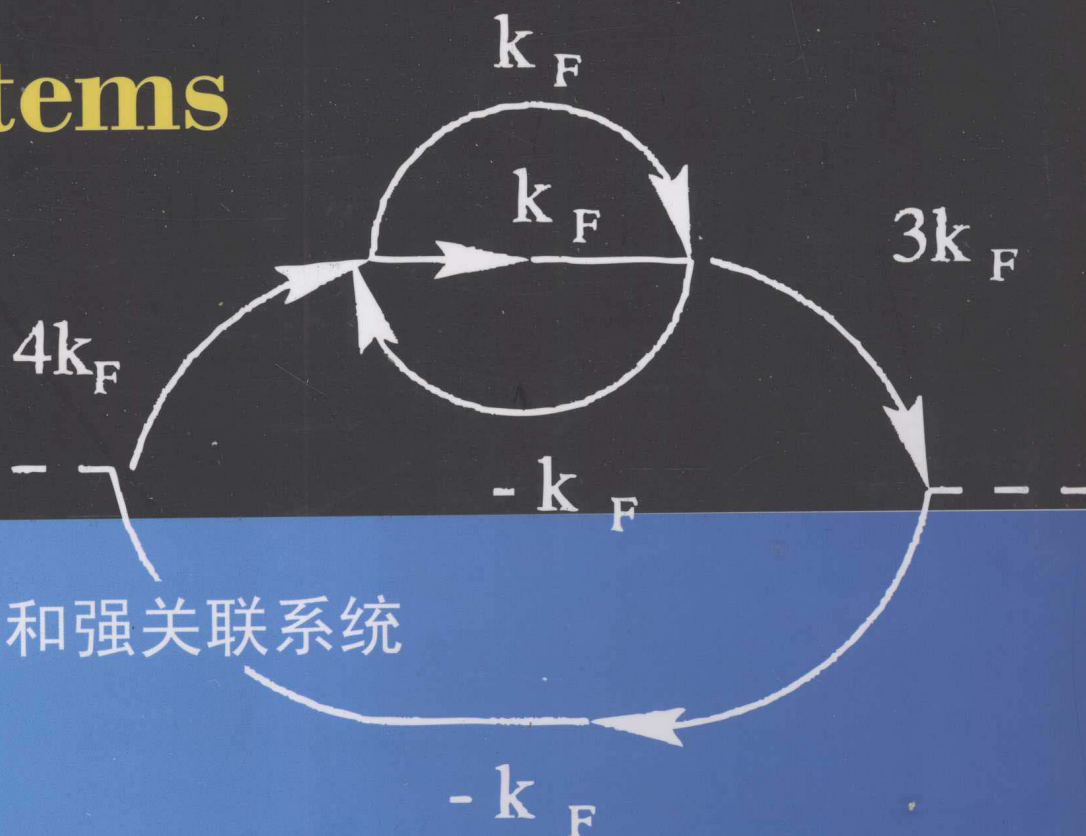# A. O. Gogolin, A. A. Nersesyan, A. M. Tsvelik

Bosonization and Strongly Correlated Systems

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Bosonization and Strongly Correlated Systems

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This volume provides a detailed account of bosonization. This important technique represents one of the most powerful nonperturbative approaches to many-body systems currently available.

The first part of the book examines the technical aspects of bosonization. Topics include one-dimensional fermions, the Gaussian model, the structure of Hilbert space in conformal theories, Bose–Einstein condensation in two dimensions, non-Abelian bosonization, and the Ising and WZNW models. The second part presents applications of the bosonization technique to realistic models including the Tomonaga–Luttinger liquid, spin liquids in one dimension and the spin-1/2 Heisenberg chain with alternative exchange. The third part addresses the problems of quantum impurities. Chapters cover potential scattering, the X-ray edge problem, impurities in Tomonaga–Luttinger liquids and the multi-channel Kondo problem. This book will be an excellent reference for researchers and graduate students working in theoretical physics, condensed matter physics and field theory.

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Preface

We used to think that if we know one, we knew two, because one and one are two. We are finding that we must learn a great deal more about 'and'.
  Sir Arthur Eddington, from *The Harvest of a Quiet Eye*, by A. Mackay

The behaviour of large and complex aggregations of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviours requires research which I think is as fundamental in its nature as any other.
                    P. W. Anderson, from *More is Different* (1972)

High energy physics continues to fascinate people inside and outside of science, being perceived as the 'most fundamental' area of research. It is believed somehow that the deeper inside the matter we go the closer we get to the truth. So it is believed that 'the truth is out there' – at high energies, small distances, short times. Therefore the ultimate theory, Theory of Everything, must be a theory operating at the smallest distances and times possible where there is no difference between gravitational and all other forces (the Planck scale). All this looks extremely revolutionary and complicated, but once a condensed matter physicist has found time and courage to acquaint himself with these ideas and theories, these would not appear to him utterly unfamiliar. Moreover, despite the fact that the two branches of physics study objects of vastly different sizes, the deeper into details you go, the more parallels you will find between the concepts used. In many cases the only difference is that models are called by different names, but this has more to do with funding than with the essence. Sometimes differences are more serious, but similarities still remain, for example, the Anderson–Higgs phenomenon in particle theory is very similar to the Meissner effect in superconductivity; the concept of 'inflation' in cosmology is taken from the physics of first order phase

transitions; the hypothetical 'cosmic strings' are similar to magnetic field vortex lines in type II superconductors; the Ginzburg–Landau theory of superfluid He^3 has many features in common with the theory of hadron–meson interaction etc. When you realize the existence of this astonishing parallelism, it is very difficult not to think that there is something very deep about it, that here you come across a general principle of Nature according to which same ideas are realized on different space-time scales, on different hierarchical 'layers', as a Platonist would put it. This view puts things in a new perspective where truth is no longer 'out there', but may be seen equally well in a 'grain of sand' as in an elementary particle.

In this book we are going to deal with the area of theoretical physics where the parallels between high energy and condensed matter physics are especially strong. This area is the theory of strongly correlated low-dimensional systems. Below we will briefly go through these parallelisms and discuss the history of this discipline, its main concepts, ideas and also the features which excite interest in different communities of physicists.

The problems of strongly correlated systems are among the most difficult problems of physics we are now aware of. By definition, strongly correlated systems are those ones which cannot be described as a sum of weakly interacting parts. So here we encounter a situation when the whole is greater than its parts, which is always difficult to analyse. The well-known example of such a problem in particle physics is the problem of strong interactions – that is a problem of formation and structure of heavy particles – hadrons (with proton and neutron being the examples) and mesons. In popular literature, which greatly influences minds outside physics, one may often read that particles constituting atomic nuclei consist in their turn of 'smaller', or 'more elementary', particles called quarks, coupled together with gluon fields. However, invoking images and using language quite inadequate for the essence of the phenomenon in question this description more confuses than explains. The confusion begins with the word 'consist' which here does not have the same meaning as when we say that a hydrogen atom consists of a proton and an electron. This is because a hydrogen atom is formed by electromagnetic forces and the binding energy of the electron and proton is small compared to their masses: $E \sim -\alpha^2 m_e c^2$, where $\alpha = e^2/hc \approx 1/137$ is the fine structure constant and m_e is the electron mass. The smallness of the dimensionless coupling constant α obscures the quantum character of electromagnetic forces, yielding a very small cross section for processes of transformation of photons into electron–positron pairs. Thus α serves as a small parameter in a perturbation scheme where in the first approximation the hydrogen atom is represented as a system of just two particles. Without

small α quantum mechanics would be a purely academic discipline.* One cannot describe a hadron as a quantum mechanical bound state of quarks, however, because the corresponding fine structure constant of the strong interactions is not small: $\alpha_G \sim 1$. Therefore gluon forces are of essentially quantum nature, in the sense that virtual gluons constantly emerge from vacuum and disappear, so that the problem involves an infinite number of particles and therefore is absolutely non-quantum-mechanical. It turns out, however, that the proton and neutron have the same quantum numbers as a quantum mechanical bound state of three particles of a certain kind. Only in this sense can one say that a 'proton consists of three quarks'. The reader would probably agree that this is a very nontraditional use of this word. So it is not actually a statement about the material content of a proton (as a wave on a surface of the sea, it does not have any *permanent* material content), but about its symmetry properties, that is to what representation of the corresponding symmetry group it belongs.

It turns out that reduction of dimensionality may be of a great help in solving models of strongly correlated systems. Most nonperturbative solutions presently known (and only nonperturbative ones are needed in physics of strongly correlated systems) are related to (1 + 1)-dimensional quantum or two-dimensional classical models. There are two ways to relate such solutions to reality. One way is that you imagine that reality on some level is also two dimensional. If you believe in this you are a string theorist. Another way is to study systems where the dimensionality is artificially reduced. Such systems are known in condensed matter physics; these are mostly materials consisting of well separated chains, but there are other examples of effectively one-dimensional problems such as problems of solitary magnetic impurities (Kondo effect) or of edge states in the quantum Hall effect. So if you are a theorist who is interested in seeing your predictions fulfilled during your life time, condensed matter physics gives you a chance.

At present, there are two approaches to strongly correlated systems. One approach, which will be only very briefly discussed in this book, operates with exact solutions of many-body theories. Needless to say not every model can be solved exactly, but fortunately many interesting ones can. So this method can provide a treasury of valuable information.

The other approach is to try to reformulate complicated interacting models in such a way that they become weakly interacting. This is the idea of bosonization which was pioneered by Jordan and Wigner in 1928 when they established equivalence between the spin $S = 1/2$ anisotropic Heisenberg chain and the model of interacting fermions (we shall discuss

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* With only bodiless spirits to discuss it, for sure, because there would not be stable complex atoms to form bodies.

this solution in detail in the text). Thus in just two years after introduction of the exclusion principle by Pauli it was established that in many-body systems the wall separating bosons from fermions might become penetrable. The example of the spin-1/2 Heisenberg chain has also made it clear that a way to describe a many-body system is not unique, but is a matter of convenience.

If the anisotropy is such that the coupling between the z-components of spins vanishes, the fermionic model becomes noninteracting. Thus, at least at this point, the excitation spectrum (and hence thermodynamics) can be easely described. However, since spins are expressed in terms of the fermionic operators in a nonlinear and nonlocal fashion, the problem of correlation functions remains nontrivial to the extent that it took another 50 years to solve it.

The transformation from spins to fermions completes the solution only for the special value of anisotropy; at all other values fermions interact. Interacting fermions in $(1 + 1)$-dimensions behave very differently from noninteracting ones. It turns out however, that in many cases interactions can be effectively removed by the second transformation – in the given case from the fermions to a scalar massless bosonic field. This transformation is called bosonization and holds in the continuous limit, that is for energies much smaller than the bandwidth. So at such energies the spin $S = 1/2$ Heisenberg chain can be reduced to a bunch of oscillators.

The spin $S = 1/2$ Heisenberg chain has provided the first example of 'particles transmutation'. We use these words to describe a situation when low-energy excitations of a many-body system differ drastically from the constituent particles. Of course, there are elementary cases when constituent particles are not observable at low energies, for example, in crystalline bodies atoms do not propagate and at low energies we observe propagating sound waves – phonons; in the same way in magnetically ordered materials instead of individual spins we see magnons etc. These examples, however, are related to the situation where the symmetry is spontaneously broken, and the spectrum of the constituent particles is separated from the ground state by a gap. Despite the fact that continuous symmetry cannot be broken spontaneously in $(1 + 1)$-dimensions and therefore there is no finite order parameter even at $T = 0$, spectral gaps may form. This nontrivial fact, known as dynamical mass generation, was discovered by Vaks and Larkin in 1961.

However, one does not need spectral gaps to remove single electron excitations since they can be suppressed by overdamping occurring when $T = 0$ is a critical point. In this case propagation of a single particle causes a massive emission of soft critical fluctuations. Both scenarios will be discussed in detail in the text.

The fact that soft critical fluctuations may play an important role in

Fig. 1.   A. I. Larkin and I. E. Dzyaloshinskii.

(1 + 1)-dimensions became clear as soon as theorists started to work with such systems. It also became clear that the conventional methods would not work. Bychkov, Gor'kov and Dzyaloshinskii (1966) were the first who pointed out that instabilities of one-dimensional metals cannot be treated in a mean-field-like approximation. They applied to such metals an improved perturbation series summation scheme called 'parquet' approximation (see also Dzyaloshinskii and Larkin (1972)). Originally this method was developed for meson scattering by Diatlov, Sudakov and Ter-Martirosian (1957) and Sudakov (1957).

It was found that such instabilities are governed by quantum interference of two competing channels of interaction – the Cooper and the Peierls ones. Summing up all leading logarithmic singularities in both channels (the *parquet* approximation) Dzyaloshinskii and Larkin obtained differential equations for the coupling constants which later have been identified as Renormalization Group equations (Solyom (1979)). From the flow of the coupling constants one can single out the leading instabilities of the system and thus conclude about the symmetry of the ground state. It turned out that even in the absence of a spectral gap a coherent propagation of single electrons is blocked. The charge–spin separation – one of the most striking features of a one-dimensional liquid of interacting electrons – had already been captured by this approach.

The problem the diagrammatic perturbation theory could not tackle

Fig. 2. A. A. Abrikosov.

was that of the strong coupling limit. Since phase transition is not an option in $(1 + 1)$-dimensions, it was unclear what happens when the renormalized interaction becomes strong (the same problem arises for the models of quantum impurities as the Kondo problem where similar singularities had also been discovered by Abrikosov (1965)). The failure of the conventional perturbation theory was sealed by P. W. Anderson (1967) who demonstrated that it originates from what he called 'orthogonality catastrophy': the fact that the ground state wave function of an electron gas perturbed by a local potential becomes orthogonal to the unperturbed ground state when the number of particles goes to infinity.[†] That was an indication that the problems in question cannot be solved by a partial summation of perturbation series. This does not prevent one from trying to sum the entire series which was brilliantly achieved by Dzyaloshinskii and Larkin (1974) for the Tomonaga–Luttinger model using the Ward identities. In fact, the subsequent development followed the spirit of this work, but the change in formalism was almost as dramatic as between the systems of Ptolemy and Copernicus.

As it almost always happens, the breakthrough came from a change of the point of view. When Copernicus put the Sun in the centre of the coordinate frame, the immensely complicated host of epicycles was transformed into an easily intelligeble system of concentric orbits. In a similar way a transformation from fermions to bosons (hence the term *bosonization*) has provided a new convenient basis and leads to a radical

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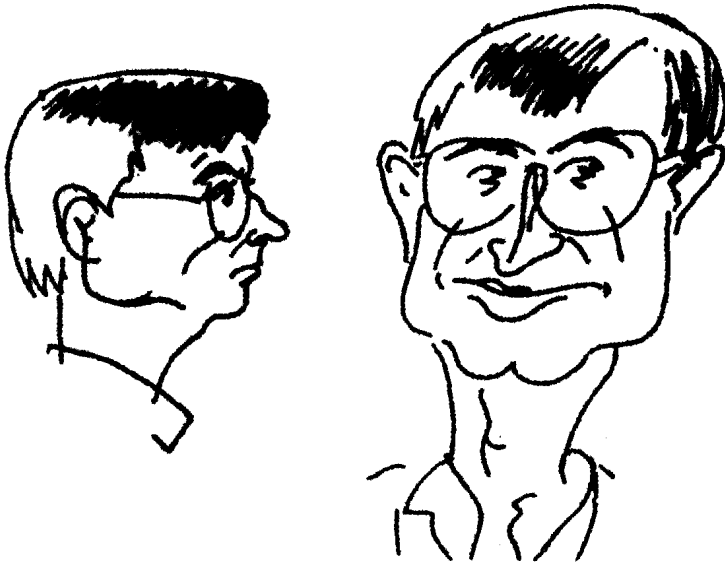[†] Particle transmutation includes orthogonality catastrophy as a particular case.

Fig. 3. Alan Luther.

simplification of the theory of strong interactions in $(1 + 1)$-dimensions. The bosonization method was conceived in 1975 independently by two particle and two condensed matter physicists – Sidney Coleman and Sidney Mandelstam, and Daniel Mattis and Alan Luther respectively.[‡] The focal point of their approach was the property of Dirac fermions in $(1 + 1)$-dimensions. They established that correlation functions of such fermions can be expressed in terms of correlation functions of a free bosonic field. In the bosonic representation the fermion forward scattering became trivial which made a solution of the Tomonaga–Luttinger model a simple exercise.

The new approach had been immediately applied to previously untreatable problems. The results by Dzyaloshinskii and Larkin were rederived for short range interactions and generalized to include effects of spin. It was then understood that the low-energy sector in one-dimensional metallic systems might be described by a universal effective theory later christened 'Luttinger-' or 'Tomonaga–Luttinger liquid'. The microscopic description of such a state was obtained by Haldane (1981), the original idea, however, was suggested by Efetov and Larkin (1975). Many interesting applications of bosonization to realistic quasi-one-dimensional metals had been considered in the 1970s by many researches.

Another quite fascinating discovery was also made in the 1970s and

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[‡] The first example of bosonization was considered earlier by Schotte and Schotte (1969).

Fig. 4.   Victor Emery.

concerns particles with fractional quantum numbers. Such particles appear as elementary excitations in a number of one-dimensional systems, with a typical example being spinons in the antiferromagnetic Heisenberg chain with half-integer spin. A detailed description of such systems will be given in the main text; here we just present an impressionistic picture.

Imagine that you have a magnet and wish to study its excitation spectrum. You do it by flipping individual spins and looking at propagating waves. Naturally, since the minimal change of the total spin projection is $|\Delta S^z| = 1$ you expect that a single flip generates a particle of spin-1. In measurements of dynamical spin susceptibility $\chi''(\omega, q)$ an emission of this particle is seen as a sharp peak. This is exactly what we see in conventional magnets with spin-1 particles being magnons.

However, in many one-dimensional systems instead of a sharp peak in $\chi''(\omega, q)$, we see a continuum. This means that by flipping one spin we create at least two particles with spin-1/2. Hence fractional quantum numbers. However, excitations with fractional spin are subject to topological restriction – in the given example this restriction tells us that the particles can be produced only in pairs. Therefore one can say that the elementary excitations with fractional spin (spin-1/2 in the given example) experience

Fig. 5. P. W. Anderson.

'topological confinement'. Topological confinement puts restriction only on the overall number of particles leaving their spectrum unchanged. Therefore it should be distinguished from dynamical confinement which occurs, for instance, in a system of two coupled spin-1/2 Heisenberg chains (see Chapter 21). There the interchain exchange confines the spinons back to form $S = 1$ magnons giving rise to a sharp single-magnon peak in the neutron cross section which spreads into the incoherent two-spinon tail at high energies.

An important discovery of non-Abelian bosonization was made in 1983–4 by Polyakov and Wiegmann (1983), Witten (1984), Wiegmann (1984) and Knizhnik and Zamolodchikov (1984). The non-Abelian approach is very convenient when there are spin degrees of freedom in the problem. Its application to the Kondo model done by Affleck and Ludwig in their series of papers (see references in Part III) has drastically simplified our understanding of the strong coupling fixed point.

The year 1984 witnessed another revolution in low-dimensional physics. In this year Belavin, Polyakov and Zamolodchikov published their fundamental paper on conformal field theory (CFT). CFT provides a unified approach to all models with gapless linear spectrum in $(1 + 1)$-dimensions.

Fig. 6. Andreas Ludwig.

It was established that if the action of a $(1 + 1)$-dimensional theory is quantizable, that is its action does not contain higher time derivatives, the linearity of the spectrum guarantees that the system has an infinite dimensional symmetry (conformal symmetry). The intimate relation between CFT and the conventional bosonization had became manifest when Dotsenko and Fateev represented the CFT correlation functions in terms of correlators of bosonic exponents (1984). In the same year Cardy (1984) and later Blöte, Cardy and Nightingale (1986) found the important connection between finite size scaling effects and conformal invariance.

Both non-Abelian bosonization and CFT are steps from the initial simplicity of the bosonization approach towards complexity of the theory of integrable systems. Despite the fact that correlation functions can in principle be represented in terms of correlators of bosonic exponents, the Hilbert space of such theories is not equivalent to the Hilbert space of free bosons. In order to make use of the bosonic representation one must exclude certain states from the bosonic Hilbert space. It is not always convenient to do this directly; instead one can calculate the correlation functions using the Ward identities. It is the most important result of CFT that correlation functions of critical systems obey an infinite number of

Fig. 7. Alexander Belavin.

the Ward identities which have the form of differential equations. Solving these equations one can uniquely determine all multi-point correlation functions. This approach is a substitute for the Hamiltonian formalism, since the Hamiltonian is effectively replaced by Ward identities for correlation functions. Conformally invariant systems being systems with infinite number of conservation laws constitute a subclass of exactly solvable (integrable) models.

After many years of intensive development the theory of strongly correlated systems became a vast and complicated area with many distingushed researchers working in it. Different people have different styles and different interests – some are concerned with applications and some with technical developments. There is certainly a gap between those who develop new methods and those who apply them. As an example we can mention the Ising model which has been very extensively studied, but scarcely used in applications. Meanwhile, as will be demonstrated later in the text, this is the simplest theory among those which remain solvable outside of criticality.

This book is an attempt to breach the gap between mathematics of strongly correlated systems and its applications. In our work we have

been inspired by the idea that the theory in $(1 + 1)$-dimensions, though being but a small subsector of a global theory of strongly correlated systems, may give an insight for more important and general problems and give the reader a better vision of 'the Universe as a great idea'. The reader will judge whether our attempt is successful.

In conclusion we say several words about the structure and style of this book. The reader should keep in mind that we shall frequently and without much discussion switch between Hamiltonian and Lagrangian formalisms. As a consequence the same notations will stand for operators in the first case and for number (or Grassmann number) fields in the second case. Please beware of this and watch what formalism is used to avoid confusion. We shall also frequently use the field theory jargon: for example, electronic densities are often called currents. Bear in mind that the essence of things does not depend on how they are called, and be indulgent.

The book contains three parts – in the first one we discuss the method, in the second part describe its applications to some interesting $(1 + 1)$-dimensional systems, and in the third part discuss nonlinear quantum impurities. There are important topics which we do not cover; some being even very important – such as applications of bosonization in more than one spatial dimension and the boundary conformal theory. The only reason for omitting these topics is our ignorance.

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