authorship of R. Duncan Luce and Howard Raiffa. The third volume, to be published simultaneously with or shortly after this volume by the Free Press, is edited by R. Duncan Luce. Professor Luce, now at the University of Pennsylvania, was previously at Columbia University where he played a central role in the mathematics program of the Bureau of Applied Social Research. Professor Raiffa, now at Harvard University, and previously at Columbia University, also played an important role in the Bureau program. The editor for this volume, who served as Mathematics Program Director for the Bureau, is now at Stanford University and his two collaborators also are now gone from Columbia University; James Coleman is at Johns Hopkins University and Ernest Adams is at the University of California in Berkeley. In a way, then, Columbia has produced progeny for other centers.

These three volumes are composed of analytical surveys of the uses of mathematics in several social science disciplines. Some of these surveys were completed as long as three or four years ago but publishing difficulties delayed their appearance. Thus they may suffer from some sins of omission because of the recent impetus in research in these subjects. Emphasis in the surveys is on the uses of mathematical techniques and concepts which demonstrate the effectiveness of mathematical thinking in the resolution of substantive problems in social science. No attempt has been made to inte-

grate the subject matter of the surveys, either between volumes or within volumes.

In planning the volumes, it was recognized that not all areas where mathematics has been successfully used in social science could be included. However, it was hoped that by detailed and critical surveys in some specific areas the power of mathematical thinking in social sciences could be promoted to the extent that (1) mathematical and social science curricula would jointly undergo necessary revisions and that (2) social science research would become more of a science than an art.

The topics discussed in the present volume are the choices of the authors and the editor. The opening chapter by James Coleman on small group behavior and the following chapter by Ernest Adams on individual choice behavior represent areas of social science where the uses of mathematics are quite recent. The final chapter, by the editor, on factor analysis, represents a topic which has been undergoing mathematical therapy for half a century. This may give one explanation for the relatively small length of the last chapter as compared with the two preceding chapters. Other explanations are too painful to contemplate.

It is clear that this volume is not a text book in the ordinary sense. However, it should be quite useful in courses that are currently being developed which feature the applications of mathematics in social science. It has been used successfully in a graduate course at Columbia University on mathematical models in the social sciences. It should be useful also to the many research workers in social science whose formal training has been of the non-quantitative type; and to mathematicians and statisticians who have an enthusiasm for applications in social science and the development of techniques for social science problems.

I would like to thank my Columbia colleagues — Professor Paul F. Lazarsfeld, for providing the bubbling intellectual fervor and drive which made the whole program possible; and Professors T. W. Anderson, E. Nagel, H. Raiffa and W. Vickery, for their help and counsel in guiding the program through the completion of the three volumes. Many thanks are extended to R. Duncan Luce for the strong and wise guidance he gave to research performed for the Behavioral Models Project and for his own stimulating research, both of which made the three volumes possible. I would be more than remiss in my duty if at this point I did not cite the Office of Naval Research for its understanding, general helpfulness, and generous financial support of the Project from 1952—1956.

HERBERT SOLOMON
Stanford, California
September 1, 1959

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PART ONE

The Mathematical Study of Small Groups

By JAMES S. COLEMAN

JOHNS HOPKINS UNIVERSITY

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Prefatory Remarks(*)

THIS SURVEY consists of an intensive examination of four kinds of uses of mathematics in the study of small groups. These four approaches are not the only uses of mathematics in this area; no attempt at exhaustiveness has been made. On the other hand, they do represent distinctly different approaches, and thus give some idea of the different ways in which the tools of mathematics have been used to aid the study of small groups.

There are three parts to the survey. The first and shortest of the three is a brief attempt to locate mathematical models of small group behavior in the context of small group studies generally. This is not an overview of research and theory in the area of small groups, but only an attempt to place the work to be examined in the larger framework of theory and research which includes non-mathematical work. The second, which constitutes the main body of the survey, examines each of four uses of mathematics in detail. The third is a comparison and evaluation of the four approaches, suggesting the direction in which each is leading, and speculating about the kinds of results which may reasonably be expected from each.

(*) I am grateful to Duncan Luce for discussion, comment, and criticism throughout the period of preparation of this paper, and to Anatol Rapoport and Herbert Simon for their comments on an earlier draft.

I

INTRODUCTION

1. MATHEMATICS AND SMALL GROUP RESEARCH

IN RECENT YEARS, there has emerged a focus of research and theory in social science with the "small group" as its center. This work has covered a wide range of interests, attempting to answer such questions as:

Is the group a better problem-solving entity than the individuals who compose it, taken separately?

What is the effect of the group on the opinions of individuals within it?

What makes a group more or less cohesive?

How does the communication structure in a group affect its performance in carrying out tasks?

These and other questions have occupied the attention of numerous social psychologists and sociologists over the past fifteen or twenty years, and some answers have been forthcoming as a consequence of their work.

Systematic attention for small groups began in the early part of this century. One of the first men to emphasize the importance of small, face-to-face groups in modern social science was Charles Horton Cooley ([1925], p. 23-28), whose concept of "primary group" is widely used even today. Cooley saw one of the major problems of society to be the socialization of the young: the transformation into civilized beings of the infant "savages" who invade society by being born into it.(*) Cooley suggested that certain kinds of associations with other people, that is, intimate, face-to-face, continuing associations, were the means by which such socialization takes

(*) Although Cooley was not the first to pose this as a problem, most social theorists before his time had taken it for granted, and not raised the question. The work which developed from Cooley's beginning is a good example of the advance in a science which comes not from giving answers but from raising questions, seeing as problems things which before were simply taken as "obvious." This is a step in the building of a science which is too often overlooked in the search for answers to already-posed questions. The *change* in a question, that is, perceiving the problem in a new light or posing a new question can often pave the way to solving problems which were previously unresolvable. Conant [1950] gives good examples of this in his *Harvard Case Histories in Experimental Science*. See "Robert Boyle's Experiments in Pneumatics," p. 14, where Conant indicates the effect of raising a question about the similarity of the atmosphere to a "sea."

place. He saw these "primary group" relationships as a fundamental building block in society, serving as the basis for more elaborate forms of social organization.

Despite Cooley's important contribution toward small group theory, however, the major work in this direction has been relatively recent, within the past twenty years. Beginning in the late 1939's an awakening of interest in social psychology, and with it, in the study of small group processes, came about. Today there exists a voluminous literature in the field.

Small group behavior has become an important focus of social and psychological research.

The Role of Mathematics. But the use of mathematics in the study of small groups has followed some steps behind the resurgence of interest in small groups themselves. This is to be expected, for mathematics is often used to formalize theories or laws which have been previously developed through experimentation and field work. In any science, it ordinarily follows after empirical investigations. However, in this branch of social science it seems that mathematical developments have lagged even farther behind empirical work than one might expect. This is particularly surprising in view of the numbers of mathematically-inclined social scientists who have become interested in developing mathematical models of small group behavior. Workers at Harvard, MIT, Columbia, Michigan, and at many other centers have approached the study of small groups with the intention of developing mathematical models to describe their behavior, but there exist only a few such models in published form as a consequence of these intentions.

This state of affairs immediately raises the question: Why has there been no real proliferation of mathematical models? The answers to this question—for there appear to be several partial answers—lie in diverse areas. To see what these different answers are, it will be helpful to examine closely the characteristic goals of investigations in small group behavior.

The Goals of Research: Showing that a Relationship Exists. Probably the most common goal of investigation, both field and experimental, has been to demonstrate the importance of one factor in affecting one or more other factors. The Lippitt "leadership climates" study [1940], for example, demonstrated the importance of the kind of leadership—democratic, authoritarian, or laissez-faire—for group functioning (*i.e.*, for morale,