The Concept of Probability

Fundamental Theories of Physics

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Proceedings of the Delphi Conference, October 1987, Delphi, Greece

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Fundamental Theories of Physics

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PREFACE

This volume contains articles from invited speakers at a meeting which took place in Delphi, during the week of October 12-16, 1987. The theme of the meeting was "The concept of probability" and was organized by the "Group of Interdisciplinary Research" (Physics Department, University of Athens) and the Theoretical and Physical Chemistry Institute of the National Hellenic Research Foundation, Athens. (The Group of Interdisciplinary Research organized two previous Meetings, 1) on the Concept of physical reality (1982) and 2) on the question of determinism in Physics (1984)).

This small gathering, which was attended by scientists, mathematicians and philosophers from more than 22 countries, took place on the occasion of the 100th year from the birthday of E.Schrödinger.

As the father of wave-mechanics, Schrödinger thrushed us into an era of physics where knowledge of the Ψ-function is considered, for most situations, as the ultimate aim and the ultimate truth. Yet, he, as well as another towering figure of 20th century physics, A.Einstein, never really felt confortable with the interpretation of the meaning of Ψ and of the information that it contains. With Einstein playing the leading role a debate about concepts and interpretation started as soon as quantum mechanics was born. Central theme to this debate is the concept of probability, a concept which permeates- explicitly or implicitly- all science and even our decision making in everyday life.

The articles cover a broad spectrum of thought and results - mathematical, physical, epistemological, experimental, specific, general,- many of them outside the accepted norm. Regardless of their degree of validity, we hope that the mosaic of ideas, information, arguments and proposals which they contain, will prove to be a useful and timely addition to the existing literature on probability and its physical implications.

It is our pleasure to thank the speakers and all the participants for their contribution to a succesful and pleasant meeting. We are grateful for the financial support from the Greek Ministries of Culture and of Science and Technology, as well as from the National Hellenic Research Foundation and the International Center at Delphi.

Finally, we are thankful to Professor Thomas Brody for his invaluable help in the preparation of this volume as well as to Dr.G.Roussopoulos for his help in the organization of the conference.

Athens, September 1988

E.I.Bitsakis

C.A.Nicolaides

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Part 1

Following Schrödinger's Thoughts

From the book "LETTERS ON WAVE MECHANICS" ed.K.Przibram for the Austrian Academy of Sciences(1963) . Translated by M.J.Klein, Phil.Library, N.Y. (1967)

Innsbruck, Innrain 55
18 November 1950

Dear Einstein,

It seems to me that the concept of probability is terribly mishandled these days. Probability surely has as its substance a statement as to whether something is or is not the case- an uncertain statement, to be sure. Bur nevertheless it has meaning only if one is indeed convinced that the something in question quite definitely either is or is not the case. A probabilistic assertion presupposes the full reality of its subject. No reasonable person would express a conjecture as to whether Caesar rolled a five with his dice at the Rubicon. But the quantum mechanics people sometimes act as if probabilistic statements were to be applied just to events whose reality is vague.

The conception of a world that really exists is based on there being a far-reaching common experience of many individuals, in fact of all individuals who come into the same or a similar situation with respect to the object concerned. Perhaps instead of "common experience" one should say "experiences that can be transformed into each other in a simple way". This proper basis of reality is set aside as trivial by the positivists when they always want to speak only in the form: if "I" make a measurement then "I" "find" this or that. (And that is to be the only reality).

It seems to me that what I call the construction of an external world that really exists is identical with what you call the describability of the individual situation that occurs only once-different as the phrasing may be. For it is just because they prohibit our asking what really "is", that is, which state of affairs really occurs in the indi-

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vidual case, that the positivistis succeed in making us settle for a king of collective description. They accuse us of metaphysical heresy if we want to adhere to this "reality". That should be countered by saying that the metaphysical significance of this reality does not matter to us at all. It comes about for us as, so to speak, the intersection pattern of the determinations of many-indeed of all conceivable-individual observers. It is a condensation of their findings for economy of thought, which would fall appart without any connections if we wanted to give up this mode of thought before we have found an equivalent that at least yields the same thing. The present quantum mechanics supplies no equivalent. It is not conscious of the problem at al; it passes it by with blithe disinterest.

It is probably justified in requiring a transformation of the image of the real world as it has been constructed in the last 300 years, since the re-awakening of physics, based on the discovery of Galileo and Newton that bodies determine each other's accelerations. That was taken into account in that we interpreted the velocity as well as the position as instantaneous properties of anything real. That worked for a while. And now it seems to work no longer. one must therefore go back 300 years and reflect on how one could have proceeded differently at that time, and how the whole subsequent development would then be modified. No wonder that puts us into boundless confusion!
Warmest regards!

Yours

E.Schrödinger

SCHRÖDINGER'S THOUGHTS ON PERFECT KNOWLEDGE

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What is one is one
What is not one is also one

Chuang Tsu

ABSTRACT. Perfect knowledge of the many-body system is contained in the wavefunction but, as Schrödinger has already emphasized, best possible knowledge of a whole does not include the best possible knowledge of its parts. Separability from the point of view of quantum mechanics is discussed. General "entangled" systems are analysed in terms of knowledge. If the state vector is defined for a whole system its parts are described only by mixed density operators. Correlations violating Bell's inequality are necessary to avoid superluminal signaling and result from the lack of independent reality of subsystems. Model calculations on two separated atoms and on non-interacting gas show that the perfect knowledge of the whole system or the total wavefunction is not sufficient to calculate local properties without actually solving the local problem.

1. INTRODUCTION

The history of modern civilization spans roughly about 5000 years. Civilization is clearly in its infancy because a single lifetime may still span as much as 2% of its history. Only a few men have been privileged to contribute so much to the development of human thought as Erwin Schrödinger, whose centenary of birth we are now celebrating. He has introduced the concept of wavefunction to physics and never stopped bothering himself with its meaning. Quantum Mechanics (QM) is a strange theory already at the level of single-particle phenomena but, accepting the wave-particle dualism in one form or another [1], one can still form a reasonable picture of reality. It is the holistic nature of QM at the multi-particle level or, technically speaking, the existence of multi-particle nonfactorizable states, that is more bothering. A few months after the famous Einstein, Podolsky and Rosen (EPR) paper [2] appeared Schrödinger made a profound analysis of the problem [3]. He summarized the conclusion

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in one sentence: "Best possible knowledge of a whole does <u>not</u> include best possible knowledge of its parts - and that is what keeps coming back to haunt us".

It is my feeling that we have just started to explore the consequences of this statement. The present paper is an attempt to look at it from modern perspective. In the second section separability in QM is discussed. Next, two sections deal with the spatially extended quantum states, EPR paradox and Bell inequality. In section 5 general "entangled systems" are considered. Finally Schrödinger's statement about the knowledge of a whole is exemplified in section 6 on a model of non-interacting gas.

2. SEPARABILITY IN QUANTUM MECHANICS

The question "When can one consider two physical systems to be separate?" is rather subtle. Naively, if I could isolate one system and perform experiments on this system without influence from the second system I would call it "a separate system". From the QM point of view such a definition would be naive indeed: how can one be sure that the results of experiments are really not determined by what happens to the other system? Maybe the wavefunction of my system was changed by someone experimenting with the second system? The only way to know is by computing correlations between the results of measurements on the two systems and checking if these correlations are trivial, i.e. if they can be explained assuming that the systems are independent. I will come to this point in the next section.

Leaving aside the subtleties concerned with isolation of physical systems let us consider the question of separation. The common knowledge is that "when two systems interact, their ψ - functions... consist, to mention this briefly, at first simply of the product of the two individual functions...", as Schrödinger [3] has put it. Quite recently Rosen [4], discussing separability, made a similar statement. Is it really true?

There is no smooth connection between distinguishability and indistinguishability, and thus between the simple product of two functions and a total function with a definite permutational symmetry. If there was, is, or will be a possibility of an interaction the product function is a false start. No matter how small, the interaction makes the total Hamiltonian symmetric and forces a definite permutational symmetry on the total wave function. By setting the interaction to zero one switches off the possibility of any interaction in the future. But, to rephrase the famous remark of Pauli [5], "Was Gott vereint hat, soll der Mensch nicht trennen" (what God has united men should not separate). We cannot start with the product function without "playing God's role".

<u>Formal proof</u>: consider two systems, S_A and S_B, with N_A and N_B particles, respectively. Each system is described by its own function, χ_A antisymmetric in N_A particles and χ_B in N_B . It is easy to show that the product function $\chi=\chi_A\chi_B$ is always "far" from the antisymmetric function $\psi=\hat{A}\chi$ looking at the norm $\|\chi-\psi\|$. If χ and ψ are normalized then the antisymmetrizer $\hat{A}$ is

$$(1) \qquad \hat{A} = (N_A!N_B!/N!)^{½} \sum_P \epsilon(P)P$$

where $N = N_A + N_B$ and P is either identity or permutes particles of S_A with those of S_B. Therefore

$$(2) \qquad ||\chi-\psi||^2 = \langle\chi-\hat{A}\chi|\chi-\hat{A}\chi\rangle = 2-2\mathrm{Re}\langle\chi|\hat{A}\chi\rangle \geq 2-2^{½}$$

because the integral $\langle\chi|\hat{A}\chi\rangle = (N_A!N_B!/N!)^{½}$ is not smaller than $2^{-½}$.

Quantum mechanics is thus unable to describe two separated systems. The minority attitude towards this fact is to beat the drum: "quantum mechanics is dead", as Piron does [6]. His criticism is based primarily on his own quantum logic approach and on the axiomatic approach of Aerts [7], who emphasized many times that "it is impossible in quantum mechanics to describe two separated physical systems". Aerts presented a theory of the quantum-logic type that allows for such a description. The majority adopts a "so what?" attitude. As long as QM calculations agree with experimental results everything is O.K. Axiomatic approaches are not important because they do not give numbers. I would like to adopt here the Middle Way: with the "so what" attitude we may easily overlook things that are important but we never dreamed of. On the other hand it is too soon to mourn over the death of quantum mechanics before we will find predictions in direct conflict with experimental results. After all, separation may be just one of these illusions acquired in the childhood. It is simply impossible to measure correlations among more than a few bodies but concentrating on just two particles correlations proving their nonseparability should be seen. Let us look closer at the consequences of quantum mechanics.

3. SPATIALLY EXTENDED QUANTUM STATES

Suppose that two spatially separated particles are in a pure quantum state. There are basically two ways in which such a state could be prepared. First, by breaking one system into fragments, for example in the two-photon cascade emission or photodissociation of H_2 into H + H in a triplet state, spatially extended systems in pure states are obtained. A second possibility, peculiar to QM, is to prepare the two systems in such a way that in future they will fit together into a single system in a definite way. As an example one may think of a particle or an odd-electron atom with total spin $S = ½$ crossing a magnetic field that separates $M_s = +½$ states. If one has two atoms in $|S=½, M_s=+½\rangle$ states coming from the opposite directions even before they start to interact they must be in $|S=1, M_s=1\rangle$ combined state. It is not a question of interaction at an earlier time but of the wave functions that we are allowed to write. The first possibility of forming a spatially separated quantum state is due to the common past and the second due to the common future [8].

Are there observable consequences of the existence of such quantum states? Obvious consequences were noted by Einstein, Podolsky and Rosen [2] and by Schrödinger [3] already in 1935. There are also less obvious consequences noted by Bell [9] in 1965. To see the obvious consequences let us write the state of the two systems as

(3) $|\psi\rangle = 2^{-1/2}(|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle)$

where $|a_k\rangle$ ($|b_k\rangle$) are states of the system S_A (S_B). If a measurement on S_A finds the system in the state $|a_k\rangle$ then S_B should be in $|b_k\rangle$, so a measurement on one system selects the state of the other system! This is the essence of the EPR paradox, a feature of quantum mechanics entirely unacceptable to Einstein [10]. This is also the reason why the definition of separated systems given previously is naive. It is tempting to assume that the state $|\psi\rangle$ describes an ensemble of systems, half of them in the $|a_1\rangle|b_1\rangle$ and half in $|a_2\rangle|b_2\rangle$ state. Einstein himself has advocated such interpretation. Unfortunately this would remove all interference and exchange effects. Superposition of states is not just the question of our knowledge but of potential reality.
Formal argument against ensemble interpretation of $|\psi\rangle$ is the following: density operator ρ_m corresponding to the two ensembles $|a_1\rangle|b_1\rangle$ and $|a_2\rangle|b_2\rangle$ is:

(4) $\rho_m = \frac{1}{2}(|a_1\rangle|b_1\rangle\langle a_1|\langle b_1| + |a_2\rangle|b_2\rangle\langle a_2|\langle b_2|)$

This operator does not correspond to the pure state (3) nor any other pure state because it is not idempotent ($\rho_m^2 \neq \rho_m$). It means that ρ_m does not represent the maximal knowledge about the total system. It misses precisely the correlation between the two systems, included in the true density operator $|\psi\rangle\langle\psi|$

(5) $\rho = \rho_m + \frac{1}{2}(|a_1\rangle|b_1\rangle\langle a_2|\langle b_2| + |a_2\rangle|b_2\rangle\langle a_1|\langle b_1|)$

One may still think that there is some physical mechanism that reduces the pure state ρ to the mixture ρ_m as the distance between the two systems S_A and S_B increases, thus localizing the states around S_A and S_B. A proposition in this spirit has been most recently advocated by Piccioni [11], but it has a longer history. Einstein himself suggested that "the current formulation of the many-body problem in quantum mechanics may break down when particles are far enough apart" (in a private communication to Bohm [12]). The possibility of such a localization process has been also considered by other authors [1]. In particular some physicist believe that when the wave packets of the two systems are localized and do not overlap there should be no correlations between measurements on the two systems, i.e. the reduced density operator ρ_m should be used. In this context Bohm and Hiley [13] analyzed the anisotropy measurements of the gamma rays, showing that pure states extend over macroscopic distances, more than an order of magnitude exceeding the width of the wave packet associated with each photon. The situation is qualitatively different from that which one finds in the self-interference experiments of Jánossy-Náray type or in the neutron interferometry (see the review on "empty waves" by Selleri [14]) because the experiments involving single-particle states are easily explained using the wave picture. The wavefunction in the two-particle case exists in 6-dimensional space and cannot be pictured as a real wave.

Less obvious consequences of the existence of the extended pure states were found by J.S. Bell. I will discuss them briefly now.

4. EPR AND BELL INEQUALITY REVISITED

Let E_{ab} be the usual correlation coefficient between simultaneous measurements [cf. 15]. Under a very general assumptions of locality and realism one may then prove the Bell inequality: combination of four correlation coefficients with the parameters (in particular angles) (ab), (ab'), (a'b) and (a'b') must not exceed 2:

$$(6)\quad C(a,b,a',b') = E_{ab} - E_{ab'} + E_{a'b} + E_{a'b'}$$
$$|C(a,b,a',b')| \le 2, \quad |E_{ab}| \le 1$$

Correlations computed with the extended quantum states (3) violate this inequality. In singlet and triplet spin states correlation coefficients are

$$|0,0\rangle \text{ state: } E_{ab} = -\cos(a-b)$$
$$(7)\quad |1,0\rangle \text{ state: } E_{ab} = -\cos(a+b)$$
$$|1,\pm1\rangle \text{ state: } E_{ab} = \cos a \cos b$$

In the singlet state inequality (6) is strongly violated because $C(0,45,90,135) = 2\cdot 2^{1/2}$. The same violation is observed in the triplet $|10\rangle$ state. Thus the existence of pure, spatially extended, nonfactorizable states is manifested by non-trivial correlations between the results of measurements on the two systems, correlations greater than allowed by Bell's inequality. Correlations that do not violate the Bell inequality are trivial if the two systems interacted in the past because one can explain them easily by local realistic models.

Is it possible to find non-trivial correlations between the systems that never interacted? In triplet $|1,\pm1\rangle$ states correlations are just as large as allowed by the Bell inequality, with $C(a,b,a',b')$ reaching at most 2. This is not surprising because $|1,+1\rangle = |\alpha\alpha\rangle$ and $|1,-1\rangle = |\beta\beta\rangle$, both are factorizable and therefore there are no interference terms in the density matrix [cf. 16]. But these are the only pure states that we know how to prepare externally! Mixture of all 4 states does not give any correlation, as one may expect. Going to higher spins, like $s = 3/2$, does not help either. Thus unless there is a way to prepare a pure state or a non-trivial mixture there will be no way to observe EPR correlations without previous interaction.

5. ENTANGLED SYSTEMS

Let us look from a more general point of view at the problem of "entanglement", as Schrödinger [3] calls it, manifested in EPR correlations violating local realism. If such correlations exist, as all experiments performed so far seem to indicate, then the ensemble interpretation of quantum mechanics cannot be maintained and local modifications of quantum mechanics are not admissible.