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Control Systems Theory and Applications for Linear Repetitive Processes

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Preface

Repetitive processes, also termed multipass processes in the early literature, are characterized by a series of sweeps, termed passes, through a set of dynamics where the duration, or length, of each pass is finite. On each pass an output, or pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This so-called unit memory property is a special case of the more general situation where it is the previous M passes which contribute to the dynamics of the current one. The positive integer M is termed the memory length and such processes are simply termed non-unit memory.

The concept of a repetitive process was first introduced in the early 1970's as a result of work in The University of Sheffield, UK on the modelling and control of long-wall coal cutting and metal rolling operations. In these applications, productive work is undertaken by a series of passes through a set of dynamics defined over a finite duration, or pass length, which is the first distinguishing feature of a repetitive process. As the process evolves from given initial conditions, an output sequence of pass profiles is produced and it was observed that this could include a first pass profile which had acceptable dynamics along the pass but subsequent passes contained oscillations which grew, or increased in amplitude, severely from pass-to-pass.

Further investigation in the long-wall coal cutting case established that the deterioration in performance after the first pass was due to the effects of the previous pass profile on the production of the current one. In this application, the machine which undertakes the cutting operation rests on the previous pass profile during the production of the current one and its weight alone clearly means that it will most certainly influence the next pass profile, i.e. the output dynamics on any pass acts as a forcing function on, and hence contributes to, the dynamics of the next pass. This is the second distinguishing feature of a repetitive process, i.e. it is possible to generate oscillations which increase in amplitude from pass-to-pass. Such behavior is clearly not acceptable and requires appropriate control action.

Recognizing the unique control problem here, the first approach to the design of control laws was to write down a simplified mathematical model and then make use of standard, termed 1D here, control action. The essence of such an approach is to use a single variable to convert the mathematical model of the process under consideration into that for an equivalent infinite length single pass process in which the relationships between variables are expressed only in terms of the so-called total distance traversed. This then led to the design and evaluation of control schemes for this and other examples such as metal rolling.

The analysis employed in this early work was somewhat application oriented and it was necessary to impose the assumptions that (i) the pass length is 'long' (but finite) and hence the effects of the initial conditions at the start of each pass can be ignored, and (ii) the effects of the previous pass dynamics can be represented by a long delay term. Intuitively, however, the resetting of the initial conditions before the start of each new pass could act as a form of stabilizing action and hence prevent the growth of disturbances. This and the need for a generally applicable control theory led to the development of an alternative approach to stability analysis which does not require the above assumptions and, in particular, takes full account of the interaction between successive pass profiles over the finite pass length. This stability theory is based on an abstract model of the process dynamics in a Banach space setting which includes a wide range of examples with linear dynamics (and a constant pass length) as special cases.

In this abstract model, the critical contribution from the previous pass dynamics to those of the current one is expressed in terms of a bounded linear operator mapping a Banach space into itself and the stability theory is expressed in terms of spectral and induced norm properties of this operator. Hence, unlike the initial approach, this setting provides a rigorous general purpose basis for the control related analysis of linear constant pass length repetitive processes. This is all the more important with the later emergence of other applications and, in particular, those termed algorithmic where adopting a repetitive process setting for analysis either has clear advantages over alternatives or indeed provides the only viable approach.

The stability theory based on the abstract model setting shows that this property for these processes is much more involved than first envisaged. In particular, it shows that the structure of the initial conditions at the start of each new pass is critical to the dynamics which evolve (both along the pass and pass-to-pass) and, critically, they cannot be neglected. Hence, at best, the original approach to the analysis and control of these processes can only be correct under very special circumstances. Moreover, two distinct stability properties can be defined and physically justified, where the essential difference between them is a direct result of the finite pass length.

Given the unique control problem, so-called asymptotic stability demands that the sequence of pass profiles converge to a steady or so-called limit profile which, in turn, is equivalent to demanding bounded-input bounded-output stability (defined in terms of the norm on the underlying function space) over the finite pass length. This, however, does not guarantee that the resulting limit profile has acceptable along the pass dynamics. For example, certain practically relevant sub-classes produce a limit profile which is described by a 1D (differential or discrete) unstable linear state-space model but over a finite duration such a model is guaranteed to produce a bounded response and hence satisfy the definition of asymptotic stability (for repetitive processes).

Stability along the pass removes this difficulty by demanding the bounded-input bounded-output property uniformly, i.e. independent of the pass length. Moreover, asymptotic stability is a necessary condition for stability along the pass and for certain sub-classes of major interest in terms of applications the resulting conditions can be tested by direct application of 1D linear systems tests. Missing, however, is the ability to use these tests, e.g. in the frequency domain, as a basis for control law design.

If the dynamics along the pass are described by a (matrix) discrete linear state equation, it can be shown that stability along the pass (for one particular case of pass initial conditions) of the resulting so-called discrete linear repetitive process is equivalent to bounded-input bounded-output stability of 2D discrete linear systems described by well known and extensively studied state-space models. This, in turn, suggests that the theory for these 2D systems should be directly applicable to discrete linear repetitive processes. Note, however, that this equivalence is only present in the case of the simplest possible boundary conditions and there is no corresponding result for linear repetitive processes whose along the pass dynamics are described by a (matrix) linear differential equation.

By the mid 1990's, the stability theory and associated tests for differential and discrete linear repetitive processes was well developed but there was much yet to be done before their full power in terms of applications could be exploited to the maximum extent. This prompted an expanded research effort into areas of systems theory such as controllability, observability, robust stability, optimal control, and the structure and design of control laws (or controllers) with and without uncertainty in the process description. This monograph gives the results of this work and also its application to, in the main, iterative learning control which is one of the major algorithmic applications for repetitive process systems theory.

The following chapter provides the essential background in terms of examples, their modelling as special cases of the abstract model, the links with certain classes of 2D discrete linear systems and delay differential systems, the development of a 1D equivalent model for the dynamics of discrete linear repetitive processes, and a 2D transfer-function matrix description of the dynamics of differential and discrete processes. The two currently known algorithmic applications for repetitive processes are also introduced by showing how their dynamics fit naturally into the repetitive process setting. This is followed by a chapter giving the abstract model based stability theory and its application in terms of computable tests and (in some relevant cases) the extraction of information concerning expected performance in the presence of stability.

Chapters 4 and 5 give further development of the existing stability theory and tests in two basic directions for the sub-classes of discrete and differential linear repetitive processes which have (currently) the most relevance in terms of applications. This leads to new interpretations of stability in the form of so-called 1D and 2D Lyapunov equations which provide computable information concerning expected performance and also, via Linear Matrix Inequalities (LMIs) and Lyapunov functions, algorithms for control law design to ensure stability and performance. Chapter 6 deals with the case when there is uncertainty in the defining state-space model.

The remaining chapters focus on systems theoretic properties and control law (or controller) design. In Chap. 7, controllability and observability for both differential and discrete linear repetitive processes is treated. As in the theory of 2D/nD discrete linear systems, the situation here is more complex than for 1D linear systems and it is also important to note that some of the properties defined for discrete processes have no 2D linear systems counterparts. In the differential case, the analysis is much less well developed and requires further work to be undertaken.

In Chap. 8, a substantial body of results on control law (or controller) design are developed and illustrative examples given. A major part of these relate to the development of design algorithms which can be computed using LMIs and cover the cases when stability and stability plus performance respectively are required. These control laws are, in general, activated by a combination of current and previous pass information. Moreover, they have a well grounded physical basis, a feature which is not always present in 2D/nD systems. The performance objectives considered include that of forcing the process under control action to be stable along the pass with a resulting limit profile which has acceptable properties as a 1D linear system, which again has a well grounded physical basis.

Linear quadratic optimal control is an obvious approach to the control of the processes considered here, where a cost function can be formed by taking the usual quadratic cost along each pass and then summing over the passes (either for the finite number of passes to be completed or else to infinity). Here it is shown (by a straightforward extension of familiar 1D theory) that such a cost function can be minimized by a state control law which cannot be implemented because it is not causal. It is, however, subsequently shown that a causal solution to this problem does exist but further work is required on the computational aspects.

Chapter 9 deals with control law (or controller) design for robustness and performance. These are the first ever results in this key area and build on those in the previous chapter in terms of the structure of the laws used and LMI based computations. The uncertainty structures considered are expressed in terms of perturbations to the defining process state-space model. This is followed by an H_∞ based design and, in the final section, H_2 and mixed H_2/H_∞ approaches.

Iterative learning control (ILC) is a major application area for repetitive process theory and this is the subject of Chap. 10. The results given range from those previously known, which highlight a major performance trade-off inherent in ILC, through to the very latest analysis supported by experimental results from application to a conveyor system and a gantry robot. Finally, Chap. 11 summarizes the current state of the art and discusses areas for possible future research, where this latter aspect includes both further development of the results reported in this monograph and also extensions to the structure of the models currently considered to capture essential dynamics not included in any of those studied to-date.

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Contents

1 Examples and Representations

Summary. This chapter first introduces the unique features and control problems for repetitive processes by reference to two physical examples – long-wall coal cutting and metal rolling. Two so-called algorithmic examples are considered next, i.e. problem areas where adopting a repetitive process approach to modelling and analysis has clear advantages over alternatives. All these examples are shown to be special cases of the general abstract model in a Banach space setting on which the stability theory for linear repetitive processes is based. Finally, the links at the modelling/structural level with well known 2D discrete and standard, termed 1D in this setting, linear systems are detailed.

1.1 Examples and Control Problems

1.1.1 Long-wall Coal Cutting

The unique characteristic of a repetitive process can be illustrated by considering machining operations where the material or workpiece involved is processed by a sequence of passes of the processing tool. Assuming the pass length $\alpha < \infty$ to be constant, the output vector, or pass profile, $y_k(t)$, $0 \leq t \leq \alpha$ (where t denotes the independent spatial or temporal variable) generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t)$, $0 \leq t \leq \alpha$, $k \geq 0$.

These processes have their origins in the mining and metal rolling industries where the first to be identified was long-wall coal cutting , which was the most satisfactory, and commonly used, method of mining coal in Great Britain. Even though coal mining in Great Britain is now a much reduced industry in comparison to former times, this example can still be used to illustrate the 'basic mechanics' of a repetitive process and the essential unique control problem. This is treated next, starting with a brief description of the long-wall coal cutting process. We use the notation of the original treatment of this example in [50, 51].

Figures 1.1 and 1.2 illustrate the basic operation of the long-wall coal cutting process in which the coal cutting machine is hauled along the entire length of the coal face riding on the semi-flexible structure of the armored face conveyor, denoted A.F.C., which transports away the coal cut by the rotating drum. In the simplest mode of operation, these machines only cut

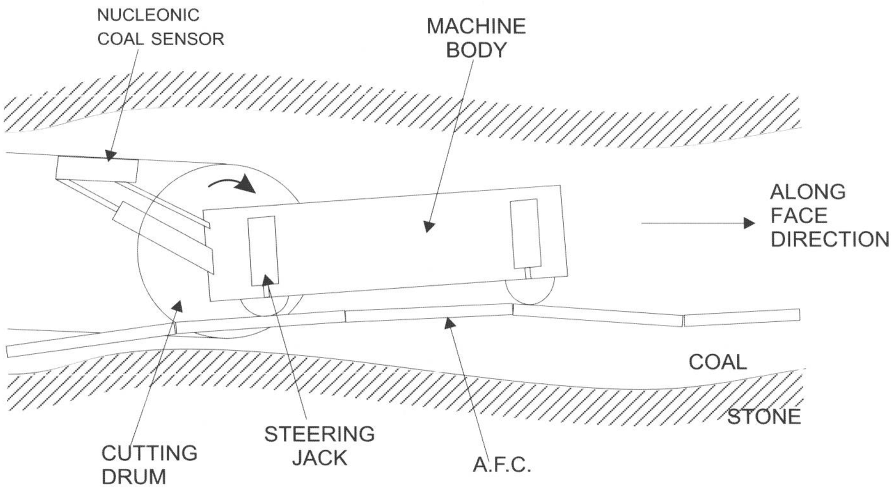

Fig. 1.1. Side elevation of coal cutting machine

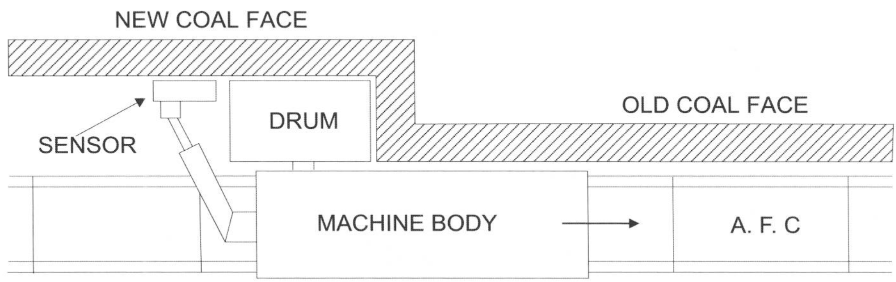

Fig. 1.2. Plan view of coal cutting machine

in one direction, left to right in Figs. 1.1 and 1.2, and they are hauled back in reverse at high speed for the start of each new pass of the coal face.

Between passes, the conveyor is 'snaked' forward using hydraulic rams, as illustrated in Fig. 1.3, so that the machine now rests on the newly cut floor, i.e. the pass profile produced during the previous pass. During the cutting operation, the machine's drum may be raised or lowered with respect to the A.F.C. by using hydraulically operated jacks (illustrated schematically in Fig. 1.1) to tilt the machine body about a datum line on the drum (also termed the face) side. The objective of this operation is the vertical steering of the entire long-wall installation (machine, conveyor and roof support units) to maintain it within the undulating confines of the coal seam (or layer). A nucleonic coal sensor, situated some distance behind the cutting drum, provides the primary control signal by measuring either the floor or ceiling thickness of coal left by the machine (penetration of the stone/coal interface is to be avoided on both economic and safety grounds).

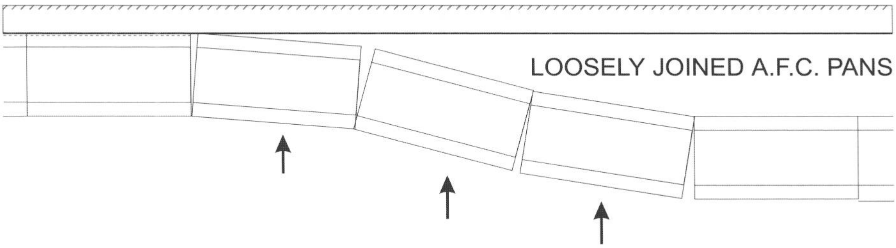

Fig. 1.3. Snaking of conveyor during pushover stage

In order to obtain a simplified mathematical model of this process, consider the idealized side elevation and plan shown in Figs. 1.4 and 1.5 respectively. Here the constants F, R and W represent the feet spacing, drum offset, and width of the machine (and drum) respectively, the variable $J_{k+1}(t)$ represents the controlled drum deflection, $Y_{k+1}(t)$, $e_{k+1}(t)$ denote the coal floor thickness and the height of the A.F.C. above a fixed datum plane respectively, X is the transport delay, or lag, by which the coal floor sensor lags behind the cutting drum, $Z_{k+1}(t)$ denotes the height of the stone/coal interface above the same fixed datum plane as the A.F.C., and $\beta_{k+1}(t)$ denotes the longitudinal tilt of the machine. (The skids labelled A,B,C and D respectively in these last two figures represent the mountings used to fix the machine body to the conveyor and are not relevant to the analysis here.) Suppose also that all angular deflections are small. Then elementary geometrical considerations immediately yield the following description of the coal cutting process dynamics over $0 \leq t \leq \alpha$, (where α denotes the finite and assumed constant pass length)

$$\begin{aligned} Y_{k+1}(t) + Z_{k+1}(t) &= e_{k+1}(t+R) + W\gamma_{k+1}(t+R) \\ &+ R\beta_{k+1}(t) + J_{k+1}(t) \end{aligned} \tag{1.1}$$

where γ denotes the transverse tilt of the machine.

The transverse and longitudinal tilts of the machine are also those of the supporting conveyor structure and are given by

$$\gamma_{k+1}(t) = \frac{(e_{k+1}(t) - e_k(t))}{W} \tag{1.2}$$

and

$$\beta_{k+1}(t) = \frac{(e_{k+1}(t) - e_{k+1}(t+F))}{F} \tag{1.3}$$

respectively. Suppose also that the A.F.C. moulds itself exactly onto the cut floor on which it rests – the so-called 'rubber conveyor' assumption. Then

$$e_{k+1}(t) = k_2(Y_k(t) + Z_k(t)) \tag{1.4}$$

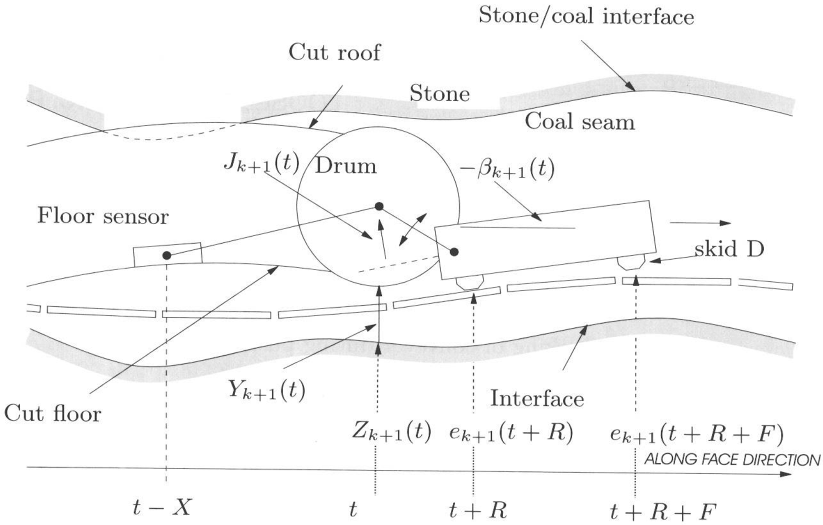

Fig. 1.4. Side elevation with variables labelled

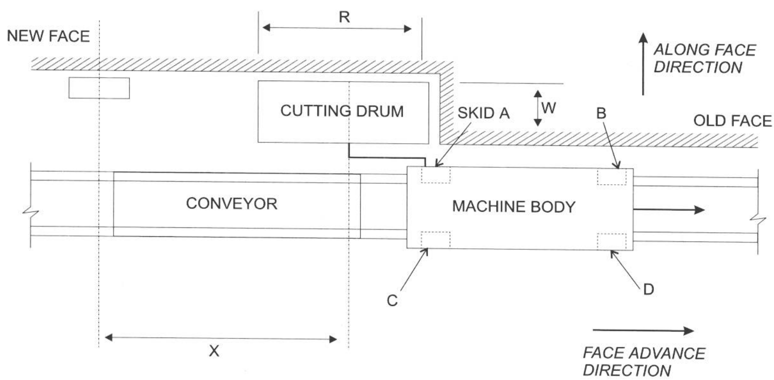

Fig. 1.5. Plan view with variables labelled

where k_2 is a positive real constant. This completes the description of the open-loop system in this case.

One approach to controlling this system is to manipulate the variable $J_{k+1}(t)$ from a delayed measurement of the coal floor thickness $Y_{k+1}(t-X)$. More commonly, however, the roof coal thickness was used since it can be related to $Y_{k+1}(t-X)$ on the assumption that the seam thickness is constant. Suppose also that the sensor and actuator dynamics can be neglected (to a first approximation) and a so-called fixed drum shearer is used, i.e. $R=0$. Then a possible control law in this case takes the form

$$J_{k+1}(t) = k_1(R_{k+1}(t) - Y_{k+1}(t-X)) - W\gamma_{k+1}(t),\ 0 \le t \le \alpha \tag{1.5}$$

where k_1 is a positive real constant and $R_{k+1}(t)$ is a new external reference vector taken to represent the desired coal thickness on pass $k+1$.

Suppose now, for simplicity, that the variable $Z_{k+1}(t)$ is set equal to zero. Then combining (1.1)–(1.5) yields the following description of the controlled process dynamics over $0 \le t \le \alpha,\ k \ge 0$,

$$Y_{k+1}(t) = -k_1 Y_{k+1}(t-X) + k_2 Y_k(t) + k_1 R_{k+1}(t),\ X > 0 \tag{1.6}$$

with assumed pass initial conditions

$$Y_{k+1}(t) = 0,\ -X \le t \le 0,\ k \ge 0 \tag{1.7}$$

Figure 1.6 shows the response of this controlled process in the case when $k_1 = 0.8,\ k_2 = 1,\ X = 1.25,\ \alpha = 10$ to a downward step change in $R_{k+1}(t)$ applied at $t = 0$ on each pass, i.e. $R_{k+1}(t) = -1,\ 0 \le t \le 10,\ k \ge 0$. Note here that the oscillations grow, or increase in amplitude, severely from pass-to-pass (i.e. in the k direction). Consequently the deterioration in performance after the first pass must be due to the effects of the cut floor profile on the previous pass. In other words, the output dynamics on any pass acts (by the basic system geometry) as a forcing function (or disturbance) on, and hence contributes to, the dynamics of the next pass, i.e. the shape of the floor profile produced on the next pass of the cutting machine along the coal face. This interaction between successive pass profile dynamics is a unique characteristic of all repetitive processes and in cases such as that of Fig. 1.6 appropriate control action is clearly required.

If the example under consideration is single-input single-output (SISO) and the dynamics are assumed to be linear, an obvious intuitive approach to

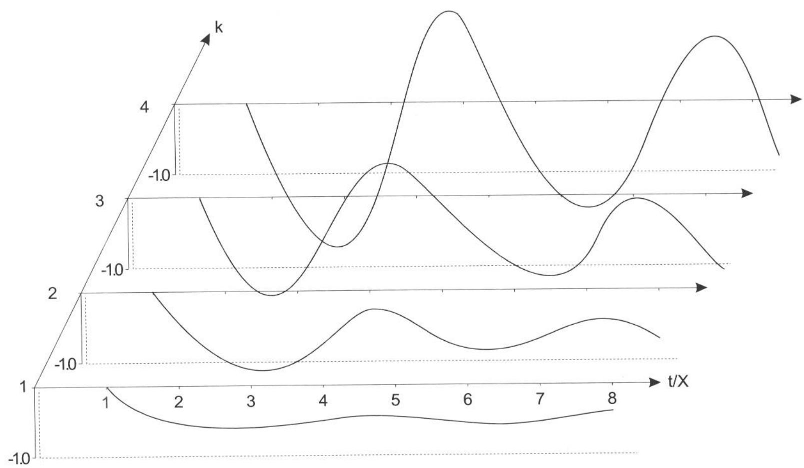

Fig. 1.6. Closed-loop system negative unit step response