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Jacques Stern (Ed.)

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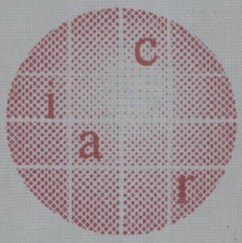
International Conference on the Theory and Application of Cryptographic Techniques
Prague, Czech Republic, May 1999
Proceedings



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Jacques Stern
Ecole Normale Supérieure
45, rue d'Ulm, F-75230 Paris 05, France
E-mail: Jacques.Stern@ens.fr

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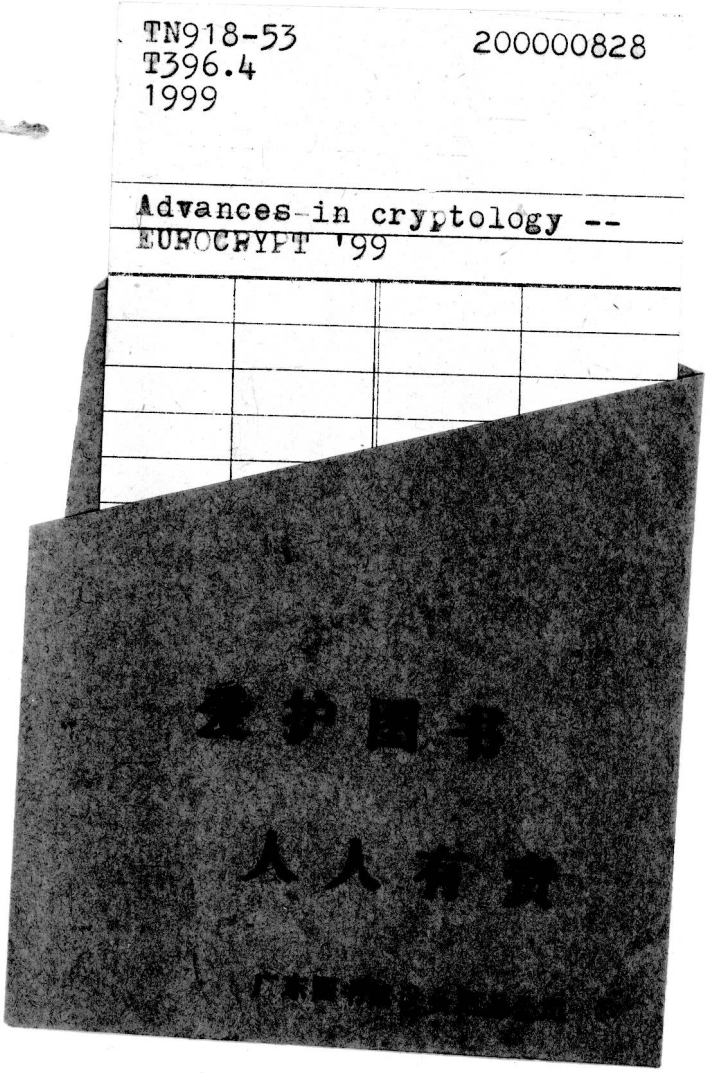

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Preface

EUROCRYPT '99, the seventeenth annual Eurocrypt Conference, was sponsored by the International Association for Cryptologic Research (IACR), in cooperation with the Group of Cryptology within the Union of Czech Mathematicians and Physicists. The General Chair, Jaroslav Hruby, was responsible for the overall organization of the conference in the beautiful city of Prague. Let me mention that it was a pleasure to work together: although we were in different locations, we managed to stay in close contact and maintain a smooth organization of the conference.

The Program Committee, consisting of 21 members, considered 120 papers and selected 32 for presentation. In addition, Ross Anderson kindly agreed to chair the traditional rump session for informal short presentations of new results. These proceedings include the revised versions of the 32 papers accepted by the Program Committee. These papers were selected on the basis of originality, quality, and relevance to cryptography. As a result, they should give a proper picture of how the field is evolving. Revisions were not checked and the authors bear full responsibility for the contents of their papers.

The selection of papers was a difficult and challenging task. Each submission was refereed by at least three reviewers and most had four reports or more. I wish to thank the program committee members, who did an excellent job. In addition, I gratefully acknowledge the help of a large number of colleagues who reviewed submissions in their areas of expertise. They are: Michel Abdalla, Josh Benaloh, Charles Bennett, Simon Blackburn, Matt Blaze, Christian Cachin, Jan Camenisch, Ran Canetti, Benny Chor, Galdi Clemente, Jean-Sébastien Coron, Paolo D'Arco, Anand Desai, Uri Feige, Marc Fischlin, Roger Fischlin, Matt Franklin, Steven Galbraith, Rosario Gennaro, Pierre Girard, Dieter Gollmann, Shai Halevi, Helena Handschuh, Yuval Ishai, Markus Jakobsson, Mike Just, Ted Krovetz, Kaoru Kurosawa, Eyal Kushilevitz, Keith Martin, Barbara Masucci, Johannes Merkle, Daniele Micciancio, Victor S. Miller, Fauzan Mirza, Serge Mister, Peter L. Montgomery, Tal Mor, David M'Raïhi, Luke O'Connor, Andrew Odlyzko, Wakaha Ogata, Koji Okada, Pascal Paillier, Pino Persiano, David Pointcheval, Bart Preneel, Tal Rabin, Omer Reingold, Phil Rogaway, Ludovic Rousseau, Berry Schoenmakers, Peter Shor, Jean-Pierre Seifert, Othmar Staffelbach, Ugo Vaccaro, Serge Vaudenay, Ruizhong Wei, Mike Wiener, Rebecca Wright, Xian-Mo Zhang, and Robert Zuccherato. I apologize for any inadvertent omission.

I also wish to thank my PhD students Phong Nguyen, Thomas Pornin, and Guillaume Poupard, who helped me a great deal at various steps of the whole process. Their computer skills and the time and effort they invested were a crucial ingredient of my ability to run the program committee. Thomas ran the electronic submission phase and was able to print all postscript files, including those produced by non-standard word processors. Guillaume opened a private

FTP server and Web site for PC members, and Phong did the editing work, both in paper and in electronic form. I hope I did not distract them too much from their research, but they were kind enough to tell me they had learnt a lot. Thanks also to Joelle Isnard and Nadine Riou, who organized the PC meeting in Paris.

Following the example of CRYPTO '98, EUROCRYPT '99 was the first of the Eurocrypt series with electronic submissions. The electronic submission option was a clear choice for almost all authors, with only 5% of the papers submitted by regular mail. I believe that the time has come to make e-submission mandatory, but it will be the choice of future Crypto and Eurocrypt PC chairs. I wish to thank Joe Kilian, who forwarded us the electronic submission software used for CRYPTO '98 and helped us run it. This software was originally developed by ACM's SIGACT group and I thank the ACM for allowing us to use their system.

Finally, I wish to thank the all authors who submitted papers for making this conference possible by creating the scientific material, and especially the authors of accepted papers. I would also like to thank the publisher, Springer-Verlag, for working within a tight schedule in order to produce these proceedings in due time.

February 1999

Jacques Stern
Program Chair
EUROCRYPT '99

EUROCRYPT '99

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Cryptanalysis of RSA with Private Key d Less than $N^{0.292}$

Dan Boneh* and Glenn Durfee**

Computer Science Department, Stanford University, Stanford, CA 94305-9045
{dabo,gdurf}@cs.stanford.edu

Abstract. We show that if the private exponent d used in the RSA public-key cryptosystem is less than $N^{0.292}$ then the system is insecure. This is the first improvement over an old result of Wiener showing that when $d < N^{0.25}$ the RSA system is insecure. We hope our approach can be used to eventually improve the bound to $d < N^{0.5}$.

1 Introduction

To provide fast RSA signature generation one is tempted to use a small private exponent d. Unfortunately, Wiener [10] showed over ten years ago that if one uses $d < N^{0.25}$ then the RSA system can be broken. Since then there have been no improvements to this bound. Verheul and Tilborg [9] showed that as long as $d < N^{0.5}$ it is possible to expose d in less time than an exhaustive search; however, their algorithm requires exponential time as soon as $d > N^{0.25}$.

In this paper we give the first substantial improvement to Wiener's result. We show that as long as $d < N^{0.292}$ one can efficiently break the system. We hope our approach will eventually lead to what we believe is the correct bound, namely $d < N^{0.5}$. Our results are based on the seminal work of Coppersmith [2].

Wiener describes a number of clever techniques for avoiding his attack while still providing fast RSA signature generation. One such suggestion is to use a large value of e. Indeed, Wiener's attack provides no information as soon as $e > N^{1.5}$. In contrast, our approach is effective as long as $e < N^{1.875}$. Consequently, larger values of e must be used to defeat the attack. We discuss this variant in Section 5.

2 Overview of Our Approach

Recall that an RSA public key is a pair $\langle N, e\rangle$ where $N = pq$ is the product of two n-bit primes. For simplicity, we assume $\gcd(p-1, q-1) = 2$. The corresponding private key is a pair $\langle N, d\rangle$ where $e \cdot d \equiv 1 \bmod \frac{\phi(N)}{2}$ where $\phi(N) = N - p - q + 1$.

* Supported by DARPA.
** Supported by Certicom and an NSF Graduate Research Fellowship.

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Note that both e and d are less than $\phi(N)$. It follows that there exists an integer k such that

$$ed + k\left(\frac{N+1}{2} - \frac{p+q}{2}\right) = 1. \tag{1}$$

Writing $s = -\frac{p+q}{2}$ and $A = \frac{N+1}{2}$, we know:

$$k(A+s) \equiv 1 \pmod{e}.$$

Throughout the paper we write $e = N^\alpha$ for some α. Typically, e is of the same order of magnitude as N (e.g. $e > N/10$) and therefore α is very close to 1. As we shall see, when α is much smaller than 1 our results become even stronger.

Suppose the private exponent d satisfies $d < N^\delta$. Wiener's results show that when $\delta < 0.25$ the value of d can be efficiently found given e and N. Our goal is to show that the same holds for larger values of δ. By equation (1) we know that

$$|k| < \frac{2de}{\phi(N)} \leq 3de/N < 3e^{1+\frac{\delta-1}{\alpha}}.$$

Similarly, we know that

$$|s| < 2N^{0.5} = 2e^{1/2\alpha}.$$

To summarize, taking $\alpha \approx 1$ (which is the common case) and ignoring constants, we end up with the following problem: find integers k and s satisfying

$$k(A+s) \equiv 1 \pmod{e} \quad \text{where} \quad |s| < e^{0.5} \text{ and } |k| < e^\delta. \tag{2}$$

The problem can be viewed as follows: given an integer A, find an element "close" to A whose inverse modulo e is "small". We refer to this is the *small inverse problem*. Clearly, if for a given value of $\delta < 0.5$ one can efficiently list all the solutions to the small inverse problem, then RSA with private exponent smaller than N^δ is insecure (simply observe that given s modulo e one can factor N immediately, since $e > s$). Currently we can solve the small inverse problem whenever $\delta < 1 - \frac{1}{2}\sqrt{2} \approx 0.292$.

Remark 1. A simple heuristic argument shows that for any $\epsilon > 0$, if k is bounded by $e^{0.5-\epsilon}$ (i.e. $\delta < 0.5$) then the small inverse problem (equation (2)) is very likely to have a unique solution. The unique solution enables one to break RSA. Therefore, the problem encodes enough information to prove that RSA with $d < N^{0.5}$ is insecure. For $d > N^{0.5}$ we have that $k > N^{0.5}$ and the problem will no longer have a unique solution. Therefore, we believe this approach can be used to show that $d < N^{0.5}$ is insecure, but gives no results for $d > N^{0.5}$.

The next section gives a brief introduction to lattices over $\mathbb{Z}^n$. Our solution to the small inverse problem when α is close to 1 is given in Section 4. In Section 5 we give a solution for arbitrary α. Section 6 describes experimental results with the algorithm.