MATHEMATICS
first course

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The characters that appear on the cover represent the first Hindu-Arabic numerals introduced in Europe. Note the similarity of the characters to our present numbers.

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Preface

The mathematics experiences of junior high school students should be a very important part of their education. Junior high school mathematics serves as a bridge between the study of arithmetic in the elementary school and the study of algebra and geometry in the senior high school. Mathematics at this level also provides much of the foundation for the study of high school science. If an understanding of mathematics is obtained in junior high school, the student will be much better prepared for the great variety of applications of mathematics which he will want to make in his more advanced studies in many fields and in experiences throughout life.

Mathematics, First Course and *Mathematics, Second Course* are planned for the seventh and eighth grades for all students. There is sufficient material to challenge the most gifted, and careful attention is given to the needs of those who have not been so successful in earlier mathematical study. The emphasis in these texts on unifying principles, the use of the inductive method, and the bringing together of ideas of arithmetic, geometry, and algebra should enable the student to study mathematics with greater enjoyment and greater success. This will be due in considerable part to the opportunity for the pupil to develop further understanding of previously studied materials, and of the many new topics which are included.

The materials are written in the spirit of modern mathematics whenever feasible, with attention given to structure and to some of the newer vocabulary, such as sets and number statements. The recommendations of the Commission on Mathematics for the Junior High School have been followed. Numeration, bases other than ten, has been introduced early and is made use of where appropriate in exercises throughout the text. The choice of new topics is based on an experimental study carried out at a major university laboratory school by one of the authors even before much of the current widespread work on secondary school curriculum had been started.

Special features of the text are the Quick Quizzes, which appear throughout both courses, and the Appendix materials, which contain many exercises both for maintenance of skills and for the introduction to new topics which might not be included in a basic course for all students. There is less emphasis on the so-called social applications of mathematics than is traditional, although additional work is provided on these topics by means of appendices. Time taken from these traditional topics is given to the interpretation of principles of number systems and ideas of geometry which are fundamental to the study of mathematics at this level.

The authors wish to express appreciation particularly to the many students who have been in their junior high school classes and to the many teachers with whom they have had the privilege of exchanging ideas in local and state curriculum committees, teachers' meetings, and meetings of the National Council of Teachers of Mathematics. The authors wish to thank especially the following reviewers: Mrs. Lina Walters, Paterson State College, Paterson, New Jersey; Dr. James H. Zant, Professor of Mathematics, Oklahoma A & M College, Stillwater, Oklahoma; Dr. John J. Kinsella, School of Education, New York University, New York, New York; James L. Chedester, Principal, Pattonville High School, Pattonville, Missouri.

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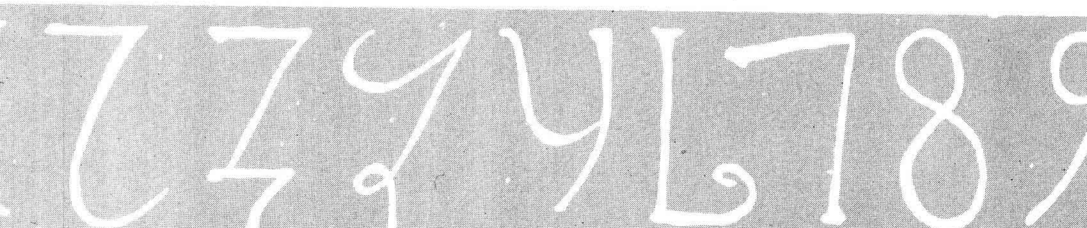

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Contents

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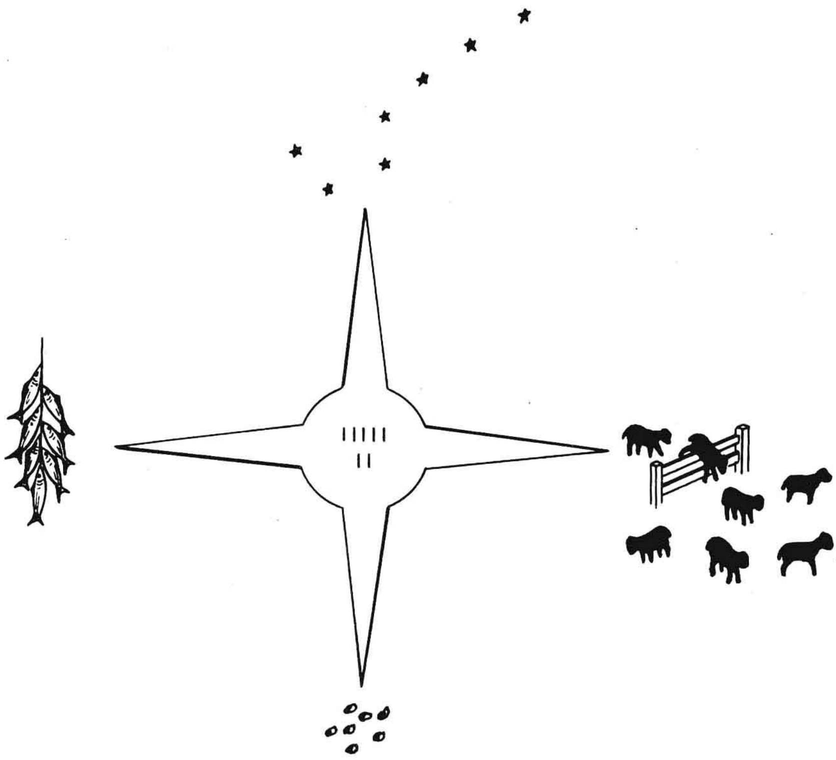
The Basis of Numbers

Arithmetic is a language of numbers. It is a system of ideas invented by people to help solve problems involving numbers. Our arithmetic of today developed slowly through history.

The daily life of the earliest man did not require a bookkeeping system based upon numbers. His possessions, since they were meager, were not extensive enough to demand a record of his wealth.

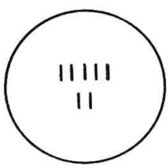
As man gained more property it became wise for him to keep records. For instance, he may have had "many" sheep and "many" tools among his wealth. Since "many" does not indicate an exact amount, a more definite system was needed. One way a shepherd kept an account of his sheep was to let one stone represent one sheep. The sheep were counted by using the principle of one-to-one correspondence, that is, there was one stone for each sheep and one sheep for each stone. The counting resulted in a collection of stones. The total collection indicated the number of sheep in the flock.

In communicating his wealth to other people, such as merchants or the tax collector, it was necessary for man to have a spoken word and a written expression to indicate the size of the pile of stones. The spoken word or the written expression may be referred to as a verbal expression. In our discussion, the pile of stones will be referred to as a "model group." The pile of stones is an imitation of the flock of sheep. The written name of the model group is a *numeral.*



Examples

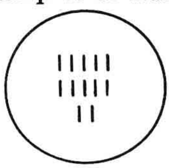
(a)



The marks represent a model for counting. One may name this group by the verbal expressions "seven" or "five plus two." A numeral for the model is "7," or "5 + 2."

(b) The feet of a bird may be thought of as a model. The verbal expressions could be "two," "a pair," or "one plus one." Examples of numerals for the model group are "2" or "1 + 1."

(c)



This model could be called a "dozen," "twelve," or "ten and two more." Numerals to name the model could be "12" or "10 + 2."

EXERCISES—Represent the following model by (a) a verbal expression and (b) a numeral.

Example xxxxx
xxx (a) Eight or five plus three.
(b) 8

1. xxxxx

2. xxxxxx
xxxx

3. xxxx
xxxx
xxxx

4. xxxxxx
xxxxxx
xxxxxx
xxxxxx

5. xxxxxxxxxx
xxxxxxxxxx

6. oooooo
oooooo
oooooo
oooooo
oooo

7. ooooooooooooo
ooooooooooooo

8. xxxxxxxxxx
xxxxx

The Egyptian Numerals

Two systems of writing numerals, besides our own, will be studied in this chapter. The study of these systems will reveal the role of symbols and the role of the principles involved in using these symbols to represent number.

The Egyptians had an advanced civilization many years before the birth of Christ. Among their accomplishments was a set of number symbols. The following table illustrates our verbal expression of the model and the Egyptian numeral for the model:

Table I

One	𓏤
Ten	𓎆
One hundred	𓍢
One thousand	𓆼
Ten thousand	𓂭
One hundred thousand	𓆐
One million	𓁨

Notice that the Egyptians did not use a symbol for a model group for each quantity that they wished to express. For example, they did not have a single numeral for twelve. Accepting the principles of (a) repetition and (b) addition, the seven numerals could be used to write numerals for any quantities.

The system followed a simple pattern. One stroke (|) was "one"; two strokes (||), using the principles of repetition and addition, meant "two," that is, one plus one; three strokes (|||) using the two principles, meant "three," and so on to nine (|||||||||). Ten was written as ∩ . This symbol replaced the ten strokes.

Examples

1. Represent the number in the group using Egyptian numerals.

xxxxxx
xxxxxx
x

Explanation: This follows from ∩ = 10 and | = 1 and by applying the principles of repetition and addition, ∩ + | + | + | would be written ∩||| . Notice that the plus sign was understood in the writing of the numeral. Also, the symbol with the largest value was written first.

2. Change 9∩∩∩||||||| from the Egyptian system to our own system.

9∩∩∩||||||| = 100 + 10 + 10 + 10 + 7 = 137

3. Write 236 in the Egyptian system.

236 = 99∩∩∩||||||

EXERCISES—In Exercises 1 through 5, represent the number of the items in the groups by using Egyptian numerals.

1. xxxxx

2. xxxx
xxxx

3. xxxxxx
xxxxxxxx

4. oooooo
oooooo
oooooo
ooooo

5. oooooooooo
oooooooooo
oooooooooo
ooooo

In Exercises 6 through 9, change the numerals from the Egyptian system to our own system.

6. ∩∩∩∩∩|||

7. 99∩||

8. 𓆼9||||

9. 999999∩∩∩∩∩∩∩|||||||||

In Exercises 10 through 14, write the numerals in the Egyptian system.

10. Fourteen.

11. Three hundred eight.

12. Nineteen fifty-seven.

13. 2351

14. 6004

15. What number relationships exist among the seven models used in the Egyptian numeration system?

The Roman Numerals

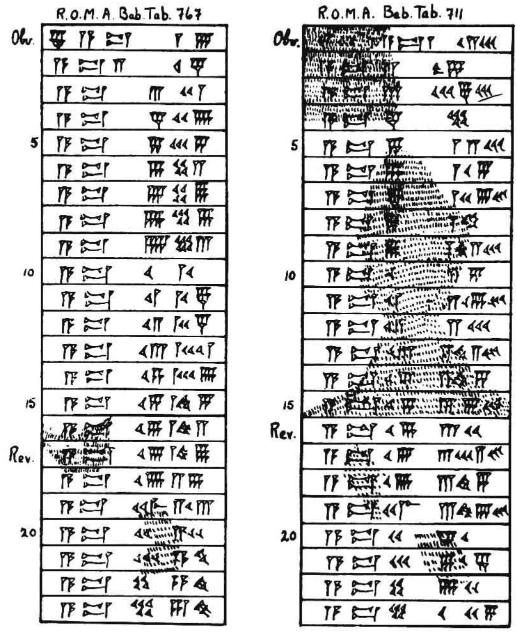
The Romans used seven symbols in their numeration system, most of which were different from the ones accepted by the Egyptians. The numerals used for the Roman model groups have changed during the years. The table below shows the modern adaptation of the symbols used by the Romans.

Table II

One	I
Five	V
Ten	X
Fifty	L
One hundred	C
Five hundred	D
One thousand	M

Like the Egyptian system, the model groups did not represent all quantities that one would want to count and record. In order to have numerals for these quantities, the Romans used the principles of (a) repetition, (b) addition, (c) subtraction, and (d) multiplication.

In the numeral III, repetition and addition are used to write $1 + 1 + 1$. In the numeral IV, subtraction is used to write four, that is, five minus one. When subtraction is used, the symbol representing the number to be subtracted is written before the symbol representing the larger numeral. A bar over a numeral multiplies the number represented by the numeral by one thousand. For instance, $\overline{\text{V}}$, would be five thousand and $\overline{\text{XLIII}}$ would be forty-three thousand.



Babylonian multiplication tables.

One notices that a numeration system requires numerals for the model groups together with a set of principles. Both the symbols and the principles are arbitrarily chosen. For other systems of numeration such as the Greek and Babylonian systems, the same is true. A set of numerals together with principles are arbitrarily chosen to make the system operational.

Examples

1. Represent the group using Roman numerals.

xxxxxx xxxxxx	XII	This number is represented with one ten, X, plus two ones, II. The principles of repetition and addition are used.
ooooo ooooo oooo	XIV	This number is represented with one ten, X, plus four ones. The subtraction principle is used to write four, that is, five minus one.

2. Change the Roman numerals to a numeral in our system.

$$\text{LXXIV} = 50 + 10 + 10 + 4 = 74$$

3. Write the following in Roman numerals.

Three hundred twenty-four = CCCXXIV

EXERCISES—In Exercises 1 through 4, represent the groups in the Roman number system.

1. ooooo
   ooo

2. oooooo
   oooooo
   oooooo

3. xxxxxxxxxx
   xxxxxxxxxx
   xxx

4. xxxxx xxxxx
   xxxxx xxxxx
   xxxxx xxxx

In Exercises 5 through 9, change the Roman numerals to numerals in our own system.

5. XXVIII
6. CX
7. CIV
8. DCCIX
9. MCDII

In Exercises 10 through 13, write the numbers in the Roman system.

10. Sixty-eight.
11. Ninety-nine.
12. Four hundred seventy.
13. 1957.
14. Explain how the principles of the Roman system are used to write 3109.
15. Explain how the principles of the Roman system are used to write 42,300.
16. What relationships exist among the seven numerals of the Roman system?

The Hindu-Arabic Numerals

The numeration system we use today is called the "Hindu-Arabic" system. Generally it is agreed that the Hindu people first used a set of nine symbols together with the principles of (a) addition and (b) place value. Arabian scholars traveling through India noticed this unusual system and introduced it into their homeland. After many years someone invented zero as the tenth symbol. The zero, used as a place holder, was a most important invention in making the whole system more useful.