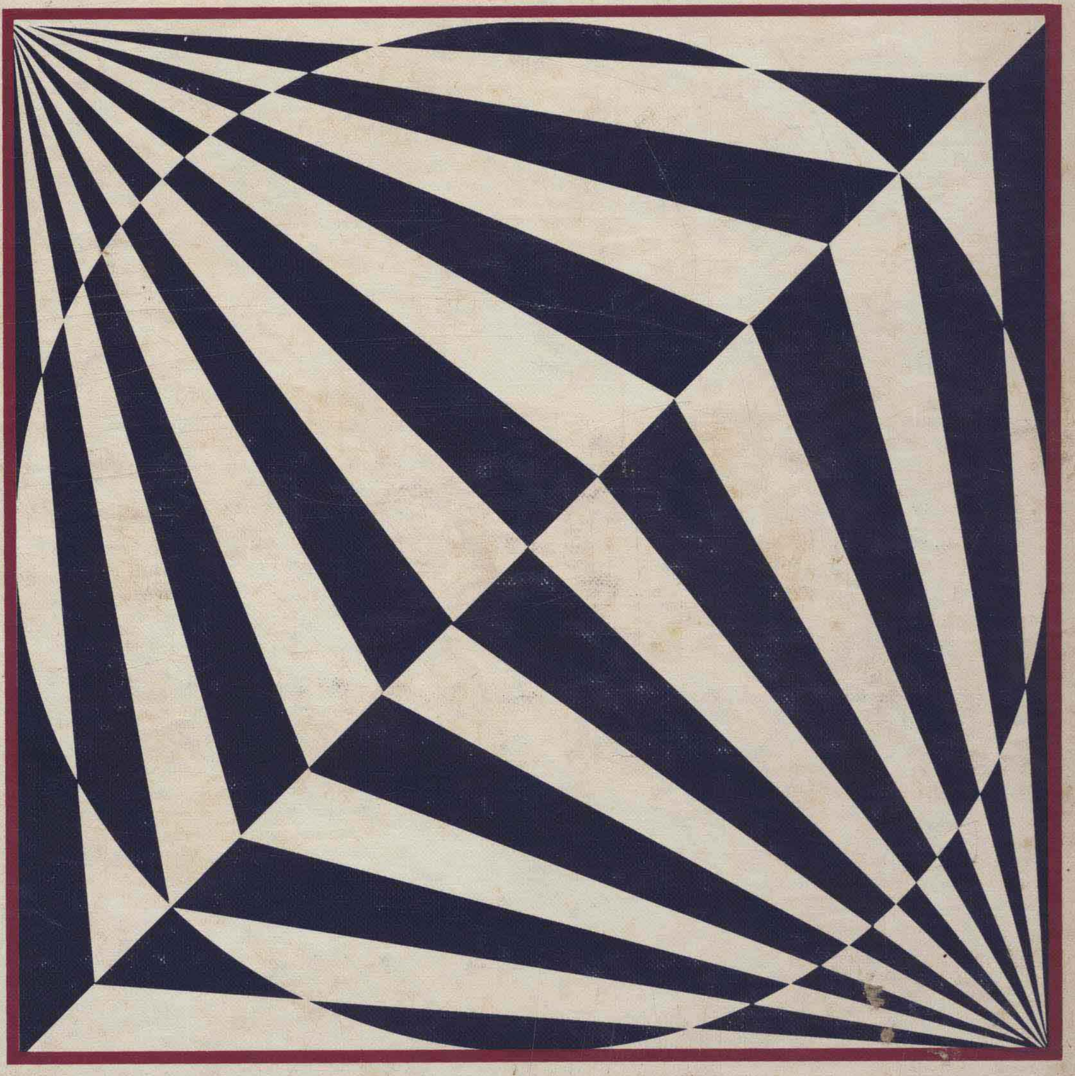
essentials of technical mathematics

RICHARD S. PAUL ▪ M. LEONARD SHAEVEL



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which are structured to relate to the examples and material in the text. Each exercise set has been designed to include a reasonable number of problems which the student can master without difficulty in order to fix in his mind the basic concepts under study. As a result, the student will quickly gain some degree of confidence. The relevance of the subject matter to the engineering technologies is indicated by the illustrative applications in the examples, the material of the text, and the exercises. These applications require no prior knowledge on the part of the student.

Chapter 0, not necessarily intended as part of a formal course, provides a very brief review of some fundamental arithmetic operations and geometrical concepts; it has been included for the convenience of the student. With the exception of Chapter 0, each chapter contains a review section which includes a programmed-style review covering essential mathematical concepts, and numerous review exercises.

Answers to the odd-numbered problems, all review exercises, and all slide-rule problems appear at the end of the book.

By relegating the unusually thorough descriptions of slide-rule operations to the appendix, they may be presented at any convenient point in the mathematics program without interrupting the continuity of the text. They also form a convenient reference for the student.

Available from the publisher is an extensive instructor's manual which contains answers to all exercises and detailed solutions to a great many of them, including all applied problems.

We express our thanks to Keuffel and Esser Company for the photographs, material, and exercises on the slide rule which have been adapted to our needs and appear in the appendix; and to Cary Baker and Bob Duchacek and the entire Prentice-Hall staff for their cooperation and assistance.

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contents

7 trigonometry 192

8 systems of linear equations 243

9 exponents and radicals 271

10 variation 296

11 complex numbers 308

12 quadratic equations 342

13 exponential and logarithmic functions 370

14 graphs of the trigonometric functions 406

0

preliminary topics

0-1
INTRODUCTION

Although it is assumed that you have had some prior preparation in mathematics, a brief review of certain arithmetical operations and selected geometrical concepts may be useful. This chapter provides such a review for your convenience; it is not meant to be a summary of all prerequisites for a course of this type. The topics covered are those to which an immediate second exposure may be beneficial. We urge you to devote whatever time is necessary to those topics in which you need review.

0-2
BASIC TERMINOLOGY

Whole numbers are those numbers which are used in counting, such as 0, 1, 2, 13, 44, and 610; they are also called **integers**. Whole numbers are either **even** or **odd**, depending, respectively, on whether they are divisible by 2 or not. Thus, 2, 10, and 1564 are even integers and 1, 3, 29, and 97 are odd integers.

A **prime** number is a whole number greater than 1 that is divisible only by itself and 1. Thus, 2, 3, 5, 7, 11, 13, and 17 are prime numbers, but 15 is not, since 15 is divisible by 3 and 5 as well as 15 and 1.

Whole numbers are represented by combinations of the ten **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. An analysis of the numeral 234 would be that the digit 4 occupies the units place and represents four 1's (4), the digit 3 occupies the tens place and represents three groups of 10 (30), and the digit 2 occupies the hundreds place and represents two groups of 100 (200).

It seems reasonable to assume that you are totally familiar with addition, subtraction, multiplication, and division of whole numbers, and so we dispense with any further discussion of these operations. It is appropriate, however, to state two important properties that are valid for all types of numbers:

1. *The addition and multiplication of a group of numbers can be performed in any order.*

Thus, $2 + 3 + 4 = 4 + 2 + 3 = 4 + 3 + 2 = 9$. Similarly, $(3)(2)(4) = (2)(3)(4) = (4)(3)(2) = 24$. Since the numbers 2, 3, and 4 when multiplied together give 24, each of 2, 3, and 4 is called a **factor** of 24, and 24 is called the **product** of 2, 3, and 4.

2. *The operations of subtraction and division must be performed in the given order.*

Thus, $6 \div 3$ is not the same as $3 \div 6$. In $6 \div 3 = 2$, 6 is called the **dividend**, 3 is called the **divisor**, and 2 is the **quotient**.

In computations involving more than one operation, multiplications and divisions should be performed first, followed by the additions and subtractions. For example,

a. $6 + (6)(2) = 6 + 12 = 18$.

b. $12 - (2)(2) = 12 - 4 = 8$.

c. $(12)(2) + 6 \div 3 = 24 + 2 = 26$.

However, any series of operations within parentheses should be performed first. Thus,

d. $12 + (2 + 3) - (4)(6 - 3)$
$= 12 + 5 - (4)(3)$
$= 12 + 5 - 12$
$= 17 - 12$
$= 5$.

e. $16 + (12 \div 3)(4 - 3)$
$= 16 + (4)(1)$
$= 16 + 4$
$= 20.$

To avoid any ambiguity in part c above, $6 \div 3$ can be written $\frac{6}{3}$, and it is preferable to write

$$(12)(2) + \tfrac{6}{3} \qquad \text{instead of} \qquad (12)(2) + 6 \div 3.$$

EXERCISE 0-2

Perform the indicated operations in Problems 1–20.

1. $10 + (2)(4) - (8 \div 2)$

2. $(12 - 4)(6) + 2(5 + 1)$

3. $1600 - (5)(210 \div 2)(2)$

4. $(2 \div 2) + 7(6)(3 - 1) - 17$

5. $(6 + 4 - 5) \div (2 + 3 - 4) + (7)(6 + 2)$

6. $(36 \div 6)(6 \div 2)(1 + 2) - 6$

7. $(2 + 16)(8 - 2) \div (5 - 2)(1)$

8. $(12 - 4)(3 + 1) + 2(16 \div 4) - 12$

9. $(223)(21)(84)$

10. $(1062)(2)(56)$

11. $8525 \div 341$

12. $10{,}209 \div 123$

13. $(621)(4)(16) \div 8$

14. $26{,}451 + 26{,}245 + 86{,}216 - 726 + 624 - 18{,}707$

15. $576 \div (2 + 16)$

16. $8622 \div (13 - 4)$

17. $(12{,}604)(865)$

18. $33{,}454 \div 86$

19. $1{,}864{,}200 \div 17{,}925$

20. $1{,}003{,}005 \div 6471$

21. The tens and units digits of the number 862 are reversed and the new number is subtracted from 862. What is the result?

22. What digit must be changed in 6257 to make it an even number?

23. If the dividend and quotient of a division are both 6, what is the divisor?

24. The product of three factors is 504. Two of the factors are 6 and 7. Find the third factor.

25. In the number 62,417, what place does the 6 occupy?

0-3 COMMON FRACTIONS

A common fraction such as $\frac{2}{5}$ consists of the **numerator** 2 and the **denominator** 5. It can be considered as denoting the division of the **dividend** 2 by the **divisor** 5 and, as a result, is often referred to as being the **quotient** of 2 divided by 5, or, simply, the quotient $\frac{2}{5}$.

Fractions are of two types, depending on the relative values of the numerator and denominator. If the numerator is less than the denominator, such as in $\frac{3}{8}$, the fraction is said to be a **proper** fraction and its value is less than 1. If, on the other hand, the numerator is equal to or greater than the denominator, the fraction, which has a value of at least 1, is said to be **improper.** Fractions such as $\frac{3}{3}$, $\frac{16}{7}$, and $\frac{214}{161}$ are examples of improper fractions. Any improper fraction can be expressed as the sum of an integer and a proper fraction, called a **mixed number.** For example,

$$\frac{23}{6} = 3 + \frac{5}{6} = 3\frac{5}{6}.$$

Here the improper fraction $\frac{23}{6}$ is equivalent to the mixed number $3\frac{5}{6}$.

To write an improper fraction as a mixed number, it is only necessary to divide the numerator by the denominator, thus obtaining the integer part, and add to this the fraction whose numerator is the remainder of the division and whose denominator is the denominator of the original fraction. Thus,

a. $\frac{16}{13} = 1 + \frac{3}{13} = 1\frac{3}{13}$.

b. $\frac{14}{3} = 4 + \frac{2}{3} = 4\frac{2}{3}$.

c. $\frac{27}{4} = 6 + \frac{3}{4} = 6\frac{3}{4}$.

A fundamental principle used in computations involving fractions is that *multiplying or dividing both the numerator and denominator of a fraction by the same nonzero number does not change the value of the fraction.* In effect, the fraction is being multiplied by 1. Thus,

a. $$\frac{3}{8} = \frac{(3)(4)}{(8)(4)} = \frac{12}{32}.$$

b. $12 = \frac{12}{1} = \frac{(12)(2)}{(1)(2)} = \frac{24}{2}.$

c. $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}.$

d. $\frac{210}{26} = \frac{210 \div 2}{26 \div 2} = \frac{105}{13}.$

By multiplying both the numerator and the denominator of a fraction by the same number, we can express the fraction as an equivalent fraction having any desired denominator. Thus, if the fraction $\frac{5}{8}$ is to be written as an equivalent fraction whose denominator is 24, it should be clear that since the denominator must be multiplied by 3, so must the numerator.

$$\frac{5}{8} = \frac{(5)(3)}{(8)(3)} = \frac{15}{24}.$$

A fraction is said to be in **lowest terms** if the numerator and denominator have no common factor other than 1. Thus, $\frac{18}{12}$ is not in lowest terms since 18 and 12 have a common factor of 6, that is, $18 = (6)(3)$ and $12 = (6)(2)$. However, by dividing the numerator and denominator by 6 we get the equivalent fraction

$$\frac{18}{12} = \frac{18 \div 6}{12 \div 6} = \frac{3}{2},$$

which is in lowest terms. Moreover, if we write the fraction $\frac{18}{12}$ as

$$\frac{(6)(3)}{(6)(2)},$$

the division by 6 is seen to be nothing more than the familiar process of **cancellation**, whereby factors common to the numerator and denominator cancel each other, denoted by slashes (/):

$$\frac{\not{(6)}(3)}{\not{(6)}(2)} = \frac{3}{2}.$$

The process of repetitive cancellation can be used as follows, where in each step, a factor 2 is removed:

$$\frac{104}{224} = \frac{\overset{52}{\not{104}}}{\underset{112}{\not{224}}} = \frac{\overset{26}{\not{52}}}{\underset{56}{\not{112}}} = \frac{\overset{13}{\not{26}}}{\underset{28}{\not{56}}} = \frac{13}{28}.$$

In a similar fashion,

$$\frac{(12)(6)\overset{1}{\cancel{(3)}}(14)}{(22)(14)\underset{7}{\cancel{(21)}}(18)} = \frac{(12)(6)\overset{2}{\cancel{(14)}}}{(22)(14)\underset{1}{\cancel{(7)}}(18)} = \frac{(12)(6)\overset{1}{\cancel{(2)}}}{\underset{11}{\cancel{(22)}}(14)(18)}$$

$$= \frac{(12)\overset{1}{\cancel{(6)}}}{(11)(14)\underset{3}{\cancel{(18)}}} = \frac{\overset{4}{\cancel{(12)}}}{(11)(14)\underset{1}{\cancel{(3)}}}$$

$$= \frac{\overset{2}{\cancel{(4)}}}{(11)\underset{7}{\cancel{(14)}}} = \frac{2}{77}.$$

This can be written more compactly as

$$\frac{\overset{\overset{2}{\cancel{4}}}{\cancel{(12)}}\overset{1}{\cancel{(6)}}\overset{1}{\cancel{(3)}}\overset{\overset{1}{\cancel{2}}}{\cancel{(14)}}}{\underset{11}{\cancel{(22)}}\underset{7}{\cancel{(14)}}\underset{\underset{1}{\cancel{7}}}{\cancel{(21)}}\underset{\underset{1}{\cancel{3}}}{\cancel{(18)}}} = \frac{2}{77}.$$

The sum (or difference) of two fractions having the same denominator is a fraction having the same denominator as the original fractions but whose numerator is the sum (or difference) of the numerators of the original fractions. For example,

a. $\dfrac{2}{3} + \dfrac{5}{3} = \dfrac{2+5}{3} = \dfrac{7}{3}.$

b. $\dfrac{8}{9} - \dfrac{6}{9} = \dfrac{8-6}{9} = \dfrac{2}{9}.$

c. $\dfrac{7}{18} + \dfrac{3}{18} - \dfrac{5}{18} = \dfrac{7+3-5}{18} = \dfrac{5}{18}.$

d. $\dfrac{12}{27} - \dfrac{6}{27} + \dfrac{4}{27} - \dfrac{1}{27} = \dfrac{12-6+4-1}{27} = \dfrac{9}{27} = \dfrac{1}{3}.$

To add fractions when the denominators are not alike, the fractions must first be expressed as equivalent fractions, all of which have the same denominator. The appropriate denominator to choose is the smallest number such that each of the denominators is a factor of it. Such a number is called the **least common denominator**, denoted L.C.D. For example,