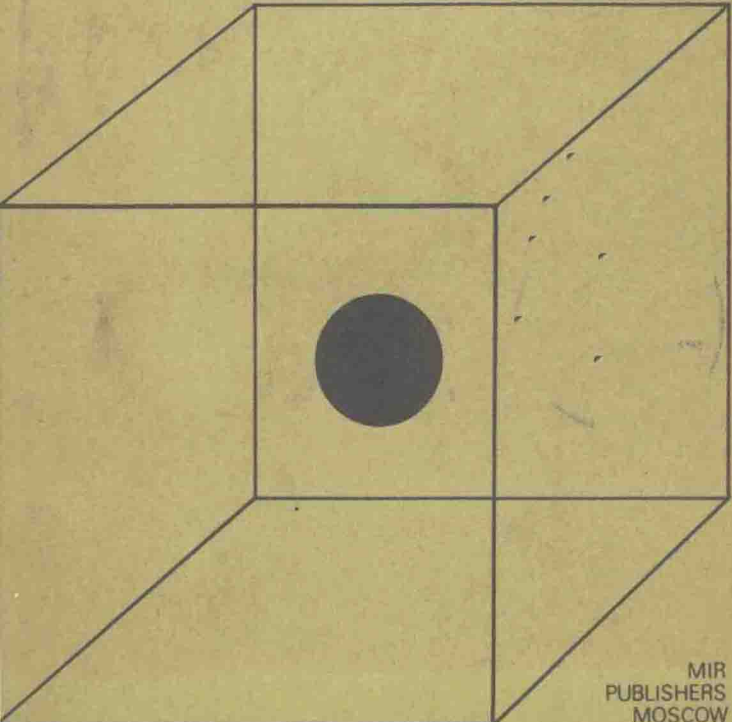
M. I. Kaganov and I. M. Lifshits

QUASI-PARTICLES



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КВАЗИЧАСТИЦЫ

ИДЕИ И ПРИНЦИПЫ КВАНТОВОЙ ФИЗИКИ ТВЕРДОГО ТЕЛА

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QUASIPARTICLES

Ideas and principles of solid state quantum physics

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FOREWORD

A number of approaches can be chosen to describe the state-of-the-arts in a sufficiently mature field of science. The choice lies with the author, and reflects his tastes, habits, and experience. This book is our attempt to present the basic concepts (or those we regard as basic) of the quantum theory of solid state, paying maximum attention to answering the question that we selected as a heading for the first chapter, viz. "What Are the Components of ..?". A favourite image the authors had invariably kept in mind was: an inquisitive boy is dismantling a toy car trying to understand what its parts are; he holds the car's skeleton in his hand, and bolts and wheels are in a pile on the floor. It was not, however, our intent to reassemble the "car" after its structure has been analyzed, and to give a detailed account of its functioning. The reader, we assume, had met with the "functioning" of solid state devices more than once; the properties of such devices make the subject of many an excellent book.

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What are the components of ..?

There exist certain concepts in science that are not easy to formulate. On their face value they are frequently accepted as obvious, although they are very profound. Careful analysis shows that these concepts embody the experience accumulated during many centuries of careful observation of nature. The statement that all material objects surrounding us possess certain structure, that we may ask "What are the components of ...?" about anything that we perceive by our senses or by means of the most sophisticated instruments, is one of such concepts. Moreover, we can expect a non-trivial, substantive answer: the object consists of molecules of atoms, ions or nucleons, photons or neutrons. In the present-day paradigm the subdivision of matter into molecules, atoms, nuclei, electrons, protons and neutrons is regarded as a truism, i. e. a self-evident truth, requiring no special proof, despite the fact that less than a century separates us from the experimental discovery of atoms and molecules. Still, we shall risk forcing an open door and take up again the question "What are the components of ...?", attempting to specify its meaning. It is known beyond doubt that all materials consist of molecules, molecules of atoms, atoms of electrons and nuclei, and nuclei of protons and neutrons; why then are we not surprised to hear that a molecule of common salt consists of sodium and chlorine *ions*, a diamond crystal consists of carbon *atoms*, and a protein macromolecule of *amino* *acids*? Our intuition tells us that he who asks "What are the components of ...?" means decomposition into components by applying minimum force. In order to decompose common salt into ions, it is sufficient to dissolve it in water which, owing to its high polarizability, decreases coulomb attractive forces between ions so that the crystal, being a giant molecule, breaks into its components ($NaCl \leftrightarrows Na^+ + Cl^-$). Much greater forces are required to tear the electrons away from sodium or chlorine ions, roughly several electron volts (ev) per each electron. Energy a million times greater (Mev) would be needed to separate a nucleon (a proton or a neutron) from a nucleus.

Therefore, a structural unit isn't something quite unambiguous, quite definite. It essentially depends on the depth of penetration into the structure. No doubt, the discovery of quarks, those hypothetical particles making up the nucleons, would be one of the fundamental discoveries of the century; it shouldn't, however, strike the imagination of the contemporaries quite so much since it is preconditioned by the whole history of prior development of physics, which with each new discovery pushes deeper into the heart of matter its elementary essence or fundamental basis, those famous "building blocks" of which everything is built.

The meaning of such concepts as elementary particle, structural unit, and so forth, is undoubtedly conditional, and characterizes either the level to which physics has progressed or the set of means (forces) applied to single out the structural unit. However, these concepts also reflect certain objective content, permitting clear quantitative evaluation. A *molecule* can be regarded as an elementary particle in those phenomena in which it acts as a *whole*, i. e. undergoes no impacts capable of decomposing it. The bonding energy of atoms in the molecule being known, we can strictly delineate the range of phenomena together with the values of parameters that describe them, in which the molecule is an elementary particle. Furthermore, it is always possible to specify the accuracy of the concept of an "elementary particle". For instance, at any temperature a molecular gas contains "fragments" of molecules, i. e. atoms and ions. The higher the temperature, the higher their concentration, and vice versa. The concentration of these "fragments" is a measure of accuracy of the concept of "elementary" particle with respect to a molecule in the gas.

This example demonstrates that although the choice of components is not unambiguous, it is unambiguously dictated by the specific physical conditions. One important remark: even in identical conditions one and the same object has to be considered consisting of one sort of elementary particles in experiments of one type, and of a different sort of particles in a different type of experiment. Let us take again the simple example of a molecular gas at not too high temperatures. Investigation of specific heat of the molecular gas shows that sufficiently high accuracy is obtained if the gas is assumed to consist of molecules only. A small admixture of ions can be simply neglected. In investigations of electric conductivity of the same molecular gas, however, we have to admit that the current in this molecular gas "consists" of ions. *When electric conductivity* of a molecular gas *is considered, the elementary particles are indeed the ions.* In such experiments neutral molecules play the role of a background. This example shows what exactly we mean by the concepts of "elementary particle", "consists of", etc.

In the case of specific heat investigation, gas molecules can be considered elementary particles because, firstly, gas energy E is, to a high degree of accuracy, the sum of energies of individual molecules; secondly, internal motions in a molecule can be neglected when energy of an individual molecule is calculated *.

The current must be considered consisting of ions because only ions transport charge in a molecular gas to which electric field is applied. Hence, the total current passing through the gas is the sum of elementary currents of individual ions (the elementary current of an ion is equal to its charge times its velocity).

Therefore, the concept of "elementary particle" essentially includes the *additivity* of something consisting of the introduced elementary particles, or portions.

The requirement of additivity of what a body consists of is demonstrated especially clearly for the concept of mass, if we take up the concept of mass defect, which is a corollary of Einstein's relationship $E = mc^2$ (E is energy, m is mass, and c is velocity of light) stating that diminishing energy of a body makes it lighter. Stability of a crystalline phase signifies that the energy of the crystal is lower than the sum of energies of the separated molecules, i. e. molecules removed to infinitely large distance from one another. As an example, let us consider a crystal of solid hydrogen. Its mass is equal to the sum of masses of molecules, to the accuracy of mass defect. The mass defect in this case is very small, about 10^{-37} g per one molecule. When a hydrogen molecule is "decomposed" into two atoms ($H_2 \leftrightarrows H + H$), mass defect is already about $5 \cdot 10^{-34}$ g, and decomposition of a hydrogen atom into a proton and an electron ($H \leftrightarrows p + e$) results in mass defect of $3 \cdot 10^{-33}$ g. This sequence of figures (10^{-37}, $5 \cdot 10^{-34}$, $3 \cdot 10^{-33}$) points

* The possibility of neglecting internal motions in certain conditions is a corollary of quantization of energy, namely of the fact that the energy of the first excited state of the molecule is substantially greater than kT (T is gas temperature and k is the Boltzmann constant).

to the accuracy of the statement "Solid hydrogen consists of ...". It is quite natural to consider the statement "Solid hydrogen consists of hydrogen molecules H_2" as the most exact. But we must emphasize that this statement became rigorously defined only because we have analyzed one definite property of the crystal, viz. its mass. If, however, we analyzed the same statement from the "standpoint" of energy, the statement would be devoid of meaning. The energy of a crystal, E_{cr}, cannot be expressed as a sum of energies of the separate non-interacting molecules, since the energy of intermolecular interactions is far from being small in comparison with kinetic energy of the molecules.

Photons

Although quantum mechanics demonstrated that waves and particles (corpuscles) are not separated by a seemingly unbridgable gap, that unsurmountable difference that was postulated by classical physics, still it is clear that electromagnetic waves and particles are qualitatively different entities. In fact, to-day we still assume without any modification the conclusion of the classical pre-quantum physics that world consists of particles and electromagnetic fields, or waves.

Quantum mechanics developed and made more profound the concepts of "wave" and "particle" by demonstrating that particles possess wave properties and waves possess corpuscle properties; it did not nullify, however, the difference between the two forms of existence of the matter (namely, field and corpuscle forms).

The space between particles, of which the material objects consist, is thus filled with electromagnetic field; this field is time-dependent, absorbed by particles and generated by the same particles. Is then the question

"What are the components of the electromagnetic field?" justified and acceptable? And if it is, what should the answer be?

It is convenient to expand the electromagnetic field into *plane monochromatic waves*, i. e. present the intensity of electric and magnetic field in any point of space at any moment of time by a sum of plane monochromatic waves with definite frequency ω and wavelength λ. Electrodynamics equations lead to a conclusion that $\omega = 2\pi c/\lambda$, where c is the velocity of light. The procedure of expansion is mathematically rigorous. Electromagnetic field in vacuum between the bodies can be expressed exactly as the superposition (sum) of plane monochromatic waves. Note that the energy of the field is then the sum of energies of the constituent plane waves. This statement is equally applied to the energy flux. Therefore, by considering a plane monochromatic wave as an elementary wave entity, we can state that any electromagnetic field consists of plane monochromatic waves.

Corpuscular properties of electromagnetic waves are revealed in the existence of minimum portions (quanta) of electromagnetic energy. A quantum of energy, ε, is proportional to wave frequency, and the proportionality factor is the famous Planck constant $\hbar \approx 10^{-27}$ erg·s. The energy E_ω of an electromagnetic wave with frequency ω is equal to an integer number of quanta $\hbar\omega$, i. e. $E_\omega = n\hbar\omega$, where $n = 1, 2, \ldots$ is an arbitrary integer. A quantum of electromagnetic energy possesses all the attributes of a quantum particle; for instance, its momentum is equal to $2\pi\hbar/\lambda$. The relationship between the energy and momentum of a quantum is especially simple: energy is proportional to momentum, $\varepsilon = cp$. We remind the reader that for an ordinary (classical) particle energy is proportional to the square of momentum: $\varepsilon = p^2/2m$, where m is the particle's mass.

Hence, we developed a corpuscular picture of the electromagnetic field. It consists of special corpuscles, viz. quanta, which were given the name *photons*. To describe the state of the electromagnetic field in, say, a resonator, means to state what types of photons are present and the number of photons of each type.

Photons are not merely a convenient method of describing the electromagnetic field. A large number of experimental facts cannot be interpreted at all if corpuscular concepts are neglected. Such is the photoelectric effect (photoeffect) in which an electron absorbs a photon, and the electrons's energy after absorption depends on the frequency of the incident light. Or the Compton scattering of light, i. e. inelastic scattering of photons by electrons. A scattered photon transfers part of its energy to an electron, which changes the frequency of the photon. In order to emphasize the corpuscular nature of the phenomenon, we can state this in a different manner: light scattering by electrons consists in absorption by an electron of a photon with energy $\hbar\omega$, and generation of a photon with energy $\hbar\omega'$; obviously, $\omega' < \omega$. The energy difference $\hbar(\omega' - \omega)$ is absorbed by the electron, i. e. recoil is observed. A photon and an electron collide as two balls with, naturally, very different masses. It is these and other similar facts that led to the formation of the concept of photons.

It may seem that the main difference between a photon and an ordinary particle lies in the dependence of energy on momentum. This notion is readily demonstrated to be wrong. The exact relativistic formula which relates energy of a particle to its momentum is $E = \sqrt{m^2c^4 + c^2p^2}$. We see that the dependence of energy on momentum for ordinary particles and photons is almost identical for very high momenta,

i. e. for $p \gg mc$ (ultrarelativistic case). Moreover, there exist particles whose properties are such that we can regard them as "ordinary", although their mass, as that of photons, is zero, and the dispersion law completely coincides with that for photons. These particles are neutrinos. More: photons in a waveguide are governed by a dispersion law coinciding with that for an ordinary relativistic particle: $\hbar\omega = \sqrt{\varepsilon_0 + c^2p^2}$, where p is the photon's momentum along the waveguide, and ε_0 is the rest energy of the photon. It is inversely proportional to squared radius of the waveguide. We can even introduce the concept of the photon's heavy mass $m^* = \beta\frac{\hbar^2}{R^2c^2}$ (β is numerical factor).

Bosons and fermions

A large number of various elementary particles are known at present. They are elementary in the sense that almost no information is available about their structure. One particle differs from a particle of a different type not only in mass, but in the whole set of properties, such as the possibility to participate in certain reactions, life time, etc.

We want to emphasize here only one of the properties of elementary particles, namely their spin, a very specific property of microparticles. In an attempt to form a classical analogy of the concept of "spin", one can imagine the particle as a submicroscopic rotating top. In accordance with their spin, all particles can be subsumed into two classes. Those particles in the first class are called *fermions*, and those in the second class are called *bosons*. A particle's spin is given by $\hbar\sqrt{s(s+1)}$, where s is either 0, or 1/2, or 1, or 3/2, etc. If $s = 0$ or 1, or 2, etc., the particle is classified as a boson, and if $s = 1/2$, or 3/2, etc., then as a fermion. Electrons, protons, neutrons, neutrinos are fermions, while photons and π-mesons are bosons.

Spin is an individual property, i. e. a property of each *individual* particle, but at the same time it determines the particles' behaviour in an *ensemble* (*collective*). For instance, no two fermions can be found in the same quantum state. This statement is referred to as the Pauli exclusion principle. In essence, this principle explains the great variety of atomic structures. Were it not for the Pauli principle, all the electrons in any atom would assemble in the lowest, so-called ground, state.

Bosons, on the contrary, tend to accumulate in one state. As for generation ("birth") of a new particle in a given state, a fermion can be generated only if the state is vacant, while the probability of boson generation is the higher the greater the number of particles in this state. Fermions are, so to speak, individualists, and bosons are collectivists.

It must be underlined that the effect of spin on behaviour of a number of particles cannot be reduced to any *force* interaction between them, and therefore no classical analog of this concept can be found.

Quantum statistics

Realization that atomic structure is inherent to matter gave birth to a complex of sciences united by the idea of interpreting the macroscopic properties on the basis of known laws governing the motion of individual particles, i. e. to statistical physics. The picture that appears is most clear in the case of gases where interactions between particles are comparatively weak; hence, the properties of gases can be completely described in terms that deal exclusively with the behaviour of an individual particle. Studying the properties of a gas, one can often assume that particles do not interact at all. A gas consisting of noninteracting particles is said to be *ideal*. It should be kept in mind, though, that interaction between particles is the sole reason for equilibrium onset in the gas. The smaller this interaction, the longer it takes for the equilibrium to set in. If, however, we consider equilibrium state, interaction can simply be ignored, provided it is small. It is important to remember that in gases all macroscopic characteristics (mass, energy, pressure, etc.) are, to a good approximation, additive. In essence, additivity is a necessary condition for *synthesizing*, i. e. for *deriving the macroscopic properties on the basis of studying the properties of individual particles*; indeed, the very concept of an "individual particle" includes certain independence, even if it be approximate, of the remaining particles of the material object. In this sense, ideal gas is one of the most important objects of investigation of the statistical physics.

Let us underline once again: *individual properties of particles include those properties which determine their behaviour in the interactions with other particles*. These properties are usually divided into those responsible for force interactions and are called *charges*, and those not related to force interaction and called *spins*. For example, a proton possesses, in addition to electric charge, both barionic charge and the charge describing the so-called weak interaction, i. e. various types of β-decay, while a neutron possesses no electric charge but has the other two charges. In addition to the ordinary spin, both protons and neutrons are characterized by the isotopic spin, which is $+1/2$ for protons and $-1/2$ for neutrons. The spin, as the charge, is an