SPACE-CHARGE WAVES

And Slow Electromagnetic Waves

by

A. H. W. BECK, B.Sc.(Eng.), A.M.I.E.E.

Standard Telecommunications Laboratories Ltd.

PERGAMON PRESS

LONDON · NEW YORK · PARIS · LOS ANGELES

1958

PERGAMON PRESS LTD.
4 & 5 Fitzroy Square, London W.1

PERGAMON PRESS, INC.
122 East 55th Street, New York 22, N.Y.
P.O. Box 47715, Los Angeles, California

PERGAMON PRESS, S.A.R.L.
24 Rue des Écoles, Paris Ve

Library of Congress Card Number 58-10735

Set in Times New Roman by Santype Ltd., Salisbury
Printed in Great Britain by Wyman and Sons Ltd.
London, Reading and Fakenham

PREFACE

MICROWAVE valves, as we know them today, are the product of extensive research carried out in considerable secrecy during the Second World War. It is hardly surprising that many different theoretical treatments were evolved, some of fairly general application, others not. It is now realized that the most satisfactory treatment is the general one based on the application of two fundamental laws of electrodynamics, Maxwell's equations and the Lorentz force law, combined with the principle of the conservation of charge which itself is a consequence of Maxwell's equations. As our knowledge progressed it became clear that it was unnecessary to use the full apparatus of the Maxwellian approach, which in most cases gives far more knowledge than we need. Instead, one may direct attention to the interaction of two wave systems: the first, that observed on the circuit in the absence of an electron beam; the second, that on the beam in the absence of circuit waves. The purpose of this book is to examine this idea and to illustrate it by describing the operation of most known microwave valves in these terms.

The book starts with a brief qualitative description of the major types of microwave valve so as to acquaint the reader with the broad phenomena to be described. A very compressed account of Maxwellian electrodynamics is then given. Next comes the application to the solution of the circuit problem. This problem may be succinctly expressed as that of providing electromagnetic waves with phase velocities very much less than that of light and field systems which lead to the largest possible electric fields in the direction of the electron beam. The reader is then asked to master Appendix 6 which deals with the several ways in which long, heavy-current electron beams are maintained in more or less cylindrical form. The following two chapters deal with the beam waves. Chapter 4 considers the general space-charge wave problem in several geometrical arrangements and for different beam-forming means. The effects of changes of mean velocity are discussed and analogies with transmission lines and filters are drawn. Power flow theorems are derived. The general theory of plasmas is also included as it is felt that the valve engineer has something to learn from the theoretical physicist dealing with high-current gas discharges and vice versa. Chapter 5, which contains a good deal of original matter, exposes the difficulties of determining the amplitudes of the infinite series of space-charge waves which are excited in general boundary value problems and gives the solutions as far as they are known. Next come four chapters on the application of the methods in the theory of various kinds of valve. Here the more widely used devices are emphasized. The very important problem of inherent noise is dealt with in the final chapter.

The Appendices deal with some matters which seemed inappropriate in the main text, such as measurements on circuits, the theory of the maintenance of electron beams and Llewellyn's electronic equations. The remaining Appendices cover some special mathematical analyses and other material involving heavy algebra, whose absence improves the text.

In the past I have been accused of using too advanced mathematics. This is incorrect. I am an engineer and not a mathematician, as will be obvious to mathematicians, and I only use mathematics to get the results required. I should be delighted if one had only to count on one's fingers to do this, but the hard and inescapable fact is that one has to use a certain amount of mathematics. The amount is really rather limited, the main items being a knowledge of the solutions of Bessel's equation and the simpler properties of Bessel functions together with a little matrix algebra. This is no harder than, say, elementary trigonometry, and any reader who is convinced he needs the knowledge could master it in a couple of days of concentrated work. To my mind it is a symptom of one of the many things wrong with British technological education that I should need to write this paragraph. American readers should disregard these sentences.

I hope this book will be useful to several classes of reader; to research students and young physicists and engineers starting to work in the microwave tube industry, to teachers as a statement of our present level of understanding, as an indication in which directions research should go and also as yet another example of the application of Maxwellian theory. Theoretical and experimental workers in several allied fields may find useful viewpoints or may be able to throw light on the unsolved problems. Lastly, more senior valve engineers will obtain a broad picture of the field and may improve their understanding of specific devices.

The literature has been surveyed up to, roughly, the end of 1957 and, while I make no claim to have included everything, I believe that all major advances are referred to.

Many people have helped me and I offer them my sincere thanks. Among my colleagues at S.T.L. Dr. E. A. Ash has read the book and has helped me to clarify several difficult points. Mr. P. E. Deering has worked with me on the lengthy and boring computations involved in multi-mode theory. The authors and organizations who have allowed reproduction of data and illustrations are separately thanked. My wife has typed the manuscript, criticized my English, and corrected the proofs. Without her the book would not have been written.

Finally, I wish to thank the management of Standard Telecommunication Laboratories, Ltd. for permission to publish the work.

ACKNOWLEDGEMENTS

THE Author and Publishers wish to acknowledge the co-operation of the following in giving permission for the reproduction of many illustrations: *Proceedings of the Physical Society*, *Proceedings of the Institute of Radio Engineers*, *Journal of Applied Physics*, *Journal of Electronics*, *Annales de Radioelectricité*, *Proceedings of the Institution of Electrical Engineers*, Cambridge University Press and the Hughes Aircraft Co.

ERRATA

p. 21. Eq. (20) For $\frac{\partial H_x}{\partial x}$ read $\frac{\partial H_z}{\partial x}$.

p. 32. Eqs. (73) and (74). For $\left(\frac{q'_{nr}}{a}\right)$ read $\left(\frac{q'_{mr}}{a}\right)$.

p. 70. Eq. (82). For $\tan^2 \frac{I_0}{2}(I_0 + \sin I_0)$ read $\tan^2 \frac{\Phi_0}{2}(\Phi_0 + \sin \Phi_0)$

p. 101. Eq. (1). For $j\omega\mathbf{B}$ read $-j\omega\mathbf{B}$.

p. 101. Eq. (4). For $jk\omega\mathbf{E} + \mathbf{J}_1$, read $jk/c\mathbf{E} + \mu_0\mathbf{J}_1$.

p. 101. Eq. (8). Insert E_z after $(\beta^2 - k^2)$.

p. 102. Eq. (16). For $\beta_{1,2} = \pm jk$ read $\beta_{1,2} = \pm k$.

p. 107. Eq. (32). For $-j\eta$ read $-j\eta\omega I_0$.

p. 111. Eq. (45). For bsn $(\gamma_r b - \gamma_r c)$ read Bsn $(\gamma_r b - \gamma_r c)$.

p. 125. Eq. (104). For $-\cos Z_1$ read $\cos Z_1$.

p. 126. Delete minus signs in the matrix equations preceding Eq. (106).

p. 152. In Eq. (201) and in the first term of Eq. (203), for $(\omega - \gamma u_0)$ read $(\omega - \gamma u_{0q})$.

p. 167. Fig. 4. Interchange captions defining solid and dotted lines.

p. 172. Eq. (36). Invert second expression for W_n.

p. 210. Last line of page. For $(\alpha_2 M)_{opt}$ read $(S\alpha_1)M_{opt}$.

p. 294. Owing to a change of notation, late in the preparation of the book, no distinction is made between the D.C. and A.C. velocities. Denote A.C. velocities by $\tilde{u}_a$, $\tilde{u}_b$. Then, in Eqs. (6), (8), (10) and (12), for u_b, u_a read $\tilde{u}_b$, $\tilde{u}_a$. In Eqs. (7), (9) and (11) for u_a read $\tilde{u}_a$.

p. 341. Eq. (9). For W_e read W.

CONTENTS

1

GENERAL INTRODUCTION

In this book my aim is to present a unified treatment of the small-signal behaviour of microwave valves. Here, the term microwave is not to be interpreted in any pedantic sense but has its broadest meaning and comprises the frequency range from roughly 1 kMc/s to the highest frequencies which can, at present, be generated as coherent radiation. This limit, in 1956, is somewhat in excess of 100 kMc/s. I exclude from the treatment valves such as triodes and tetrodes which depend on space-charge control grids. The small-signal theory of these devices was established by the end of the nineteen thirties and their future development is now a question of improved manufacturing technique. Treatments are given of all the devices which depend on velocity-modulation of an electron beam. These include klystrons, travelling-wave tubes, backward-wave oscillators and less well-known devices such as velocity-jump and space-jump amplifiers, scalloped-beam amplifiers, resistive-wall amplifiers and space-charge wave tubes. Magnetron oscillators are not discussed because their study takes us completely outside the field of small-signal phenomena and because considerable numbers of books already exist in which magnetron performance is treated in detail, although, one is forced to add, without the complete success one would like to see. The restriction to small-signal conditions should perhaps be explained. First, this condition is not too restrictive to be useful. For example, in klystrons, small-signal conditions obtain up to electronic efficiencies of over 40% and the output efficiency of a klystron working under these conditions at the lower end of our frequency range could well be over 35%. Secondly, the study of large-signal behaviour is becoming the prerogative of the computer working with high-speed machines. While I would not wish to deny the utility of the computational approach, I feel it is a matter for regret if we abandon the search for improved theories too easily. Computational programmes only give the answers to the questions asked. They do not suggest new questions, nor do they give an indication of where the inadequacies in our understanding lie. The computer is a good servant but a bad master. Therefore I hope that a restatement, re-examination and extension of existing small-signal theories may lead to further progress with large-signal theory. Lastly, small-signal theory is relevant to all the problems which one encounters in working at the upper limit of our frequency range. All valves for these frequencies except, perhaps, some magnetrons, are small-signal devices.

Most of this introductory chapter is devoted to a brief survey of the qualitative behaviour of the various types of valve mentioned above. Before starting this survey, I think it useful to give a very short history of the development of the theory of transit-time devices. In doing this I should like to make it clear that I am writing not as a scientific historian but as a working researcher who has developed along with the subject. Others, with an equal knowledge of the facts, might legitimately place the emphasis differently.

1.1. DEVELOPMENT OF TRANSIT-TIME THEORY

In the early development of valves or vacuum tubes, if you prefer it, the velocities of electrons in free-space under the influence of electrode potentials of a few hundred volts were sufficiently large to ensure that effects due to the time of transit between one electrode and another were negligible. Owing to the rapid development of broadcasting and short-wave radio communication, the operating frequency limit of transmitters and receivers was pushed up with the consequence that it became obvious that the valves of the period exerted a severe damping effect on the tuned r.f. circuits to which they were connected. These effects were partially due to lead inductances and stray capacitances, which could be reduced by improved constructional techniques, but, by the end of the nineteen-twenties, it was realized that effects due to the finite electron velocities were also important. The earliest fairly successful attempts at a theory of these effects are due to BENHAM,[1] closely followed by MÜLLER,[2] LLEWELLYN,[3] BAKKER and DE VRIES[4] and many others. These authors start with the consideration of a system of plane-parallel electrodes, infinite in lateral extent, so that the electrons move parallel to the z axis and they include only effects due to electrostatic fields. This approach is, of course, natural by virtue of the actual structures with which they wished to work. Llewellyn, who

---

[1] BENHAM; *Phil. Mag.*, 1928, **5**, 641; 1931, **11**, 457.
[2] MÜLLER; *Hockfrequenz u. Elektroakust*, 1933, **41**, 156.
[3] LLEWELLYN; *Proc I.R.E.*, 1933, **21**, 1532; B.S.T.J., 1935, **14**, 632.
[4] BAKKER and DE VRIES; *Physica*, 1934, **1**, 1045; 1935, **2**, 683.

carried the theory to its present form, uses electromagnetic theory to establish the relation

$$\dddot{Z} = \eta \frac{I}{\epsilon_0} \qquad (1)$$

where I is a function of t, but not of z. Equation (1) can then be integrated over the path of an electron to obtain the acceleration, velocity and position of the electron in terms of the instant of observation and the time of origin. The d.c. conditions yield a value for the transit time when no a.c. fields are present and the a.c. conditions can be calculated by perturbation of the d.c. conditions. More details are given by LLEWELLYN[5] and BECK[6]. This theory, systematically applied, yields a set of equations, known today as the Llewellyn electronic equations (Appendix 12), which give precise values for the a.c. currents, voltages and velocities at any plane in terms of the (known) values at an earlier plane. Slightly modified forms of the equations are due to BAKKER and DE VRIES,[7] which have the advantage that the transit angle functions are of modulus one.

The Llewellyn electronic equations are capable of solving the small-signal problems of grid-controlled valves. They do not apply to multi-velocity electron beams, except, as is fortunately the case in practice, when the velocity range is negligible by comparison with the mean velocity. Nor do they apply when electrons are given such large velocity swings that overtaking occurs, i.e., I in Eq. (1) has to be considered as a single-valued function of z and t. The equations could be elaborated to cover cylindrical diodes and triodes but the extension would be mathematically cumbersome and has not been done.

Meanwhile, electron devices which did not depend on space-charge control were worked on. The first of these was the magnetron, invented by Hull in 1921. The second was the Barkhausen-Kurz tube in which oscillations at high frequencies were generated in a cylindrical triode operated with positive grid and negative anode. The theory of these valves, at least in the structural forms in which they first appeared, is complicated and it is hardly surprising that only a rather incomplete picture of the details of their operation was obtained. A book by GROOS[8] may be taken as a good

---

[5] LLEWELLYN; *Electron Inertia Effects*, Cambridge University Press, 1941.
[6] BECK; *Thermionic Valves*, Cambridge University Press, 1953, Section 7.4.
[7] BAKKER and DE VRIES; *loc. cit.*
[8] GROOS; *Theorie und Technik der Dezimeterwellen*, S. Hirzel, Leipzig, 1938.

account of the state of knowledge in this field in the middle and late thirties.

The break-through really began with the invention by the HEILS'[9] of the generator which bears their name. This device is a genuine velocity-modulation tube but in its original form did not use a cavity resonator. The Heils' paper did not give much in the way of a theory of the device, but work was going on at the same time in several American laboratories and resulted in the almost simultaneous publication of papers describing the klystron (VARIAN and VARIAN[10]), the velocity-modulated tubes of HAHN and METCALF[11] and the inductive output amplifier of HAEFF.[12] The theory of the klystron was treated by ballistic methods by WEBSTER[13] and the theory which we shall use and develop very extensively in this book was given by HAHN[14] and RAMO.[14]

During the war klystrons, both of the straight-through and reflex type, were extensively developed in all the combatant countries. Large numbers of theoretical studies were made, mainly based on the extension and improvement of the Webster theory. Almost all workers, including myself, were relatively insensitive to the space-charge wave theory of Hahn and Ramo. There were probably several reasons for this, among which are the following: first, the treatment given by Hahn was formidable in the extreme, it included relativistic effects and was very general in nature whereas Webster's treatment was very simple and his physical model was easy to understand. Secondly, the forms of valve developed were similar to those of the Varian–Webster–Hansen group and differed greatly from those of Hahn and Metcalf, so that there was a tendency to regard Webster's theory as the only theory which applied to this design of valve. Thirdly, the conditions of excitation of the electron beam are not made very clear in the Hahn theory, while gap theory was very extensively developed and easily taken into the ballistic theory. Lastly, the Webster theory is valid for electron beams which are short in comparison with one quarter space-charge wavelength. This condition was obeyed for all the wartime valves, which used electrostatic focusing and short electron beams. The Webster theory therefore agreed with measurements at least well enough to serve as a valuable aid to design. Several books and major

---

[9] HEIL and HEIL; *Z. Phys.*, 1935, **95**, 752.
[10] VARIAN and VARIAN; *J. Appl. Phys.*, 1939, **10**, 321.
[11] HAHN and METCALF; *Proc. I.R.E.*, 1939, **27**, 106.
[12] HAEFF; *Electronics*, February 1939, 30.
[13] WEBSTER; *J. Appl. Phys.*, 1939, **10**, 501.
[14] HAHN; *Gen. Elect. Rev.*, 1939, **42**, 258; 497. RAMO; *Phys. Rev.*, 1939, **56**, 276.

articles[15–18] describe the state of knowledge at the end of the war and none makes other than cursory mention of the Hahn–Ramo theory. However, towards the end of the war KOMPFNER[19] invented the travelling-wave tube and PIERCE[20] gave an initial theory which has been widely accepted. This theory is fundamentally a ballistic theory and has to be supplemented by space-charge wave theory when the beam current exceeds very moderate values. Moreover, the practical work which went into the design of long, magnetically focused electron beams was applied to high-gain high-powered klystron amplifiers. These valves no longer behaved according to Webster's theory unless extensive approximate corrections were introduced. It was gradually realised that these corrections simply amounted to the insertion of results of space-charge wave theory and the present state of affairs may be expressed as a situation in which gap theory, developed on ballistic lines or on the Llewellyn electronic equations, is wedded to space-charge wave theory in an acceptable way. This book is an attempt to systematize the theory of several different types of valve along these lines.

To conclude this historical sketch, it should be pointed out that newer tubes, such as velocity-jump amplifiers and space-jump amplifiers, have been invented which depend entirely on space-charge wave ideas for their functioning.

We now describe the physical principles of some of the valve types mentioned above.

1.2. THE KLYSTRON

Klystrons have been described in a great deal of detail in the references cited. A two-cavity klystron amplifier consists of an electron beam which traverses an input resonant cavity (buncher), a space free of a.c. fields (drift space) and an output cavity (catcher). The beam is finally collected on a heat-dissipating electrode (collector). In ballistic terminology the operation of the device is as follows: suppose that an a.c. voltage is imposed on the terminals

15 HARRISON; *Klystron Tubes*, McGraw-Hill, 1947.

16 HAMILTON, KUPER and KNIPP; *Klystrons and Microwave Triodes*. M.I.T. Series V.7, McGraw-Hill, 1948.

17 BECK; *Velocity Modulated Thermionic Tubes*, Cambridge University Press, 1948

18 PIERCE and SHEPHERD; *Bell Syst. Tech. J.*, 1947, 26, 460.

19 KOMPFNER; *Proc. I.R.E.*, 1947, 35, 124.

20 PIERCE; *Proc. I.R.E.*, 1947, 35, 111.

of the buncher gap, electrons passing the gap in one half-cycle are accelerated, those in the other decelerated. The emergent beam is said to be velocity modulated. In the drift tube, accelerated electrons catch up with those decelerated in the preceding half-cycle so that there is an increase in electron density at the position in space of a reference electron which passed the gap centre at the instant when the field changed from negative (decelerating) to positive. As the beam progresses down the drift tube the local increase in density round the reference electron becomes more pronounced and a bunch is formed. When the bunch passes the catcher gap it induces an opposing electric field in the catcher resonator which slows all the electrons in the bunch, thereby converting battery energy into a.c. energy. The tighter the bunch and the fewer electrons left in unfavourable phases, the more efficient the energy conversion becomes.

This picture, which is that of the simplest Webster theory, is incomplete. As the beam becomes bunched in the drift space, retarding forces due to the repulsion forces between the electrons are set up. Immediately the beam is disturbed from its uniform state the repulsion forces start to grow and to reduce the change in velocity due to the modulation process. The new situation has to be described by space-charge wave theory. If we picture an electron beam and imagine that all the electrons are given the same initial a.c. velocity at a specified plane, electromagnetic theory can be used to show that space-charge waves are set up on the beam, in much the same way that waves are set up on a dielectric rod. We may thus picture the beam as a conductor on which the waves propagate. The space-charge waves arise in pairs, one wave of each pair having a phase velocity slightly greater than the electron velocity, the other slightly less. The difference depends on the plasma frequency of the electron beam and on the geometry of the system. The plasma frequency is defined through

$$\omega_p^2 = \frac{\eta I_0}{\epsilon_0 u_0 \Sigma} \tag{2}$$

and the propagation constants of the space-charge waves are

$$\gamma_1 = \frac{\omega + F\omega_p}{u_0}, \qquad \gamma_2 = \frac{\omega - F\omega_p}{u_0} \tag{3}$$

where F is the factor depending on the geometry. The actual determination of F is detailed in Chapter 4. As we shall see, practical systems for exciting space-charge waves on beams require that the initial conditions be fulfilled by infinite series of pairs of space-charge waves, each pair with a different value of F, but for the moment we consider a single pair, for simplicity. Then, the initial condition in our klystron is that the beam is suddenly given an initial velocity modulation at the catcher gap centre while the a.c. beam current is zero at the same plane. The explicit relations giving the travelling wave forms of the a.c. velocity and the a.c. current waves are then written down and superposed to give the initial conditions $u_1 = u_{10}$, $i_1 = 0$ at $z = 0$. This gives standing wave expressions for a.c. velocity and current of the form

$$u_1 = u_{10} \cos \frac{F\omega_p}{u_0} z \exp j\left(\omega t - \frac{\omega z}{u_0}\right) \tag{4}$$

$$i_1 = j \frac{I_0 u_{10}}{u_0} \cdot \frac{\omega}{F\omega_p} \sin \frac{F\omega_p}{u_0} z \exp j\left(\omega t - \frac{\omega z}{u_0}\right) \tag{5}$$

Equation (5) allows us to determine i_1 at any distance from the buncher and therefore to calculate the voltage and power induced into a catcher at such a point. In particular i_1 as a function of z reaches the maximum value at $\sin (F\omega_p/u_0)z = 1$.

$$z = \frac{\pi}{2} \cdot \frac{u_0}{F\omega_p} \tag{6}$$

The space-charge wavelength is $2\pi u_0/F\omega_p$, so Eq. (6) tells us that the current reaches its first maximum at one-quarter space-charge wavelength distance from the buncher gap, and there is nothing to be gained by making z greater than this value. This is directly contrary to simple ballistic theory which states that transconductance varies linearly with z. Moreover, at the plane of maximum current the a.c. velocity $u_1 = 0$, so that all the electrons instantaneously have their original d.c. velocity u_0. This fact actually greatly simplifies the detailed calculation of the output power.

It should be noted that the present theory tells us nothing about the maximum value which can be given to u_{10}, which we require to know before we can calculate the maximum power output. This is a consequence of the fact that the theory is linearized or small-signal. The ballistic theory, however, does give an answer to this question and one of our problems is how to make best use of these answers to supplement space-charge wave theory.