Introduction to
FEEDBACK CONTROL THEORY

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Preface

This book is based on my lecture notes for a ten-week second course on feedback control systems. In our department the first control course is at the junior level; it covers the basic concepts such as dynamical systems modeling, transfer functions, state space representations, block diagram manipulations, stability, Routh-Hurwitz test, root locus, lead-lag controllers, and pole placement via state feedback. In the second course, (open to graduate and undergraduate students) we review these topics briefly and introduce the Nyquist stability test, basic loopshaping, stability robustness (Kharitanov's theorem and its extensions, as well as $\mathcal{H}^\infty$-based results) sensitivity minimization, time delay systems, and parameterization of all stabilizing controllers for single input-single output (SISO) stable plants. There are several textbooks containing most of these topics, e.g. [7, 17, 22, 37, 45]. But apparently there are not many books covering *all* of the above mentioned topics. A slightly more advanced text that I would especially like to mention is *Feedback Control Theory*, by Doyle, Francis, and Tannenbaum, [18]. It is an excellent book on SISO $\mathcal{H}^\infty$-based robust control, but it is lacking significant portions of the *introductory* material included in our curriculum. I hope that the present book fills this gap, which may exist in other universities as well.

It is also possible to use this book to teach a course on feedback control, following a one-semester signals and systems course based on [28, 38], or similar books dedicating a couple of chapters to control-related topics. To teach a one-semester course from the book, Chapter 11 should be expanded with supplementary notes so that the state space methods are covered more rigorously.

Now a few words for the students. The exercise problems at the end of each chapter may or may not be similar to the examples given in the text. You should first try solving them by hand calculations; if you think that a computer-based solution is the only way, then go ahead and use MATLAB. I assume that you are familiar with MATLAB; for those who are not, there are many introductory books, e.g., [19, 23, 44]. Although it is not directly related to the present book, I would also recommend [52] as a good reference on MATLAB-based computing.

Despite our best efforts, there may be errors in the book. Please send your comments to: `ozbay.1@osu.edu`, I will post the corrections on the web: `http://eewww.eng.ohio-state.edu/~ozbay/ifct.html`.

Many people have contributed to the book directly or indirectly. I would like to acknowledge the encouragement I received from my colleagues in the Department of Electrical Engineering at The Ohio State University, in particular J. Cruz, H. Hemami, Ü. Özgüner, K. Passino, L. Potter, V. Utkin, S. Yurkovich, and Y. Zheng. Special thanks to A. Tannenbaum for his encouraging words about the potential value of this book. Students who have taken my courses have helped significantly with their questions and comments. Among them, R. Bhojani and R. Thomas read parts of the latest manuscript and provided feedback. My former PhD students T. Peery, O. Toker, and M. Zeren helped my research; without them I would not have been able to allocate extra time to prepare the supplementary class notes that eventually formed the basis of this book. I would also like to acknowledge National Science Foundation's support of my current research. The most significant direct contribution to this book came from my wife Özlem, who was always right next to me while I was writing. She read and criticized the preliminary versions of the book. She also helped me with the MATLAB plots. Without her support, I could not have found the motivation to complete this project.

Hitay Özbay
Columbus, May 1999

Dedication

To my wife, Özlem

Contents

Chapter 1

Introduction

1.1 Feedback Control Systems

Examples of feedback are found in many disciplines such as engineering, biological sciences, business, and economy. In a feedback system there is a process (a cause-effect relation) whose operation depends on one or more variables (inputs) that cause changes in some other variables. If an input variable can be manipulated, it is said to be a control input, otherwise it is considered a disturbance (or noise) input. Some of the process variables are monitored; these are the outputs. The feedback controller gathers information about the process behavior by observing the outputs, and then it generates the new control inputs in trying to make the system behave as desired. Decisions taken by the controller are crucial; in some situations they may lead to a catastrophe instead of an improvement in the system behavior. This is the main reason that feedback controller design (i.e., determining the rules for automatic decisions taken by the feedback controller) is an important topic.

A typical feedback control system consists of four subsystems: a *process* to be controlled, sets of *sensors* and *actuators*, and a *controller*,

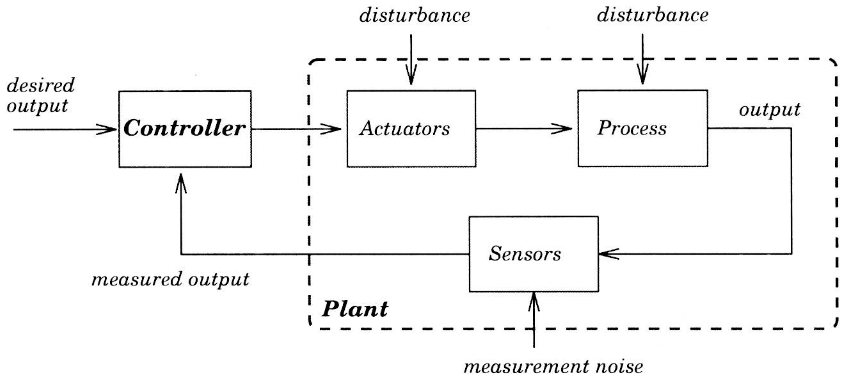

Figure 1.1: Feedback control system.

as shown in Figure 1.1. The process is the actual physical system that cannot be modified. Actuators and sensors are selected by process engineers based on physical and economical constraints (i.e., the range of signals to be measured and/or generated and accuracy versus cost of these devices). The controller is to be designed for a given *plant* (the overall system, which includes the process, sensors, and actuators).

In engineering applications the controller is usually a computer, or a human operator interfacing with a computer. Biological systems can be more complex; for example, the central nervous system is a very complicated controller for the human body. Feedback control systems encountered in business and economy may involve teams of humans as main decision makers, e.g., managers, bureaucrats, and/or politicians.

A good understanding of the process behavior (i.e., the cause-effect relationship between input and output variables) is extremely helpful in designing the rules for control actions to be taken. Many engineering systems are described accurately by the physical laws of nature. So, mathematical models used in engineering applications contain relatively low levels of uncertainty, compared with mathematical models that appear in other disciplines, where input-output relationships can be much more complicated.

In this book, certain fundamental problems of feedback control theory are studied. Typical application areas in mind are in engineering. It is assumed that there is a mathematical model describing the dynamical behavior of the underlying process (modeling uncertainties will also be taken into account). Most of the discussion is restricted to single input-single output (SISO) processes. An important point to keep in mind is that success of the feedback control depends heavily on the accuracy of the process/uncertainty model, whether this model captures the reality or not. Therefore, the first step in control is to derive a simple and relatively accurate mathematical model of the underlying process. For this purpose, control engineers must communicate with process engineers who know the physics of the system to be controlled. Once a mathematical model is obtained and performance objectives are specified, control engineers use certain design techniques to synthesize a feedback controller. Of course, this controller must be tested by simulations and experiments to verify that performance objectives are met. If the achieved performance is not satisfactory, then the process model and the design goals must be reevaluated and a new controller should be designed from the new model and the new performance objectives. This iteration should continue until satisfactory results are obtained, see Figure 1.2.

Modeling is a crucial step in the controller design iterations. The result of this step is a nominal process model and an uncertainty description that represents our confidence level for the nominal model. Usually, the uncertainty magnitude can be decreased, i.e., the confidence level can be increased only by making the nominal plant model description more complicated (e.g., increasing the number of variables and equations). On the other hand, controller design and analysis for very complicated process models are very difficult. This is the basic trade-off in system modeling. A useful nominal process model should

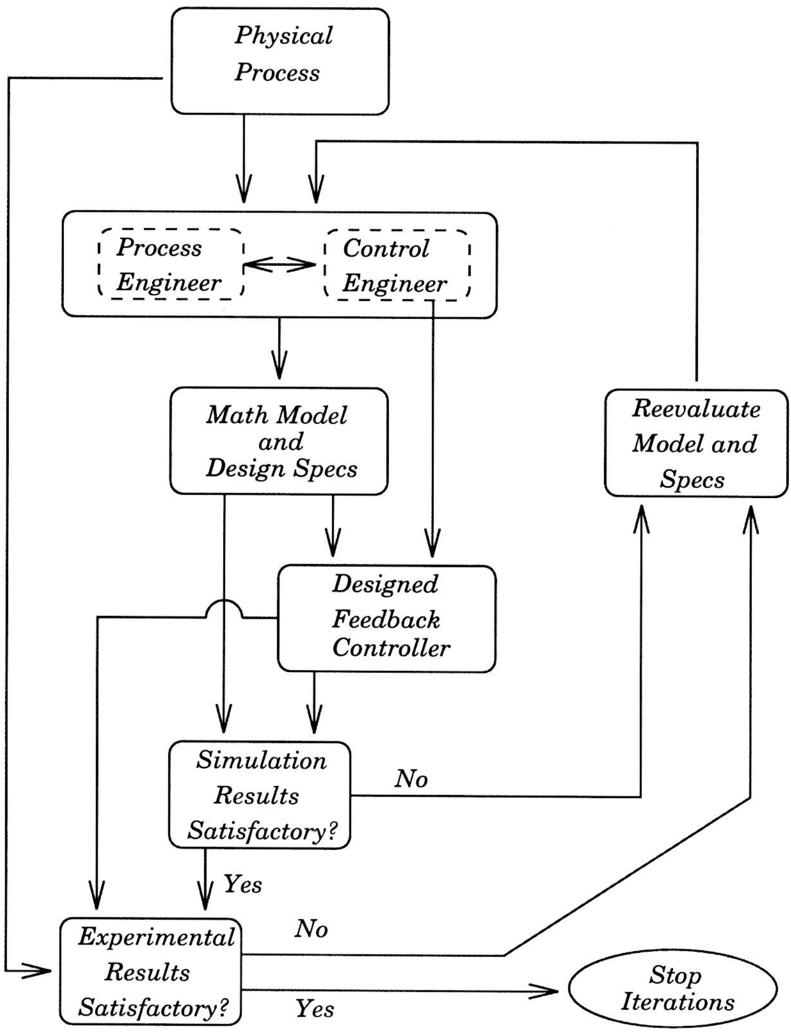

Figure 1.2: Controller design iterations.

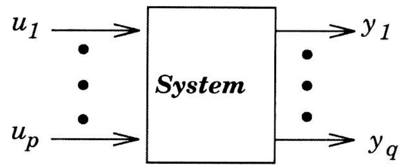

Figure 1.3: A MIMO system.

be simple enough so that the controller design is feasible. At the same time the associated uncertainty level should be low enough to allow the performance analysis (simulations and experiments) to yield acceptable results.

The purpose of this book is to present basic feedback controller design and analysis (performance evaluation) techniques for simple SISO process models and associated uncertainty descriptions. Examples from certain specific engineering applications will be given whenever it is necessary. Otherwise, we will just consider generic mathematical models that appear in many different application areas.

1.2 Mathematical Models

A multi-input-multi-output (MIMO) system can be represented as shown in Figure 1.3, where $u_1, \ldots, u_p$ are the inputs and $y_1, \ldots, y_q$ are the outputs (for SISO systems we have $p = q = 1$). In this figure, the direction of the arrows indicates that the inputs are processed by the system to generate the outputs.

In general, feedback control theory deals with dynamical systems, i.e., systems with internal memory (in the sense that the output at time $t = t_0$ depends on the inputs applied at time instants $t \leq t_0$). So, the plant models are usually in the form of a set of differential equations obtained from physical laws of nature. Depending on the operating conditions, input/output relation can be best described by linear or

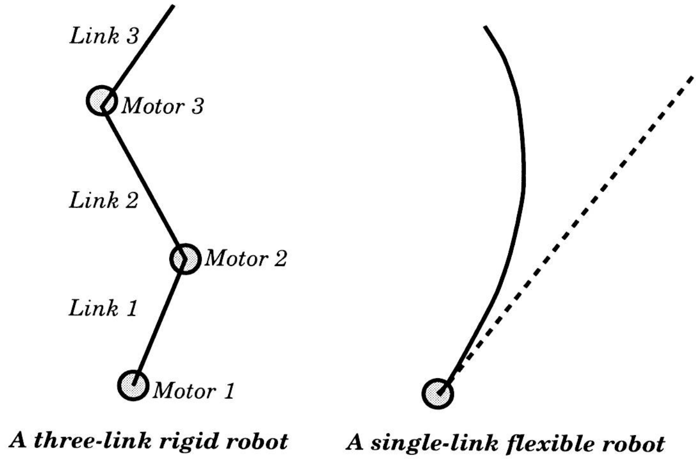

Figure 1.4: Rigid and flexible robots.

nonlinear, partial or ordinary differential equations.

For example, consider a three-link robot as shown in Figure 1.4. This system can also be seen as a simple model of the human body. Three motors located at the joints generate torques that move the three links. Position, and/or velocity, and/or acceleration of each link can be measured by sensors (e.g., optical light with a camera, or gyroscope). Then, this information can be processed by a feedback controller to produce the rotor currents that generate the torques. The feedback loop is hence closed. For a successful controller design, we need to understand (i.e., derive mathematical equations of) how torques affect position and velocity of each link, and how current inputs to motors generate torque, as well as the sensor behavior. The relationship between torque and position/velocity can be determined by laws of physics (Newton's law). If the links are rigid, then a set of nonlinear ordinary differential equations is obtained, see [26] for a mathematical model. If the analysis and design are restricted to small displacements around the upright equilibrium, then equations can be linearized without introducing too much error [29]. If the links are made of a flexible material (for example,