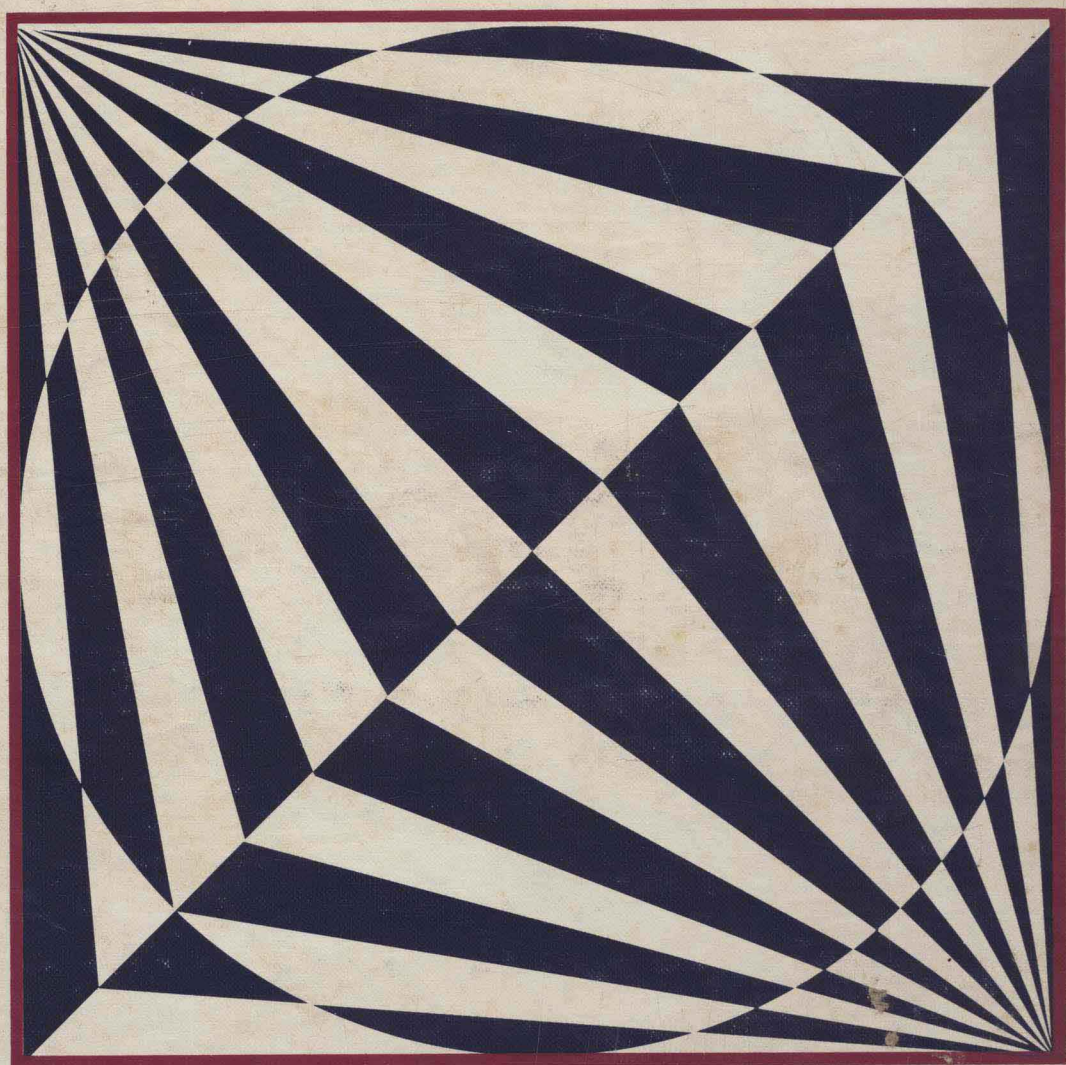


essentials of technical mathematics

RICHARD S. PAUL ■ M. LEONARD SHAEVEL



essentials of technical mathematics

PRENTICE-HALL, INC., *Englewood Cliffs, New Jersey*

PAUL, RICHARD S

Essentials of technical mathematics.

(Prentice-Hall technical mathematics series)

1. Mathematics—1961— I. Shaevel, M.

Leonard, joint author. II. Title.

QA39.2.P39 512'.1 73-16322

ISBN 0-13-288084-9

© 1974 by Prentice-Hall, Inc.

Englewood Cliffs, N. J.

All rights reserved. No part of this book
may be reproduced in any form or by any means
without permission in writing from the publisher.

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

Cover illustration courtesy of Dorine Lerner

PRENTICE-HALL INTERNATIONAL, INC., *London*

PRENTICE-HALL OF AUSTRALIA, PTY. LTD., *Sydney*

PRENTICE-HALL OF CANADA, LTD., *Toronto*

PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*

PRENTICE-HALL OF JAPAN, INC., *Tokyo*

**essentials
of
technical
mathematics**

which are structured to relate to the examples and material in the text. Each exercise set has been designed to include a reasonable number of problems which the student can master without difficulty in order to fix in his mind the basic concepts under study. As a result, the student will quickly gain some degree of confidence. The relevance of the subject matter to the engineering technologies is indicated by the illustrative applications in the examples, the material of the text, and the exercises. These applications require no prior knowledge on the part of the student.

Chapter 0, not necessarily intended as part of a formal course, provides a very brief review of some fundamental arithmetic operations and geometrical concepts; it has been included for the convenience of the student. With the exception of Chapter 0, each chapter contains a review section which includes a programmed-style review covering essential mathematical concepts, and numerous review exercises.

Answers to the odd-numbered problems, all review exercises, and all slide-rule problems appear at the end of the book.

By relegating the unusually thorough descriptions of slide-rule operations to the appendix, they may be presented at any convenient point in the mathematics program without interrupting the continuity of the text. They also form a convenient reference for the student.

Available from the publisher is an extensive instructor's manual which contains answers to all exercises and detailed solutions to a great many of them, including all applied problems.

We express our thanks to Keuffel and Esser Company for the photographs, material, and exercises on the slide rule which have been adapted to our needs and appear in the appendix; and to Cary Baker and Bob Duchacek and the entire Prentice-Hall staff for their cooperation and assistance.

Finally, we express our most grateful appreciation to Professor Frank Kocher of the Department of Mathematics of The Pennsylvania State University whose objective critique of the entire manuscript and important suggestions for its improvement are in evidence throughout the text.

RICHARD S. PAUL

M. LEONARD SHAEVEL

contents

preface

xi



preliminary topics

1

- 0-1** INTRODUCTION 1
- 0-2** BASIC TERMINOLOGY 1
- 0-3** COMMON FRACTIONS 4
- 0-4** OPERATIONS WITH DECIMALS 11
- 0-5** GEOMETRICAL CONCEPTS AND FORMULAS 15



the real number system

18

- 1-1** SETS 18
- 1-2** REAL NUMBERS AND THE REAL NUMBER LINE 21
- 1-3** INTERVALS 24
- 1-4** AXIOMS FOR THE REAL NUMBERS 26
- 1-5** THE ARITHMETIC OF REAL NUMBERS 33
- 1-6** EXPONENTS 39
- 1-7** THE ZERO EXPONENT 45
- 1-8** NEGATIVE EXPONENTS 45
- 1-9** SCIENTIFIC NOTATION 48
- 1-10** RADICALS 50
- 1-11** FRACTIONAL EXPONENTS AND RULES FOR RADICALS 52
- 1-12** REVIEW 57

2 fundamental operations with algebraic expressions 62

- 2-1** ALGEBRAIC EXPRESSIONS 62
- 2-2** SYMBOLS OF GROUPING 64
- 2-3** ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS 67
- 2-4** MULTIPLICATION OF ALGEBRAIC EXPRESSIONS 70
- 2-5** DIVISION OF ALGEBRAIC EXPRESSIONS 74
- 2-6** REVIEW 80

3 special products and factoring 83

- 3-1** SPECIAL PRODUCTS 83
- 3-2** FACTORING 89
- 3-3** REVIEW 95

4 fractions 98

- 4-1** SIMPLIFICATION OF FRACTIONS 98
- 4-2** ADDITION AND SUBTRACTION OF FRACTIONS 104
- 4-3** MULTIPLICATION AND DIVISION OF FRACTIONS 110
- 4-4** COMPLEX FRACTIONS 113
- 4-5** RATIO—CONVERSION OF UNITS 115
- 4-6** REVIEW 119

5 linear equations 121

- 5-1** TYPES OF EQUATIONS 121
- 5-2** EQUIVALENT EQUATIONS 126
- 5-3** LINEAR EQUATIONS 128
- 5-4** FRACTIONAL EQUATIONS 133
- 5-5** PROPORTION 138
- 5-6** VERBAL PROBLEMS 143
- 5-7** REVIEW 149

6 functions, graphs, and straight lines 152

- 6-1** FUNCTIONS 152
- 6-2** GRAPHS IN RECTANGULAR COORDINATES 159
- 6-3** THE SLOPE OF A STRAIGHT LINE 173
- 6-4** STRAIGHT LINES 177
- 6-5** A FINAL COMMENT ON GRAPHS 186
- 6-6** REVIEW 188

7 trigonometry

192

- 7-1** INTRODUCTION 192
- 7-2** ANGLES AND ANGULAR MEASUREMENT 193
- 7-3** TRIGONOMETRIC FUNCTIONS OF ACUTE ANGLES 200
- 7-4** TRIGONOMETRIC FUNCTIONS OF 0° , 30° , 45° , 60° , AND 90° 206
- 7-5** THE TRIGONOMETRIC TABLES—INTERPOLATION 211
- 7-6** TRIGONOMETRIC FUNCTIONS OF ANY ANGLE 216
- 7-7** SOLUTION OF THE RIGHT TRIANGLE 224
- 7-8** VECTORS 228
- 7-9** ADDITION OF VECTORS—ANALYTICAL METHOD 235
- 7-10** REVIEW 239

8 systems of linear equations

243

- 8-1** SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES 243
- 8-2** METHOD OF ELIMINATION BY ADDITION 247
- 8-3** METHOD OF ELIMINATION BY SUBSTITUTION 252
- 8-4** SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES 254
- 8-5** DETERMINANTS OF ORDER 2 256
- 8-6** DETERMINANTS OF ORDER 3 260
- 8-7** VERBAL PROBLEMS 263
- 8-8** REVIEW 267

9 exponents and radicals

271

- 9-1** A REVIEW OF EXPONENTS 271
- 9-2** FURTHER OPERATIONS WITH EXPONENTS 274
- 9-3** SOME BASIC OPERATIONS WITH RADICALS 277
- 9-4** ADDITION AND SUBTRACTION OF RADICALS 280
- 9-5** MULTIPLICATION OF RADICALS 282
- 9-6** DIVISION OF RADICALS 285
- 9-7** RADICAL EQUATIONS 291
- 9-8** REVIEW 293

10 variation

296

- 10-1** DIRECT VARIATION 296
- 10-2** INVERSE AND COMBINED VARIATION 301
- 10-3** REVIEW 305

11

complex numbers**308**

- 11-1** INTRODUCTION 308
- 11-2** THE j -OPERATOR AND COMPLEX NUMBERS 309
- 11-3** GEOMETRIC REPRESENTATION OF COMPLEX NUMBERS 314
- 11-4** OPERATIONS WITH COMPLEX NUMBERS IN RECTANGULAR FORM 321
- 11-5** OPERATIONS WITH COMPLEX NUMBERS IN TRIGONOMETRIC FORM 326
- 11-6** DE MOIVRE'S THEOREM 333
- 11-7** REVIEW 338

12

quadratic equations**342**

- 12-1** INTRODUCTION 342
- 12-2** SOLUTION OF QUADRATIC EQUATIONS BY FACTORING 343
- 12-3** COMPLETING THE SQUARE—THE QUADRATIC FORMULA 348
- 12-4** MAXIMUM AND MINIMUM OF QUADRATIC FUNCTIONS 354
- 12-5** EQUATIONS LEADING TO QUADRATIC EQUATIONS 357
- 12-6** SYSTEMS OF QUADRATIC EQUATIONS IN TWO VARIABLES 364
- 12-7** REVIEW 367

13

**exponential and
logarithmic functions****370**

- 13-1** EXPONENTIAL AND LOGARITHMIC FUNCTIONS 370
- 13-2** PROPERTIES OF LOGARITHMS 376
- 13-3** COMMON LOGARITHMS 378
- 13-4** COMPUTATIONS WITH LOGARITHMS 385
- 13-5** NATURAL LOGARITHMS 389
- 13-6** CHANGE OF BASE 391
- 13-7** LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS 393
- 13-8** EXPONENTIAL AND LOGARITHMIC EQUATIONS 396
- 13-9** LOGARITHMIC AND SEMILOGARITHMIC GRAPH PAPER 397
- 13-10** A COMMENT ON EXPONENTIAL FORM OF A COMPLEX NUMBER 402
- 13-11** REVIEW 403

14

**graphs of the
trigonometric functions****406**

- 14-1** INTRODUCTION 406
- 14-2** THE GRAPH OF THE SINE FUNCTION 406
- 14-3** THE GRAPH OF THE COSINE FUNCTION 414
- 14-4** THE GRAPH OF THE TANGENT FUNCTION 419
- 14-5** GRAPHS OF THE COTANGENT, SECANT, AND COSECANT FUNCTIONS 420
- 14-6** COMBINATIONS OF TRIGONOMETRIC FUNCTIONS—ADDITION OF ORDINATES 422
- 14-7** INVERSE TRIGONOMETRIC FUNCTIONS 425
- 14-8** THE OSCILLOSCOPE—LISSAJOUS FIGURES 427
- 14-9** REVIEW 431

15

**trigonometric formulas
and equations****434**

- 15-1** FUNDAMENTAL IDENTITIES 434
- 15-2** FUNCTIONS OF THE SUM AND DIFFERENCE OF ANGLES 440
- 15-3** DOUBLE- AND HALF-ANGLE FORMULAS 448
- 15-4** TRIGONOMETRIC EQUATIONS 453
- 15-5** REVIEW 455

16

**oblique triangles and
applications of
angular measurement****458**

- 16-1** THE LAW OF SINES 458
- 16-2** THE LAW OF COSINES 465
- 16-3** USE OF LOGARITHMS IN SOLVING TRIANGLES 470
- 16-4** APPLICATIONS OF ANGLES AND ANGULAR MEASUREMENT 471
- 16-5** REVIEW 476

17

inequalities**479**

- 17-1** LINEAR INEQUALITIES IN ONE VARIABLE 479
- 17-2** NONLINEAR INEQUALITIES 485
- 17-3** ABSOLUTE VALUE INEQUALITIES 490
- 17-4** LINEAR INEQUALITIES IN TWO VARIABLES 494
- 17-5** REVIEW 501

18

analytic geometry**504**

- 18-1** INTRODUCTION: THE CONIC SECTIONS 504
- 18-2** THE DISTANCE FORMULA 506
- 18-3** THE CIRCLE 507
- 18-4** THE PARABOLA 511
- 18-5** THE ELLIPSE 520
- 18-6** THE HYPERBOLA 525
- 18-7** SUMMARY OF CONIC SECTIONS 533
- 18-8** PARAMETRIC EQUATIONS 534
- 18-9** REVIEW 538

19

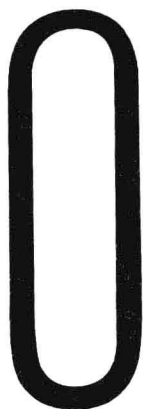
sequences and series**542**

- 19-1** SEQUENCES 542
- 19-2** ARITHMETIC AND GEOMETRIC PROGRESSIONS 545
- 19-3** SERIES 549
- 19-4** LIMITS OF SEQUENCES 555
- 19-5** THE INFINITE GEOMETRIC SERIES 561
- 19-6** THE BINOMIAL THEOREM 565
- 19-7** REVIEW 569

	appendices	573
Appendix A: THE SLIDE RULE 573		
A-1	INTRODUCTION	573
A-2	READING THE SCALES	574
A-3	THE PARTS OF THE SLIDE RULE AND DEFINITIONS	577
A-4	ACCURACY OF THE SLIDE RULE	578
A-5	THE LOCATION OF THE DECIMAL POINT	578
A-6	MULTIPLICATION	579
A-7	INTERCHANGING THE INDEXES	581
A-8	DIVISION	582
A-9	THE FOLDED SCALES—DF AND CF	583
A-10	COMBINED OPERATIONS—MULTIPLICATION AND DIVISION	585
A-11	A VISUAL SUMMARY OF SOME FUNDAMENTAL OPERATIONS	586
A-12	THE RECIPROCAL SCALES—DI, CI, AND C/F	587
A-13	SQUARE ROOTS—SCALES A AND B	589
A-14	COMBINED OPERATIONS INVOLVING SQUARE ROOTS	591
A-15	CUBE ROOTS—THE K SCALE	592
A-16	SQUARES AND CUBES	594
A-17	THE S (SINE) AND SRT (SINE, RADIAN, TANGENT) SCALES	595
A-18	THE T (TANGENT) SCALE	598
A-19	RADIANS—SMALL ANGLES	599
A-20	THE LON SCALES	601
A-21	POWERS OF e	604
Appendix B: TABLE OF EXPONENTIAL FUNCTIONS 607		
Appendix C: NATURAL TRIGONOMETRIC FUNCTIONS 608		
Appendix D: LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS 613		
Appendix E: COMMON LOGARITHMS 620		
Appendix F: NATURAL LOGARITHMS 638		
Appendix G: POWERS—ROOTS—RECIPROCALs 640		

answers to odd-numbered problems	642
---------------------------------------------	------------

index	681
--------------	------------



preliminary topics

0-1

INTRODUCTION

Although it is assumed that you have had some prior preparation in mathematics, a brief review of certain arithmetical operations and selected geometrical concepts may be useful. This chapter provides such a review for your convenience; it is not meant to be a summary of all prerequisites for a course of this type. The topics covered are those to which an immediate second exposure may be beneficial. We urge you to devote whatever time is necessary to those topics in which you need review.

0-2

BASIC TERMINOLOGY

Whole numbers are those numbers which are used in counting, such as 0, 1, 2, 13, 44, and 610; they are also called **integers**. Whole numbers are either **even** or **odd**, depending, respectively, on whether they are divisible by 2 or not. Thus, 2, 10, and 1564 are even integers and 1, 3, 29, and 97 are odd integers.

A **prime** number is a whole number greater than 1 that is divisible only by itself and 1. Thus, 2, 3, 5, 7, 11, 13, and 17 are prime numbers, but 15 is not, since 15 is divisible by 3 and 5 as well as 15 and 1.

Whole numbers are represented by combinations of the ten **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. An analysis of the numeral 234 would be that the digit 4 occupies the units place and represents four 1's (4), the digit 3 occupies the tens place and represents three groups of 10 (30), and the digit 2 occupies the hundreds place and represents two groups of 100 (200).

It seems reasonable to assume that you are totally familiar with addition, subtraction, multiplication, and division of whole numbers, and so we dispense with any further discussion of these operations. It is appropriate, however, to state two important properties that are valid for all types of numbers:

1. *The addition and multiplication of a group of numbers can be performed in any order.*

Thus, $2 + 3 + 4 = 4 + 2 + 3 = 4 + 3 + 2 = 9$. Similarly, $(3)(2)(4) = (2)(3)(4) = (4)(3)(2) = 24$. Since the numbers 2, 3, and 4 when multiplied together give 24, each of 2, 3, and 4 is called a **factor** of 24, and 24 is called the **product** of 2, 3, and 4.

2. *The operations of subtraction and division must be performed in the given order.*

Thus, $6 \div 3$ is not the same as $3 \div 6$. In $6 \div 3 = 2$, 6 is called the **dividend**, 3 is called the **divisor**, and 2 is the **quotient**.

In computations involving more than one operation, multiplications and divisions should be performed first, followed by the additions and subtractions. For example,

- a. $6 + (6)(2) = 6 + 12 = 18$.
- b. $12 - (2)(2) = 12 - 4 = 8$.
- c. $(12)(2) + 6 \div 3 = 24 + 2 = 26$.

However, any series of operations within parentheses should be performed first. Thus,

- d. $12 + (2 + 3) - (4)(6 - 3)$
 $= 12 + 5 - (4)(3)$
 $= 12 + 5 - 12$
 $= 17 - 12$
 $= 5$.

$$\begin{aligned}\text{e. } & 16 + (12 \div 3)(4 - 3) \\ & = 16 + (4)(1) \\ & = 16 + 4 \\ & = 20.\end{aligned}$$

To avoid any ambiguity in part c above, $6 \div 3$ can be written $\frac{6}{3}$, and it is preferable to write

$$(12)(2) + \frac{6}{3} \quad \text{instead of} \quad (12)(2) + 6 \div 3.$$

EXERCISE 0-2

Perform the indicated operations in Problems 1–20.

1. $10 + (2)(4) - (8 \div 2)$
2. $(12 - 4)(6) + 2(5 + 1)$
3. $1600 - (5)(210 \div 2)(2)$
4. $(2 \div 2) + 7(6)(3 - 1) - 17$
5. $(6 + 4 - 5) \div (2 + 3 - 4) + (7)(6 + 2)$
6. $(36 \div 6)(6 \div 2)(1 + 2) - 6$
7. $(2 + 16)(8 - 2) \div (5 - 2)(1)$
8. $(12 - 4)(3 + 1) + 2(16 \div 4) - 12$
9. $(223)(21)(84)$
10. $(1062)(2)(56)$
11. $8525 \div 341$
12. $10,209 \div 123$
13. $(621)(4)(16) \div 8$
14. $26,451 + 26,245 + 86,216 - 726 + 624 - 18,707$
15. $576 \div (2 + 16)$
16. $8622 \div (13 - 4)$
17. $(12,604)(865)$
18. $33,454 \div 86$
19. $1,864,200 \div 17,925$
20. $1,003,005 \div 6471$
21. The tens and units digits of the number 862 are reversed and the new number is subtracted from 862. What is the result?
22. What digit must be changed in 6257 to make it an even number?

23. If the dividend and quotient of a division are both 6, what is the divisor?
24. The product of three factors is 504. Two of the factors are 6 and 7. Find the third factor.
25. In the number 62,417, what place does the 6 occupy?

0-3

COMMON FRACTIONS

A common fraction such as $\frac{2}{5}$ consists of the **numerator** 2 and the **denominator** 5. It can be considered as denoting the division of the **dividend** 2 by the **divisor** 5 and, as a result, is often referred to as being the **quotient** of 2 divided by 5, or, simply, the quotient $\frac{2}{5}$.

Fractions are of two types, depending on the relative values of the numerator and denominator. If the numerator is less than the denominator, such as in $\frac{3}{8}$, the fraction is said to be a **proper** fraction and its value is less than 1. If, on the other hand, the numerator is equal to or greater than the denominator, the fraction, which has a value of at least 1, is said to be **improper**. Fractions such as $\frac{3}{3}$, $\frac{16}{7}$, and $\frac{214}{161}$ are examples of improper fractions. Any improper fraction can be expressed as the sum of an integer and a proper fraction, called a **mixed number**. For example,

$$\frac{23}{6} = 3 + \frac{5}{6} = 3\frac{5}{6}.$$

Here the improper fraction $\frac{23}{6}$ is equivalent to the mixed number $3\frac{5}{6}$.

To write an improper fraction as a mixed number, it is only necessary to divide the numerator by the denominator, thus obtaining the integer part, and add to this the fraction whose numerator is the remainder of the division and whose denominator is the denominator of the original fraction. Thus,

- a. $\frac{16}{13} = 1 + \frac{3}{13} = 1\frac{3}{13}$.
- b. $\frac{14}{3} = 4 + \frac{2}{3} = 4\frac{2}{3}$.
- c. $\frac{27}{4} = 6 + \frac{3}{4} = 6\frac{3}{4}$.

A fundamental principle used in computations involving fractions is that *multiplying or dividing both the numerator and denominator of a fraction by the same nonzero number does not change the value of the fraction*. In effect, the fraction is being multiplied by 1. Thus,

$$\text{a. } \frac{3}{8} = \frac{(3)(4)}{(8)(4)} = \frac{12}{32}.$$

$$\text{b. } 12 = \frac{12}{1} = \frac{(12)(2)}{(1)(2)} = \frac{24}{2}.$$

$$\text{c. } \frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}.$$

$$\text{d. } \frac{210}{26} = \frac{210 \div 2}{26 \div 2} = \frac{105}{13}.$$

By multiplying both the numerator and the denominator of a fraction by the same number, we can express the fraction as an equivalent fraction having any desired denominator. Thus, if the fraction $\frac{5}{8}$ is to be written as an equivalent fraction whose denominator is 24, it should be clear that since the denominator must be multiplied by 3, so must the numerator.

$$\frac{5}{8} = \frac{(5)(3)}{(8)(3)} = \frac{15}{24}.$$

A fraction is said to be in **lowest terms** if the numerator and denominator have no common factor other than 1. Thus, $\frac{18}{12}$ is not in lowest terms since 18 and 12 have a common factor of 6, that is, $18 = (6)(3)$ and $12 = (6)(2)$. However, by dividing the numerator and denominator by 6 we get the equivalent fraction

$$\frac{18}{12} = \frac{18 \div 6}{12 \div 6} = \frac{3}{2},$$

which is in lowest terms. Moreover, if we write the fraction $\frac{18}{12}$ as

$$\frac{(6)(3)}{(6)(2)},$$

the division by 6 is seen to be nothing more than the familiar process of **cancellation**, whereby factors common to the numerator and denominator cancel each other, denoted by slashes (/):

$$\frac{\cancel{(6)}(3)}{\cancel{(6)}(2)} = \frac{3}{2}.$$

The process of repetitive cancellation can be used as follows, where in each step, a factor 2 is removed:

$$\frac{104}{224} = \frac{\cancel{104}}{\cancel{224}} = \frac{52}{112} = \frac{\cancel{52}}{\cancel{112}} = \frac{26}{56} = \frac{\cancel{26}}{\cancel{56}} = \frac{13}{28}.$$

In a similar fashion,

$$\begin{aligned}
 \frac{\overset{1}{(12)}\overset{(6)}{\cancel{(3)}}\overset{(14)}{(14)}}{\underset{7}{(22)}\underset{(14)}{\cancel{(21)}}\underset{(18)}{(18)}} &= \frac{\overset{2}{(12)}\overset{(6)}{\cancel{(14)}}}{\underset{1}{(22)}\underset{(14)}{\cancel{(7)}}\underset{(18)}{(18)}} = \frac{\overset{1}{(12)}\overset{(6)}{\cancel{(2)}}}{\underset{11}{(22)}\underset{(14)}{\cancel{(18)}}} \\
 &= \frac{\overset{1}{(12)}\overset{(6)}{\cancel{(6)}}}{\underset{3}{(11)}\underset{(14)}{\cancel{(18)}}} = \frac{\overset{4}{\cancel{(12)}}}{\underset{1}{(11)}\underset{(14)}{\cancel{(3)}}} \\
 &= \frac{\overset{2}{\cancel{(4)}}}{\underset{7}{(11)}\underset{(14)}{\cancel{(14)}}} = \frac{2}{77}.
 \end{aligned}$$

This can be written more compactly as

$$\frac{\overset{2}{\cancel{4}}\overset{1}{1}\overset{1}{1}\overset{2}{\cancel{2}}}{\underset{11}{(22)}\underset{7}{(14)}\underset{7}{\cancel{(21)}}\underset{3}{\cancel{(18)}}} = \frac{2}{77}.$$

The sum (or difference) of two fractions having the same denominator is a fraction having the same denominator as the original fractions but whose numerator is the sum (or difference) of the numerators of the original fractions. For example,

- a. $\frac{2}{3} + \frac{5}{3} = \frac{2+5}{3} = \frac{7}{3}.$
- b. $\frac{8}{9} - \frac{6}{9} = \frac{8-6}{9} = \frac{2}{9}.$
- c. $\frac{7}{18} + \frac{3}{18} - \frac{5}{18} = \frac{7+3-5}{18} = \frac{5}{18}.$
- d. $\frac{12}{27} - \frac{6}{27} + \frac{4}{27} - \frac{1}{27} = \frac{12-6+4-1}{27} = \frac{9}{27} = \frac{1}{3}.$

To add fractions when the denominators are not alike, the fractions must first be expressed as equivalent fractions, all of which have the same denominator. The appropriate denominator to choose is the smallest number such that each of the denominators is a factor of it. Such a number is called the **least common denominator**, denoted L.C.D. For example,