Introducing Einstein's Relativity

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Ray d'Inverno
Faculty of Mathematical Studies, University of Southampton

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1.1 Notes for the student

There is little doubt that relativity theory captures the imagination. Nor is it surprising: the anti-intuitive properties of special relativity, the bizarre characteristics of black holes, the exciting prospect of gravitational wave detection and with it the advent of gravitational wave astronomy, and the sheer scope and nature of cosmology and its posing of ultimate questions; these and other issues combine to excite the minds of the inquisitive. Yet, if we are to look at these issues meaningfully, then we really require both physical insight and a sound mathematical foundation. The aim of this book is to help provide these.

The book grew out of some notes I wrote in the mid-1970s to accompany a UK course on general relativity. Originally, the course was a third-year undergraduate option aimed at mathematicians and physicists. It subsequently grew to include M.Sc. students and some first-year Ph.D. students. Consequently, the notes, and with it the book, are pitched principally at the undergraduate level, but they contain sufficient depth and coverage to interest many students at the first-year graduate level. To help fulfil this dual purpose, I have indicated the more advanced sections (level-two material) by a grey shaded bar alongside the appropriate section. Level-one material is essential to the understanding of the book, whereas level two is enrichment material included for the more advanced student. To help put a bit more light and shade into the book, the more important equations and results are given in tint panels.

In designing the course, I set myself two main objectives. First of all, I wanted the student to gain insight into, and confidence in handling, the basic equations of the theory. From the mathematical viewpoint, this requires good manipulative ability with tensors. Part B is devoted to developing the necessary expertise in tensors for the rest of the book. It is essentially written as a self-study unit. Students are urged to attempt all the exercises which accompany the various sections. Experience has shown that this is the only real way to be in a position to deal confidently with the ensuing material. From the physical viewpoint, I think the best route to understanding relativity theory is to follow the one taken by Einstein. Thus the second chapter of Part C is devoted to discussing the principles which guided Einstein in his search for a relativistic theory of gravitation. The field equations are approached first from a largely physical viewpoint using these principles and subsequently from a purely mathematical viewpoint using the variational principle approach. After a chapter devoted to investigating the quantity which goes on the 'right-hand side' of the equations, the structure of the equations is discussed as a prelude to solving them in the simplest case. This part of the course ends by considering the experimental status of general relativity. The course originally assumed that the student had some reasonable knowledge of special relativity. In fact, over the years, a growing number of students have taken the course without this background, and so, for completeness, I eventually added Part A. This is designed to provide an introduction to special relativity sufficient for the needs of the rest of the book.

The second main objective of the course was to develop it in such a way that it would be possible to reach three major topics of current interest, namely, black holes, gravitational waves, and cosmology. These topics form the subject matter of Parts D, E, and F respectively.

Each of the chapters is supported by exercises, numbering some 300 in total. The bulk of these are straightforward calculations used to fill in parts omitted in the text. The numbers in parentheses indicate the sections to which the exercises refer. Although the exercises in general are important in aiding understanding, their status is different from those in Part B. I see the exercises in Part B as being absolutely essential for understanding the rest of the book and they should not be omitted. The remaining exercises are desirable. The book is neither exhaustive nor complete, since there are topics in the theory which we do not cover or only meet briefly. However, it is hoped that it provides the student with a sound understanding of the basics of the theory.

A few words of advice if you find studying from a book hard going. Remember that understanding is not an all or nothing process. One understands things at deeper and deeper levels, as various connections are made or ideas are seen in different contexts or from a different perspective. So do not simply attempt to study a section by going through it line by line and expect it all to make sense at the first go. It is better to begin by reading through a few sections quickly — skimming — thereby trying to get a general feel for the scope, level, and coverage of the subject matter. A second reading should be more thorough, but should not stop if ideas are met which are not clear straightaway. In a final pass, the sections should be studied in depth with the exercises attempted at the end of each section. However, if you get stuck, do not stop there, press on. You will often find that the penny will drop later, sometimes on its own, or that subsequent work will produce the necessary understanding. Many exercises (and exam questions) are hierarchical in nature. They require you to establish a result at one stage which is then used at a subsequent stage. If you cannot establish the result, then do not give up. Try and use it in the subsequent section. You will often find that this will give you the necessary insight to allow you to go back and establish the earlier result. For most students, frequent study sessions of not too long a duration are more productive than occasional long drawn out sessions. The best study environment varies greatly from one individual to another. Try experimenting with different environments to find out what is the most effective for you.

As far as initial conditions are concerned, that is assumptions about your background, it is difficult to be precise, because you can probably get by with much less than the book might seem to indicate (see §1.5). Added to which, there is a big difference between understanding a topic fully and only having some vague acquaintance with it. On the mathematical side, you certainly need to know calculus, up to and including partial differentiation, and solution of simple ordinary differential equations. Basic algebra is assumed and some matrix theory, although you can probably take eigenvalues and diagonalisation on trust. Familiarity with vectors and some exposure to vector fields is assumed. It would also be good to have met the ideas of a vector space and bases. We use Taylor's theorem a lot, but probably knowledge of Maclaurin's will be sufficient. On the Physics side, you obviously need to know Newton's laws and Newtonian gravitation. It would be helpful also to know a little about the potential formulation of gravitation (though, again, just the basics will do). The book assumes familiarity with electromagnetism (Maxwell's equations, in particular) and fluid dynamics (the Navier–Stokes equation, in particular), but neither of these are absolutely essential. It would be very helpful to have met some ideas about waves (such as the fundamental relationship $c = \lambda\nu$) and the wave equation in particular. In cosmology, it is assumed that you know something about basic astronomy.

Having listed all these topics, then, if you are still unsure about your background, my approach would be to say: try the book and see how you get on, if it gets beyond you (and it is not a level two section) press on for a bit and, if things do not get any better, then cut out. Hopefully, you may still have learnt a lot, and you can always come back to it when your background is stronger. In fact, it should not require much background to get started, for part A on special relativity assumes very little. After that you hit part B, and this is where your motivation will be seriously tested. I hope you make it through because the pickings on the other side are very rich indeed. So, finally, good luck!

1.2 Acknowledgements

Very little of this book is wholly original. When I drew up the notes, I decided from the outset that I would collect together the best approaches to the material which were known to me. Thus, to take an example right from the beginning of the book, I believe that the k-calculus provides the best introduction to special relativity, because it offers insight from the outset through the simple diagrams that can be drawn. Indeed one of the themes of this book is the provision of a large number of illustrative diagrams (over 200 in fact). The visual sense is the most immediate we possess and helps lead directly to a better comprehension. A good subtitle for the book would be, **An approach to relativity theory via space-time pictures**. The k-calculus is an approach developed by H. Bondi from the earlier ideas of A. Milne. My use of it is not surprising since I spent my years as a research student at King's College, London, in the era of Hermann Bondi and Felix Pirani, and many colleagues will detect their influences throughout the book. So the fact is that many of the approaches in the book have been borrowed from one author or another; there is little that I have written completely afresh. My intention has been to organize the material in such a way that it is the more readily accessible to the majority of students.

General relativity has the reputation of being intellectually very demanding. There is the apocryphal story, I think attributed to Sir Arthur Eddington, who, when asked whether he believed it true that only three people in the world understood general relativity, replied, 'Who is the third?'

Indeed, the intellectual leap required by Einstein to move from the special theory to the general theory is, there can be little doubt, one of the greatest in the history of human thought. So it is not surprising that the theory has the reputation it does. However, general relativity has been with us for some three-quarters of a century and our understanding is such that we can now build it up in a series of simple logical steps. This brings the theory within the grasp of most undergraduates equipped with the right background.

Quite clearly, I owe a huge debt to all the authors who have provided the source material for and inspiration of this book, However, I cannot make the proper detailed acknowledgements to all these authors, because some of them are not known even to me, and I would otherwise run the risk of leaving somebody out. Most of the sources can be found in the bibliography given at the end of the book, and some specific references can be found in the section on further reading. I sincerely hope I have not offended anyone (authors or publishers) in adopting this approach. I have written this book in the spirit that any explanation that aids understanding should ultimately reside in the pool of human knowledge and thence in the public domain. None the less, I would like to thank all those who, wittingly or unwittingly, have made this book possible. In particular, I would like to thank my old Oxford tutor, Alan Tayler, since it was largely his backing that led finally to the book being produced. In the process of converting the notes to a book, I have made a number of changes, and have added sections, further exercises, and answers. Consequently this new material, unlike the earlier, has not been vetted by the student body and it seems more than likely that it may contain errors of one sort or another. If this is the case, I hope that it does not detract too much from the book and, of course, I would be delighted to receive corrections from readers. However, I have sought some help and, in this respect, I would particularly like to thank my colleague James Vickers for a critical reading of much of the book.

Having said I do not wish to cite my sources, I now wish to make one important exception. I think it would generally be accepted in the relativity community that the most authoritative text in existence in the field is **The large scale structure of space-time** by Stephen Hawking and George Ellis (published by Cambridge University Press). Indeed, this has taken on something akin to the status of the Bible in the field. However, it is written at a level which is perhaps too sophisticated for most undergraduates (in parts too sophisticated for most specialists!). When I compiled the notes, I had in mind the aspiration that they might provide a small stepping stone to Hawking and Ellis. In particular, I hoped it might become the next port of call for anyone wishing to pursue their interest further. To that end, and because I cannot improve on it, I have in places included extracts from that source virtually verbatim. I felt that, if students were to consult this text, then the familiarity of some of the material might instil confidence and encourage them to delve deeper. I am hugely indebted to the authors for allowing me to borrow from their superb book.

1.3 A brief survey of relativity theory

It might be useful, before embarking on the course proper, to attempt to give some impression of the areas which come under the umbrella of relativity theory. I have attempted this schematically in Fig. 1.1. This is a rather partial

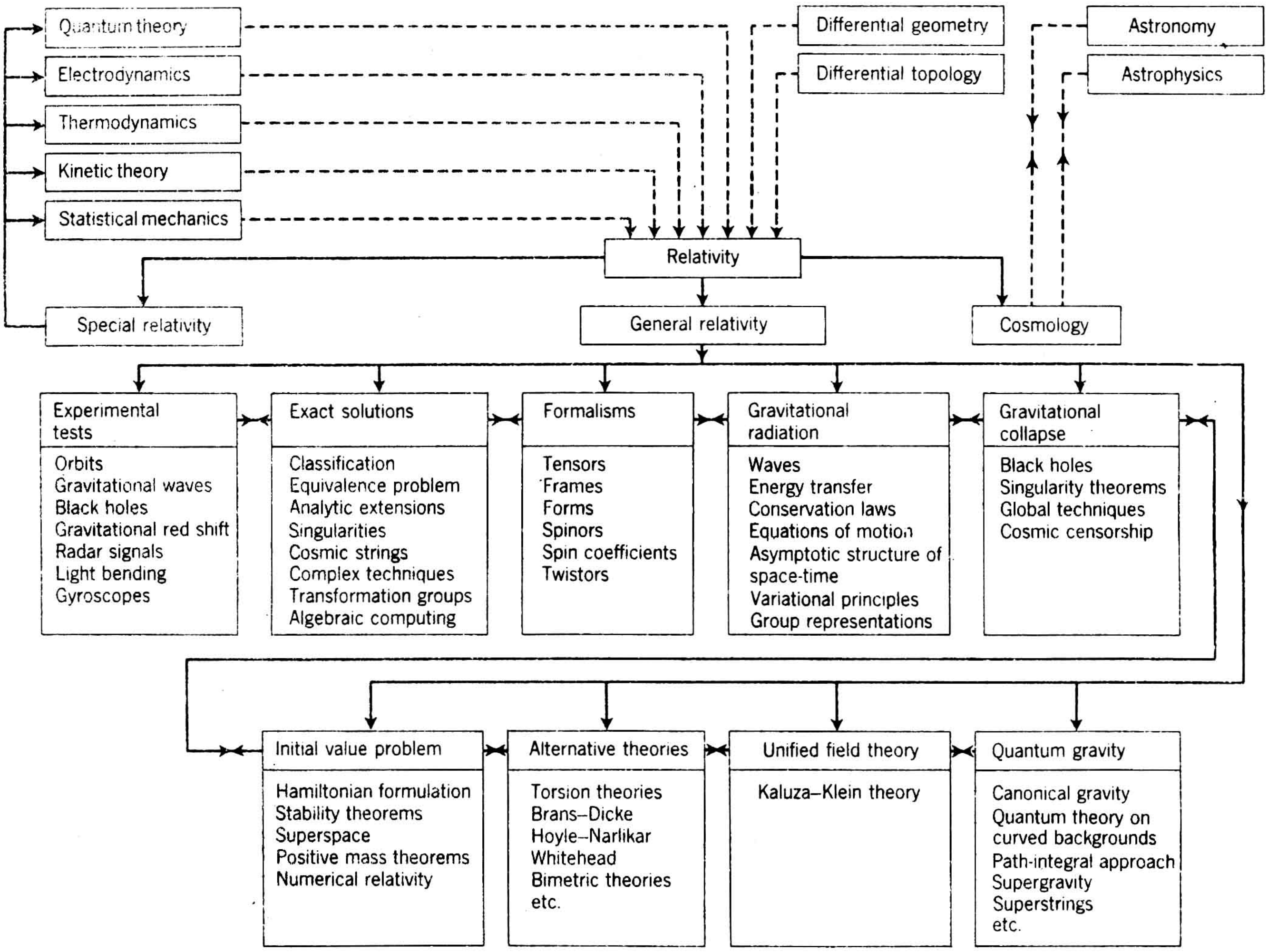

Fig. 1.1 An individual survey of relativity.

and incomplete view, but should help to convey some idea of our planned route. Most of the topics mentioned are being actively researched today. Of course, they are interrelated in a much more complex way than the diagram suggests.

Every few years since 1955 (in fact every three since 1959), the relativity community comes together in an international conference of general relativity and gravitation. The first such conference held in Berne in 1955 is now referred to as GR0, with the subsequent ones numbered accordingly. The list, to date, of the GR conferences is given in Table 1.1. At these conferences, there are specialist discussion groups which are held covering the whole area of interest. Prior to GR8, a list was published giving some detailed idea of what each discussion group would cover. This is presented below and may be used as an alternative to Fig. 1.1 to give an idea of the topics which comprise the subject.

Table 1.1

GR0	1955	Bern, Switzerland
GR1	1957	Chapel Hill, North Carolina, USA
GR2	1959	Royaumont, France
GR3	1962	Jablonna, Poland
GR4	1965	London, England
GR5	1968	Tblisi, USSR
GR6	1971	Copenhagen, Denmark
GR7	1974	Tel-Aviv, Israel
GR8	1977	Waterloo, Canada
GR9	1980	Jena, DDR
GR10	1983	Padua, Italy
GR11	1986	Stockholm, Sweden
GR12	1989	Boulder, Colorado, USA

I. Relativity and astrophysics

Relativistic stars and binaries; pulsars and quasars; gravitational waves and gravitational collapse; black holes; X-ray sources and accretion models.

II. Relativity and classical physics

Equations of motion; conservation laws; kinetic theory; asymptotic flatness and the positivity of energy; Hamiltonian theory, Lagrangians, and field theory; relativistic continuum mechanics, electrodynamics, and thermodynamics.

III. Mathematical relativity

Differential geometry and fibre bundles; the topology of manifolds; applications of complex manifolds; twistors; causal and conformal structures; partial differential equations and exact solutions; stability; geometric singularities and catastrophe theory; spin and torsion; Einstein–Cartan theory.

IV. Relativity and quantum physics

Quantum theory on curved backgrounds; quantum gravity; gravitation and elementary particles; black hole evaporation; quantum cosmology.

V. Cosmology

Galaxy formation; super-clustering; cosmological consequences of spontaneous symmetry breakdown; domain structures; current estimates of cosmological parameters; radio source counts; microwave background; the isotropy of the universe; singularities.

VI. Observational and experimental relativity

Theoretical frameworks and viable theories; tests of relativity; gravitational wave detection; solar oblateness.

VII. Computers in relativity

Numerical methods; solution of field equations; symbolic manipulation systems in general relativity.

1.4 Notes for the teacher

In my twenty years as a university lecturer, I have undergone two major conversions which have profoundly affected the way I teach. These have, in their way, contributed to the existence of this book. The first conversion was to the efficacy of the printed word. I began teaching, probably like most of my colleagues, by giving lectures using the medium of chalk and talk. I soon discovered that this led to something of a conflict in that the main thing that students want from a course (apart from success in the exam) is a good set of lecture notes, whereas what I really wanted was that they should understand the course. The process of trying to give students a good set of lecture notes meant that there was, to me, a lot of time wasted in the process of note taking. I am sure colleagues know the caricature of the conventional lecture: notes are copied from the lecturer's notebook to the student's notebook without their going through the heads of either — a definition which is perhaps too close for comfort. I was converted at an early stage to the desirability of providing students with printed notes. The main advantage is that it frees up the lecture period from the time-consuming process of note copying, and the time released can be used more effectively for developing and explaining the course at a rate which the students are able to cope with. I still find that there is something rather final and definitive about the printed word. This has the effect on me of making me think more carefully about what goes into a course and how best to organize it. Thus, printed notes have the added advantage of making me put more into the preparation of a course than I would have done otherwise. It must be admitted that there are some disadvantages with using printed notes, but this is not the place to elaborate on them.

My second conversion was to the efficacy of self-study. This is a rather elaborate title for the concept of students studying and learning on their own from books or prepared materials. It is still a surprise to me just how little of this actually goes on in certain disciplines. And yet you would think that one of the main objectives of a university education is to teach students how to use books. My experience is that, in mathematics particularly, students find this hard to do. This is not so surprising since it requires high-level skills which many do not come to university equipped with. So one needs a mechanism which encourages students to use books. My first experience was in designing a Keller-type (self-paced) self-study course, where the students study from specially prepared units and are required to pass periodic tests before they move on to new topics. This eventually led me in other courses to use a coursework component counting towards a final assessment as a mechanism for helping to get students to study on their own. I have been involved in a good deal of research into this approach and the most frequent remark students make about coursework is that 'it gets me to work'. The coursework approach was particularly important in the design of the general relativity course for reasons which I believe are worth exploring.

In the mid-1970s, there were very few undergraduate courses in general relativity in existence in the UK. Those that there were usually only got as far as the Schwarzschild solution and then stopped. This was because the bulk of the course was devoted to developing the necessary expertise in tensors and there did not seem to be any short cut. This meant, from the viewpoint of both the student and the teacher, that the course ended just as things were beginning to get really interesting. It was clear to me that what students really wanted to know about most were the topics of black holes, gravitational waves, and cosmology. So, from the outset, the object was to design a course which made this possible. It was achieved by separating out what is Part B of this book as a self-study unit on tensors. The notes were distributed at the beginning of the course and the students were instructed to begin immediately working through the self-study part and attempting all the exercises. The fact that most students put in the bulk of their efforts in their other courses towards the end of these courses helped in this respect, since they were less heavily loaded at the outset. The students were offered some optional tutorials in case they got stuck (as some undertaking individual study for the first time invariably did). The inducement for doing the exercises was that they counted towards the final assessment (by some 35 per cent currently). The deadline for completing the exercises was set for about a third of the way through the course. While the students were busy in their own time working on the tensors, the lecture course began by revising the key ideas in