

SCIENTIFIC REASONING

The Bayesian Approach



Colin Howson and Peter Urbach

SCIENTIFIC REASONING: THE BAYESIAN APPROACH

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AND

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. . . if this [probability] calculus be condemned, then the whole of
the sciences must also be condemned.

—Henri Poincaré

Our assent ought to be regulated by the
grounds of probability.

—John Locke



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First printing 1989.

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Printed and bound in the United States of America.

Library of Congress Cataloging-in-Publication Data

Howson, Colin.

Scientific reasoning : the Bayesian approach / Colin Howson and Peter Urbach.

p. cm.

Bibliography: p.

Includes index.

ISBN 0-8126-9084-2 : \$34.95. ISBN 0-8126-9085-0 (pbk.) : \$16.95

1. Science—Philosophy. 2. Reasoning. 3. Bayesian statistical decision theory. I. Urbach, Peter. II. Title.

Q175.H87 1988

501—dc19

88-25440

CIP

SCIENTIFIC REASONING

ACKNOWLEDGEMENTS

The authors are very grateful to a number of friends and colleagues who read this book in draft and whose many suggestions led to its substantial improvement. They are John Howard, Martin Knott, Dennis Lindley, and Peter Milne. We are also grateful to Larry Phillips for helpful discussions. Although all of these people would agree with some of what we have written, probably none would agree with it all. Responsibility for the views expressed herein therefore rests entirely with us.

We also express our thanks to Youssef Aliabadi, Helen Brown, Sue Burrett, Alasdair Cameron, Kurt Klappholz, Ginny Watkins, and Gay Woolven for friendly advice, research assistance, and help in preparing the manuscript, to the Suntory-Toyota International Centre for Economics and Related Disciplines for financial assistance, and to the staff of the Open Court Publishing Company for their painstaking editorial work.

Finally, we thank each other. Although we are separately responsible for particular chapters (CH: 2, 3, 9, 11; PU: 1, 4, 5, 6, 7, 8, 10), we have each benefited from regular discussions and the reading and rereading of each other's contributions and this, we believe, has produced a unified exposition of the central Bayesian ideas.

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■ PART I

Bayesian Principles

According to the Bayesian view, scientific and indeed much of everyday reasoning is conducted in probabilistic terms. In other words, when evaluating an uncertain claim, one does so by calculating the probability of the claim in the light of given information. Precisely how this is done and why it is reasonable is the topic of this book.

In Part I of the book we shall first introduce the central Bayesian idea, giving some of its intellectual and historical background. This will be Chapter 1. Then in Chapter 2 we shall present the calculus of probability, which constitutes the foundation of the Bayesian approach. This will be done in a relatively formal manner and the question of what it means to say that some hypothesis h has probability $P(h)$ will be considered in Chapter 3. The rest of the book will show how the Bayesian approach gives a penetrating insight into the nature of scientific reasoning far superior to that afforded by any of its rivals.

■ CHAPTER 1

Introduction

■ a THE PROBLEM OF INDUCTION

Hypotheses usually have a general character relative to the empirical observations they are thought to explain. For instance, Mendel's genetic theory apparently concerns all inherited characteristics in all plants and animals, whereas relatively few of these could ever have been observed. If all our information derives from empirical observation, how can we be sure that any particular explanatory theory is the correct one? This is one version of the traditional problem of induction.

It has, however, sometimes been denied that our stock of information is restricted to empirical observations, a number of philosophers having taken the view that we are also capable of cognizing important synthetic principles which enable the gap between observations and scientific theories to be bridged. Immanuel Kant (1783, p. 9), for example, who claimed that his "dogmatic slumber" had been interrupted by the problem of induction, to which he had been alerted by David Hume's brilliant exposition of it, attempted to provide a principle which was both a priori certain and sufficiently rich to guarantee the truth of the theories of physics. His effort was, however, inadequate. The principle he advocated was just that every event has a cause. Much of Kant's endeavour went into showing that this was an a priori truth, and many of his interpreters have worked hard trying to unravel just what his argument was. But whether valid or not, the principle is irrelevant to the issue at hand, which does not concern whether every event has a cause but asks the very different question: how can one be certain, in any particular case, that one has selected the correct cause of an event out of the huge, indeed infinite, number of possible causes?

Another candidate for a bridging principle between empirical observations and scientific theories is the so-called Prin-

ciple of the Uniformity of Nature, which Hume (1777, section 32) summed up in the phrase “the future will resemble the past”. It is sometimes held that when scientists advocate their theories, they are relying on this principle, at least tacitly.

However, there are two obvious reasons why the theories of science could not be established as definitely true by means of such a principle. First, as it stands, it is empty, for it fails to disclose in what respects the future is supposed to resemble the past. To perform its intended role, the principle would need to be given a specific formulation for application to each case. For example, one such formulation would need to say that, in regard to heated metals, if these have always been observed to expand in the past, then they will do so in the future. It would need a more elaborate formulation to permit the inference that *all* metals would expand if heated, as is usually assumed. But, secondly, as soon as the Uniformity of Nature Principle has been made sufficiently specific for it to connect given observations to particular general laws, its inadequacy as a basis for scientific inference becomes manifest, because its own claim to be accepted as true is now just as questionable as the scientific theory which it was designed to guarantee.

■ b POPPER'S ATTEMPT TO SOLVE THE PROBLEM OF INDUCTION

It would appear then—this is not any longer controversial—that there is no *positive* solution to the problem of induction, that is to say, no solution by whose means particular explanatory theories could be conclusively shown to be true. However, many philosophers and scientists resist the idea, embraced in recent years with particular vigour by Paul Feyerabend, that all theories are on a par and that, for example, standard scientific claims are no better and no worse than those which would commonly be dismissed as the crackpot ideas of a charlatan. Karl Popper, in particular, was concerned to resist such scepticism and put science on a rational footing. He conceded that since scientific theories are never conclusively verifiable, no positive solution exists to the problem of induction. But Popper maintained that theories may, nevertheless, have some worthwhile epistemic status and in some cases be established as epistemically superior to their rivals, this superiority supposedly being an objective feature, independent of anyone's