KENNETH STEIGLITZ

AN INTRODUCTION TO DISCRETE SYSTEMS

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with a foreword by M. E. Van Valkenburg

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FOREWORD

Professor Steiglitz has invited me to write a few words about the origins of this textbook in relationship to its general educational objectives. Beginning in 1967, the Committee on Computers in Electrical Engineering (COSINE) of the National Academy of Engineering sponsored a series of task force meetings, each charged with examining a specific area of digital systems education and making recommendations for the development of course materials. The first of these, for which I was Chairman, examined a computer-oriented first course in electrical engineering, specifically in the linear circuits area. The meeting was held at Princeton University and involved about a dozen educators, one of whom was Kenneth Steiglitz.

The Task Force group recognized the now-accepted fact that the elementary circuits course provides an excellent medium for introducing the use of the digital computer through computer applications to numerous topics within linear circuits. It differentiated two ways in which this might be done: one was described as a *transition* approach and the other as an *integrated* approach.

The transition textbook was envisioned as containing more-or-less conventional topics in linear circuits, but presented with a computer orientation, with examples drawn from the application of numerical methods. Some of the topics that might be presented in an associated software laboratory or as homework assignments include: determining roots (Newton-Raphson method), numerical integration, the solution of linear algebraic equations (Gauss elimination method), the solution of ordinary differential equations (Runge-Kutta method), perhaps the solution of simple nonlinear differential equations, general network analysis using canned programs, the determination and plotting of magnitude and phase, and determination of residues.

The integrated approach to the teaching of a computer-oriented circuits course was envisioned as one that might evolve over a period of several years. Given the widespread availability of computers, there seems little doubt that the teaching of electrical engineering should undergo an evolution, with each traditional subject reexamined as to its relevance in a computer age, and each new subject examined to see where it might best fit into an educational program. In the emerging pedagogical approach, equations should be written in discrete form as difference equations, instead of in continuous form as differential equations. Indeed, equations should seldom be used, since principles should be stated directly in algorithmic form. Many concepts and related theorems that find use in computation will not survive, or will be useful only for checking. Thus a step-by-step restructuring of subject matter is envisioned.

A number of textbooks of the transition variety have already appeared, but this textbook appears to be the first using an integrated approach. Shortly after the meeting of the Task Force, Professor Steiglitz was persuaded to teach a course, based on an integrated approach, to first-term sophomores in electrical engineering at Princeton University. This course has been developed over the years with care, and several revisions of the organization and structure of the manuscript have taken place based on the experience with students. Having been associated with the course myself from time to time, I can report on some aspects that I have observed.

The notes have been well received by the sophomores. Since their only previous formal contact with the topics in circuits has come in the brief presentation in physics, they began their study of electrical engineering directly in terms of discrete signals and an algorithmic approach; they studied without apparent difficulty in comparison to a more conventional approach. This is in itself an interesting observation. I have discussed these subjects with faculty members at other institutions and teachers of courses in industrial laboratories in which the discrete case has been presented at the graduate level after the student has had extensive instruction in the continuous case. The universal reaction has been that it is a difficult subject for students at an advanced level. It appears that the discrete case is a natural one to the uninhibited mind, but special and somewhat mysterious after a thorough grounding in continuous concepts.

At Princeton, this first course has been followed in the second term by a conventional linear circuits course, sometimes facetiously described as the special limiting case of an infinite sampling rate. The similarity of concepts, language, and conclusions for the two cases was made throughout the second course. Students who have completed this two-term sequence have since continued through their last two years and on into graduate school or to industry. I believe that they have done so at an advantage; no disadvantages have become apparent from an arrangement of topics that many might think of as unconventional.

It has been a pleasure to talk with students who have returned from summer experiences in industry after their sophomore year. They found that the topics treated in the course gave them an immediate advantage in their job assignments.

To students about to embark on their study of the topics in this textbook, I promise a rewarding experience. Discrete-time systems and the processing of discrete-time signals will grow in importance in the future, making a knowledge of these subjects indispensable to the engineer. The basic ideas will become real through the artifice of the digital filter whose operation is easily visualized (or demonstrated with laboratory equipment) and readily implemented in the laboratory in either hardware or software form. Linear graphs used to study the relationships of quantities and the enumeration of things will be found useful in a wide variety of fields important to the engineer.

To faculty members who will guide the students through the pages of the textbook, I promise a lucid presentation of important subjects prepared by an authority on the topics presented, well stocked with interesting and challenging problems, and thoroughly tested in the classroom.

M. E. Van Valkenburg
Princeton University

PREFACE

This book evolved over the past four years as notes for a one-semester, first course in electrical engineering at Princeton University. The premises behind the choice of material are that the discrete-time, digital device has become the fundamental building block of engineering systems; that first- or second-year students are often already prejudiced towards thinking in terms of continuous-time systems; and that this prejudice should be counteracted as soon as possible. The text is designed, therefore, so that it can precede a conventional circuit theory course, and the last chapter provides an introduction to circuit theory that can serve as a transition to such a course. This sequence has the advantage of introducing discrete-time linear system theory, which makes no use of differential equations or δ-functions, earlier in the curriculum than the mathematically more advanced continuous-time theory. This reinforces the student's eventual appreciation of the common fundamental notions of both theories. This arrangement has the additional advantage of providing a first course in electrical engineering that focuses on the digital computer, thus providing material that is relevant to computer science students.

The book also can be used in conjunction with a textbook in circuit theory for a "hybrid" course in discrete- and continuous-time systems. This idea—of using a pair of texts as coroutines—offers important advantages: variety of motivation, strengthening of appreciation for generality, and great flexibility for the instructor. Other possibilities are using either Part I or II separately for a one-quarter course, or omitting Chapters 8 to 10 in a faster one-quarter course for students with a background in continuous systems.

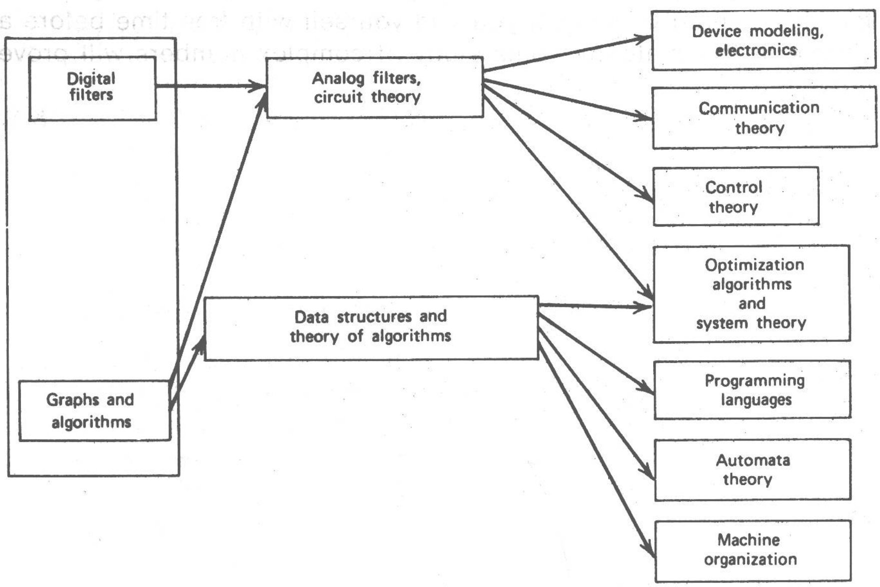
Most of the material is new to the undergraduate curriculum, and the working engineer may wish to use the text for self-study. His familiarity with the Laplace transform and computer programming should enable him to use this material as an introduction to current literature in the fields of digital signal processing and networks for communication, transmission, and transportation.

I am indebted to many people for contributions to this project. First, I thank Professor Mac E. Van Valkenburg, who first pointed out to me the need for a computer-oriented first course, and who helped and encouraged me in many ways. Without him, this book would never have been written. Dr. Godfrey Winham read many early versions of the text with great care, and his critical comments improved the result considerably. His program for the FFT is given in Chapter 6. The treatment of the Ford and Fulkerson labeling algorithm is based on a talk by Dr. Bill Rothfarb. I am indebted to Professors T. L. Booth, J. L. Bruno, S. W. Director, E. I. Jury, T. Pavlidis, and R. J. Smith for many constructive comments on the manuscript; and to G. A. Davenport, D. C. Ford, and others at Wiley for their splendid cooperation. Finally, I thank the students of Engineering 283. I hope they learned as much from me as I learned from them.

Kenneth Steiglitz

A NOTE TO THE READER

This book is an introduction to the use of the digital computer in engineering. The fundamental idea is that of the step-by-step procedure: the algorithm. In Part I, algorithms for processing signals are studied: in Part II, algorithms for solving problems associated with systems (in a general sense) are taken up. An attempt has been made to keep the material fundamental and general and, thus, to



provide a foundation for future courses, including (but not restricted to) a conventional course in circuit analysis.

Above is a block diagram with the subjects of this book, together with a rough (and somewhat arbitrary) indication of how they lead to other subjects in engineering and computer science.

Although some of the exercises are routine and are meant to test your comprehension of material, many are intended to lead deeper into the material and to challenge your wit. Especially difficult exercises are indicated by an asterisk. Answers and hints to many of the exercises are given in the Appendix.

It is assumed that you have had the equivalent of an introductory course in computer programming and are familiar with FORTRAN.

The programs given in the text have been written in WATFIV. Some of the exercises ask you to write programs; these programs can, of course, be written in any language you find convenient.

Finally, a word about mathematical requirements. The relevant mathematics for Part I is complex numbers; that for Part II is graph theory. Both subjects are introduced and explained in the text (Chapters 1 and 7). If you are not familiar with operations with complex numbers, you will find that some practice is required to gain the facility required later on. If you find yourself with free time before a course in this material, some study of complex numbers will prove helpful.

K.S.

APPENDIX

ANSWERS AND HINTS TO SELECTED EXERCISES

CHAPTER 1

1.2.2 The nth root of $z = Re^{j\theta}$ is not unique; there are n of them given by

$$R^{1/n}e^{j(\theta+2\pi k)/n},\ k=0,\ldots,n-1.$$

The nth power of $z = Re^{j\theta}$ is $R^n\, e^{jn\theta}$

1.2.3 If $z = Re^{j\theta}$, $z^* = Re^{-j\theta}$.

$$\begin{aligned}\text{Real}(z) &= \tfrac{1}{2}(z+z^*)\\ \text{Imag}(z) &= \frac{1}{2j}(z-z^*)\\ |z| &= \sqrt{zz^*}.\end{aligned}$$

1.2.5 *Hint:* use induction.

1.2.7 No.

1.2.9 Calculate in order:

$q = a+b$	1 addition
$r = c-d$	1 negation, 1 addition
$s = qr$	1 multiplication
$t = bc$	1 multiplication
$u = -t$	1 negation
$v = ad$	1 multiplication
$\text{Real}(wz) = s+u+v$	2 additions
$\text{Imag}(wz) = v+t$	1 addition.

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DIGITAL FILTERS

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