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British Library Cataloguing in Publication Data

Murray C A
Vectorial astrometry
1. Astrometry
I. Title
522 QB807

ISBN 0-85274-372-6

Consultant Editor: **Professor A J Meadows**,
Department of Astronomy, University of Leicester

Published by Adam Hilger Ltd, Techno House, Redcliffe Way, Bristol BS1 6NX.
The Adam Hilger book-publishing imprint is owned by The Institute of Physics.

Printed in Great Britain by J W Arrowsmith Ltd, Bristol.

Acknowledgments

I owe a deep debt of gratitude to Professor Robert d'E Atkinson, to whom this book is dedicated, who first stimulated my interest in astrometry more than 30 years ago when he was Chief Assistant at the Royal Observatory, Greenwich. He has profoundly influenced my own work, first as his junior collaborator and subsequently in many discussions and extensive correspondence. His own contributions to astrometric theory and instrumental design have been very significant and, in particular, it is largely due to him that we owe the recent redefinition of the pole of the celestial reference frame. His own Euclidean formulation of general relativity in a spherically symmetric field forms the basis of the development given here in the first two chapters.

Many other friends and colleagues have been kind enough to read parts of the manuscript. I am particularly grateful to Professors W H McCrea and G C McVittie for their advice and comments on earlier draft versions of Chapters 1 and 2; responsibility for the final version, however, rests entirely with me. My thanks are due to Dr B Elsmore, Dr D H P Jones, Dr J D H Pilkington, Dr A R Robbins, Dr A T Sinclair and Dr J V Wall for helpful discussions or correspondence during the course of this work. I am also very grateful to Mrs Beryl Andrews, Mrs Hazel Beney, Mrs Barbara Herbert, Mrs Anne Hicks and Miss Hilary Saunders who have shared the typing of a difficult manuscript.

Finally I must express my thanks to the Directors of the Royal Greenwich Observatory, Professor F Graham Smith and Professor A Boksenberg, who have given me considerable freedom and support in writing this book.

C A Murray
Royal Greenwich Observatory
11 January 1982

Contents

Prologue

The role of astrometry is to set up a system of space-time coordinates for describing events such as the emission of a light signal from a distant object and its reception by an observer. Within this definition can be included not only observations of direction and the derivation of linear coordinates by trigonometric methods but also the measurement of travel times of signals between observer and object.

Until the space age, astrometry was concerned only with measurement of directions and, since directions can be represented by points on the celestial sphere, the word astrometry has become virtually synonymous with spherical astronomy. All the texts on the subject which are currently available, such as for example Chauvenet (1891), Newcomb (1906), Smart (1931), Woolard and Clemence (1966), van de Kamp (1967) and McNally (1974), use spherical trigonometry as the main mathematical tool. In the past, this dependence on spherical trigonometry has led to the use, for computational convenience, of approximations which are no longer adequate.

It has been recognised since the advent of modern digital computers that many astrometric calculations, such as coordinate transformations, are more efficiently carried out using the methods of vector and matrix algebra, without the necessity for approximations; such methods were of course quite impracticable for routine use in the days of hand computation. In forming my own view that vector methods provide not only a convenient tool for computation but also give a clear physical insight in theoretical developments, I have been strongly influenced by the late Professor E A Milne's *Vectorial Mechanics*.

My original intention was to develop traditional astrometry by vector methods, but it soon became clear that, with the increased accuracy of measurements to be expected from space-borne telescopes, and the development of radar and laser ranging to the Moon and planets, a purely traditional approach based on Newtonian mechanics was no longer appropriate.

The first observational success of general relativity was the measurement of light deflection by the Greenwich and Cambridge expeditions to the eclipse of 1919; this was a purely astrometric observation, yet relativity has been generally ignored in subsequent books on astrometry. I have therefore attempted to construct a theory of astrometric observations based on general relativity. For the present purpose I have accepted Einstein's theory as it stands, and have not thought it worth while to introduce the further complication of parametrised post-Newtonian (PPN) formalism to allow for possible modifications to the theory. As an optical astrometrist it would be highly presumptuous of me to write a text on relativity *per se*, and this book is certainly not to be regarded as one; yet I believe that relativity can no longer be ignored in modern astrometric theory or practice.

In the first two chapters we develop from a relativistic standpoint the basic theory of orbital motion of a particle, and in particular that of a photon, in the Schwarzschild metric. These two chapters together provide the framework for the interpretation of the astrometric observable quantities, directions and clock times. The coordinate frames used in astrometry are closely related to the gravity field of the Earth and its orbital and rotational motion, which are discussed in Chapters 3, 4 and 5. The treatments of Keplerian motion in Chapter 1 and of the rotation of a rigid body in Chapter 3 follow closely those by Milne.

Although the rotational orientation of the Earth, as measured by universal time and polar motion, is of fundamental importance for astrometric observations, the general availability of an atomic time-scale now enables the problems arising from variability of the Earth's rotation to be separated from those of timekeeping. In Chapter 6 we discuss various time-scales and derive the relationship between the coordinate time of relativity theory and the ideal 'terrestrial time' which we identify with the IAU (1976) scale based on caesium atomic standards.

In Chapter 7 we consider the systematic effects of the atmosphere on the direction and speed of transmission of light. We make the customary approximation of spherical symmetry but an attempt has been made to correct for water vapour in a rather more rigorous way than has been usual, by allowing the scale height to be independent of that of the dry component. Order-of-magnitude estimates of the effects of ionospheric refraction are also given.

Much ground-based optical astrometry, particularly of faint objects, is carried out by photographic techniques in which relative measurements are made within a small area of sky, frequently using large telescopes; this includes not only the derivation of positions by interpolation between standard stars but also the determination of relative parallaxes and proper motions; we discuss these topics in Chapter 8. Considerable advances in this branch of astrometry have been made in recent years, with the advent of large automatic plate-measuring machines and the development of computational techniques such as the plate-overlap method.

A major problem in astrometry, which we discuss in Chapter 9, is the measurement of directions on a global scale with the ultimate aim of setting up an inertial reference frame. The traditional technique in optical astrometry is by meridian observations of bright objects, stars and the major bodies in the Solar System, which demand extreme care in calibration of the instruments and interpretation of the measurements. Observations of the motion of the local vertical are also required and we shall therefore describe the astronomical techniques now used for measuring time and the variation of latitude. In this field, astrometry is closely allied to geodesy and geophysics and the modern techniques of Doppler measurements of artificial satellites and laser ranging to satellites and the Moon now contribute very significantly to studies of the motion of the Earth's surface; however, since these measurements do not give direct information on the direction of the vertical, which is required for astrometry, we shall not discuss them in any detail.

A global coordinate system can also be constructed by radio interferometry, but it is still necessary to relate this to the reference frame of optical astrometry through observations of optical counterparts of compact radio sources.

An observational reference frame can be related to an inertial frame by comparing with theory observations of any dynamical system made over a sufficiently extended time interval. This can be done in principle from observations of Solar System bodies for which the theories have been developed to a high degree of precision, but in practice, because of limitations to the accuracy of observations of the directions to the Sun, Moon and major planets, it has been found more satisfactory to use stars. It is therefore necessary to have a model of stellar kinematics for comparison with observations. We give in Chapter 10 a description of the main features of motions in the Galaxy as presently understood, which have formed the basis for defining an inertial frame, and we conclude with a brief account of observational programmes for defining an inertial frame from observations of extragalactic objects.

My aim throughout has been to give a self-consistent mathematical formulation for the interpretation and analysis of astrometric observations, rather than to describe particular instrumental or computational techniques. Numerical values of astronomical constants have been taken from the IAU (1976) system as adopted at the General Assembly at Grenoble (IAU 1977). A development of the vector and tensor formulae which are used extensively in this book is given in Appendix A. In particular it should be noted that the prime (′) symbol is used exclusively to denote matrix transposition. Since scalar multiplication of two vectors is a special case of transpose multiplication of matrices, we use the prime, instead of the usual dot, to denote a scalar product. This and other minor deviations from the generally accepted notion are described in Appendix A.

1 Dynamical Foundations of Astrometry

Since all astrometric observations are made from a platform which is itself moving relative to the rest frame of the Solar System, it is necessary to have an accurate numerical description of this motion, and to model its consequences on the interpretation of observations.

A logical starting point for our discussion is therefore the theory of orbital motion. The complete determination of the motions of bodies in the Solar System belongs to the field of celestial mechanics and is beyond the scope of this book; for our purpose a study of elementary theory will be sufficient.

Classical astrometric techniques have been developed from the standpoint of Newtonian mechanics and the inverse square law of gravitation, and have led to a very satisfactory accordance between theory and observation. But there are small, though well determined, deviations from the predictions of Newtonian theory, two of which, the deflection of light in the gravitational field of the Sun and the excess secular motion of the planetary perihelia, notably that of Mercury, have been detected and measured by means of classical techniques. These effects now appear to be adequately accounted for by general relativity.

The very high precision which is now attainable in timekeeping has also made essential a proper relativistic treatment of timing measurements in, for example, radio interferometry and ranging by radar and laser beams.

It is now necessary therefore to re-examine from a relativistic viewpoint the assumptions upon which the whole edifice of classical astrometry has been built. In this chapter we shall develop the theory of orbital motion and discuss the definitions, physical units and basic constants which are used in astrometry and celestial mechanics. In the following chapter we shall examine the propagation of light signals in the gravitational field of the Sun, and their interpretation by an observer. These two chapters are not intended to give a complete exposition of general relativity, but merely to introduce in an elementary fashion those aspects of it which have practical consequences for high-precision astrometric measurements. Our treatment will be within a Euclidean framework, without any appeal to the elegant mathematical formalism which is usually associated with relativity.

1.1 Space-time metrics

In four-dimensional Riemannian space, the general expression for the interval $d\tau$ between two adjacent events, in terms of coordinate differentials dx^ν ($\nu = 0, \ldots, 3$), can be written

$$d\tau\, d\tau = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{1.1.1}$$

where μ, ν are dummy symbols (not exponents), and we adopt the summation convention that a repeated dummy implies summation over all values $0, \ldots, 3$. Thus (1.1.1) represents a sum of 16 terms. The numbers $g_{\mu\nu}$, which are, in general, functions of the coordinates, are said to define the metric of the space.

According to the general relativity theory, the metric depends on the distribution of matter; the $g_{\mu\nu}$ satisfy certain partial differential equations known as Einstein's equations. Such a metric is known as a 'space-time' metric.

The sequence of coordinates of a moving particle describes its 'world line' and, in particular, the world line of a particle which moves freely in the gravitational field is known as a geodesic.

It will be sufficient for our present purpose to confine attention to a static spherically symmetrical field arising from a single isolated mass. We will identify (x^1, x^2, x^3) with spatial coordinates relative to the centre of symmetry, and x^0 will be taken as 'coordinate time' and denoted by t. The assumption that the field is static implies that the $g_{\mu\nu}$ are not functions of t. Even with spherical symmetry, the radial scale can be defined to vary quite arbitrarily as a function of radius; but once this radial scale has been chosen, the differential equations describing a geodesic are completely determined.

However, we are still at liberty to choose the space upon which to map the coordinates (x^1, x^2, x^3). This is exactly analogous to the choice of a geometrical projection in constructing a two-dimensional map. It has been shown by Atkinson (1963) that the relativistic properties of a spherically symmetrical field can be described quite rigorously within the framework of a three-dimensional Euclidean space; this is perhaps not so surprising since the assumption of spherical symmetry implies that the form of the metric is unchanged by a Euclidean transformation of the spatial coordinates.

We shall adopt this point of view, and postulate a Euclidean space defined by three mutually orthogonal Cartesian axes with origin at the centre of symmetry, which represent the rest frame of the field. We define the 'coordinate vector' $\boldsymbol{x}$ and 'coordinate velocity' $d\boldsymbol{x}/dt$ to be the three-dimensional Euclidean vectors whose components are (x^1, x^2, x^3) and $(dx^1/dt, dx^2/dt, dx^3/dt)$ respectively.