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Complex Analysis Joensuu 1978

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Ilpo Laine, Olli Lehto, and Tuomas Sorvali

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Editors

Ilpo Laine
University of Joensuu
Department of Mathematics
and Physics
SF-80101 Joensuu 10 / Finland

Olli Lehto
University of Helsinki
Department of Mathematics
SF-00100 Helsinki 10 / Finland

Tuomas Sorvali
University of Joensuu
Department of Mathematics
and Physics
SF-80101 Joensuu 10 / Finland

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PREFACE

This volume consists of papers presented at the Colloquium on Complex Analysis held at the University of Joensuu, August 24 - 27, 1978. The IV Romanian-Finnish Seminar on Complex Analysis was organized as a part of this Colloquium. The major part of the contributions in this volume is related to the theory of quasiconformal and quasiregular mappings, Nevanlinna theory and complex differential equations, Riemann surfaces and potential theory.

We wish to thank the staff of the Department of Mathematics and Physics in the University of Joensuu for their cooperation in organizing these meetings and preparing this volume, Springer-Verlag for their willingness to publish this volume and, finally, Eija Faari and Riitta Laakkonen for their patient job of typing the manuscript.

Joensuu and Helsinki, April 1979,

Ilpo Laine Olli Lehto Tuomas Sorvali

OTHER LECTURES GIVEN AT THE COLLOQUIUM(§)

Ahlfors, L. V.: Beltrami differentials in several dimensions (Enseignement math., II. Sér. 24 (1978), 225 - 236)

Apostol, C.: Comments on a theorem on invariant subspaces by Scott Brown(*)

Blatter, Chr.: A two variables distortion theorem for univalent functions (Comm. math. Helv. 53 (1978), 651 - 659)

Blevins, D. K.: Conformal mappings and quasicircles

Boboc, N.: Standard H-cones(*)

Bojarski, B.: Analytic methods in the quasiconformal theory in R^n

Bshouty, D.: Löwner differential equation and quasiconformal extensions of conformal mappings

Bucur, Gh.: Standard H-cones(*)

Cegrell, U.: Construction of capacities on C^n

Colojoară, I.: On a functional calculus based on Cauchy-Pompeiu's formula(*)

Douady, A.: Projective structures on Riemann surfaces

Essén, M., Shea, D.: On the case of equality in some inequalities of A. Baernstein (to appear in Ann. Acad. Sci. Fenn.)

Fuchs, W.: On the nodes of best approximation by polynomials in the Chebychev sense

Gackstatter, F., Laine, I.: Zur Theorie der gewöhnlichen Differentialgleichungen im Komplexen (to appear in Ann. Polon. Math.)(*)

Gehring, F. W.: Remarks on the Schwarzian derivative

Gowrisankaran, K.: Construction of inner functions of polydiscs (to appear in Ann. Inst. Fourier)

(§) A reference indicates a related article.

(*) A contribution for the IV Romanian-Finnish Seminar on Complex Analysis

Hengartner, W., Gauthier, P.: Uniform approximation and simultaneous interpolation

Huber, A.: Isometric and conformal sewing (Comm. Math. Helv. 50 (1975), 179 - 186, ibid. 51 (1976), 319 - 331, and a forthcoming article)

Kiselman, Chr.: On the density of plurisubharmonic functions: a short proof of Siu's theorem (to appear in Bull. Soc. math. France)

Lelong, P.: An inverse function theorem for plurisubharmonic functions (to appear in Séminaire P. Lelong)

Matsuda, M.: Algebraic differential equations of the first order free from parametric singularities from the differential-algebraic standpoint (to appear in J. math. Soc. Japan)

Menke, K.: Näherung der Lösung des Dirichlet Problems durch ein Interpolationsverfahren

Meyer, G.: On the zeros of exponential polynomials (to appear in Arch. Math.)

Mues, E.: Über die Werteverteilung von Differentialpolynomen

Netanyahu, E., Schiffer, M. M.: On the monotonicity of some functionals in the family of univalent functions (to appear in Israel J. Math.)

Ohtsuka, M.: On type problem of Riemann surfaces

Osgood, B.: A univalent criterion for multiply-connected domains

Palka, B.: Quasiconformally homogeneous domains

Rickman, S.: Omitted values, counting function and equidistribution of quasiregular mappings (to appear in Acta math.)(*)

Rubel, L. A.: First-order conformal invariants

Sakai, M.: Analytic functions with finite Dirichlet integrals on Riemann surfaces (to appear in Acta math.)

Schwarz, B.: Disconjugacy of complex second-order matrix differential systems

Siciak, J.: On holomorphic extendability of functions on generic real analytic submanifolds (to appear in Bull. Acad. Polon. Sci.)

Siddiqi, J. A.: Nonquasianalytic classes of functions and uniform approximation on arcs by exponential sums

Sontag, A.: On the existence of substantial boundary points for extremal quasiconformal maps with angular dilatation

Stoica, L.: Axiomatic approach to potential theory associated with elliptic degenerated operators(*)

Vaaler, J.: An inequality for the volume of a centrally sliced cube in R^n (to appear in Pacific J. Math.)

Voiculescu, D., Bercovici, H.: Tensor operations on characteristic functions of C_o-contractions (to appear in Acta Sci. math.)(*)

Winkler, J.: Zur Existenz ganzer Funktionen bei vorgegebener Menge der Nullstellen und Einsstellen

COLLOQUIUM PARTICIPANTS

Ahlfors, L. V.	Harvard University, Cambridge, MA, U.S.A.
Alastalo, Hannu (*)	University of Joensuu, Finland
Anderson, Glen D.	Michigan State University, East Lansing, MI, U.S.A.
Andreian Cazacu, Cabiria (*)	Universitatea din Bucureşti, Romania
Apostol, Constantin (*)	INCREST, Bucureşti, Romania
Arsene, Grigore (*)	INCREST, Bucureşti, Romania
Aulaskari, Rauno (*)	University of Joensuu, Finland
Bakken, Ivar	Universitetet i Tromsø, Norway
Bayoumi, Aboubakr	Uppsala Universitet, Sweden
Becker, Jochen	Technische Universität Berlin, Federal Republic of Germany
Berg, Christian	Københavns Universitet, Denmark
Blatter, Christian	Eidgenössische Technische Hochschule, Zürich, Switzerland
Blevins, Donald K.	University of Florida, Gainesville, FL, U.S.A.
Boboc, Nicu (*)	Universitatea din Bucureşti, Romania
Bojarski, Bogdan	Uniwersytet Warszawski, Warszawa, Poland
Brennan, James	University of Kentucky, Lexington, KY, U.S.A.
Bshouty, Daoud	Eidgenössische Technische Hochschule, Zürich, Switzerland
Bucur, Gheorghe (*)	INCREST, Bucureşti, Romania
Campbell, Douglas	Brigham Young University, Provo, UT, U.S.A.
Caraman, Petru (*)	Universitatea "Al. I. Cuza", Iaşi, Romania
Cegrell, Urban	Uppsala Universitet, Sweden
Coeuré, G.	Université de Lille, France
Colojoară, Ion (*)	Universitatea din Bucureşti, Romania
Cornea, Aurel (*)	INCREST, Bucureşti, Romania
Douady, Adrien	Université Paris-Sud, Orsay, France
Drasin, David	Purdue University, West Lafayette, IN, U.S.A.
Earle, Clifford J.	Cornell University, Ithaca, NY, U.S.A.
Erkama, Timo (*)	University of Helsinki, Finland
Essén, Matts	Kungliga Tekniska Högskolan, Stockholm, Sweden
Farkas, Hershel	Hebrew University, Jerusalem, Israel
Flondor, Paul (*)	Institutul Politehnic Bucureşti, Romania

(*) A member of the IV Romanian-Finnish Seminar on Complex Analysis

Frank, Günter	Fernuniversität Hagen, Federal Republic of Germany
Fuchs, Wolfgang	Cornell University, Ithaca, NY, U.S.A.
Fuglede, Bent	Københavns Universitet, Denmark
Fuji'i'e, Tatsuo	Kyoto University, Japan
Gackstatter, Fritz	Rheinisch-Westfälische Technische Hochschule, Aachen, Federal Republic of Germany
Gauld, David	University of Auckland, New Zealand
Gauthier, Paul	Université de Montréal, Canada
Geatti, Laura	Università di Pisa, Italy
Gehring, Frederick W.	University of Michigan, Ann Arbor, MI, U.S.A.
Ghişa, Dorin(*)	Universitatea din Timişoara, Romania
Göktürk, Zerrin	Boğaziçi Üniversitesi, Bebek-Istanbul, Turkey
Gowrisankaran, Kohur	McGill University, Montreal, Canada
Granlund, Seppo(*)	Helsinki University of Technology, Finland
Grassman, E. G.	University of Calgary, Alberta, Canada
Grunsky, Helmut	Universität Würzburg, Federal Republic of Germany
Gussi, Gheorghe(*)	INCREST, Bucureşti, Romania
Haario, Heikki(*)	University of Helsinki, Finland
Hag, Kari	Norges Tekniske Høgskole, Trondheim, Norway
Hag, Per	Norges Laererhøgskole, Trondheim, Norway
Harmelin, Reuven	Technion-Israel Institute of Technology, Haifa, Israel
Hellerstein, Simon	University of Wisconsin, Madison, WI, U.S.A.
Hengartner, Walter	Université Laval, Québec, Canada
Hennekemper, Wilhelm	Universität Dortmund, Federal Republic of Germany
Hubbard, John	Cornell University, Ithaca, NY, U.S.A.
Huber, Alfred	Eidgenössische Technische Hochschule, Zürich, Switzerland
Hyvönen, Jaakko(*)	University of Joensuu, Finland
Ikegami, Teruo	Osaka City University, Japan
Ivaşcu, Dumitru(*)	Universitatea din Bucureşti, Romania
Jackson, Howard L.	McMaster University, Hamilton, Canada
Janßen, Klaus	Universität Düsseldorf, Federal Republic of Germany
Johnson, Raymond	Howard University, Washington, DC, U.S.A.
Kahramaner, Suzan	İstanbul Üniversitesi, Turkey
Kangasaho, Jukka(*)	University of Joensuu, Finland

Kiikka, Maire(*)	University of Helsinki, Finland
Kiltinen, John O.	Northern Michigan University, Marquette, MI, U.S.A.
Kiselman, Christer	Uppsala Universitet, Sweden
Knab, Otto	Universität Karlsruhe (TH), Federal Republic of Germany
Korevaar, Jacob	Universiteit van Amsterdam, Netherlands
Kortram, Ronald	Universiteit te Nijmegen, Netherlands
Kotman, Larry	University of Wisconsin, La Crosse, WI, U.S.A.
Kra, Irwin	State University of New York, Stony Brook, NY, U.S.A.
Kuusalo, Tapani(*)	University of Jyväskylä, Finland
Labrèche, Martine	Université de Montréal, Canada
Lahtinen, Aatos(*)	University of Helsinki, Finland
Laine, Ilpo(*)	University of Joensuu, Finland
Latvamaa, Esko(*)	Technical College of Joensuu, Finland
Launonen, Eero(*)	Technical College of Kuopio, Finland
Ławrynowicz, Julian	Instytut Matematyczny PAN, Łódź, Poland
Lelong, Pierre	Université Paris VI, France
Lindfors, Ilpo(*)	University of Joensuu, Finland
Louhivaara, I. S.(*)	Freie Universität Berlin, Federal Republic of Germany and University of Jyväskylä, Finland
Maeda, Fumi-Yuki	Hiroshima University, Japan
Martens, Henrik H.	Norges Tekniske Høgskole, Trondheim, Norway
Martio, Olli(*)	University of Helsinki, Finland
Matsuda, Michihiko	Osaka University, Japan
Mattila, Pertti(*)	University of Helsinki, Finland
Menke, Klaus	Universität Dortmund, Federal Republic of Germany
Meyer, Gottfried	Universität Würzburg, Federal Republic of Germany
Miles, Joseph	University of Illinois at Urbana-Champaign, Urbana, IL, U.S.A.
Miniowitz, Ruth	Technion-Israel Institute of Technology, Haifa, Israel
Mues, Erwin	Universität Karlsruhe (TH), Federal Republic of Germany
Nevanlinna, Rolf(*)	Academy of Finland, Helsinki, Finland
Netanyahu, Elisha	Technion-Israel Institute of Technology, Haifa, Israel
Niemi, Hannu(*)	University of Helsinki, Finland
Nguyen-Xuan-Loc	Université Paris-Sud, Orsay, France
Näätänen, Marjatta(*)	University of Helsinki, Finland

Ohtsuka, Makoto	Hiroshima University, Japan
Oja, Kirsti (*)	Helsinki University of Technology, Finland
Ortel, Marvin	University of Hawaii, Honolulu, HI, U.S.A.
Osgood, Brad	University of Michigan, Ann Arbor, MI, U.S.A.
Palka, Bruce	University of Texas, Austin, TX, U.S.A.
Pearcy, Carl	University of Michigan, Ann Arbor, MI, U.S.A.
Peschl, Ernst	Universität Bonn, Federal Republic of Germany
Pesonen, Martti (*)	University of Joensuu, Finland
Pfluger, Albert	Eidgenössische Technische Hochschule, Zürich, Switzerland
Piranian, George	University of Michigan, Ann Arbor, MI, U.S.A.
Pirinen, Aulis (*)	Helsinki University of Technology, Finland
Radu, Nicolae (*)	Universitatea din Bucureşti, Romania
Reich, Edgar	University of Minnesota, Minneapolis, MN, U.S.A.
Reimann, Martin	Universität Bern, Switzerland
von Renteln, Michael	Universität Giessen, Federal Republic of Germany
Rickman, Seppo (*)	University of Helsinki, Finland
Riihentaus, Juhani (*)	Technical College of Oulu, Finland
Rubel, Lee A.	University of Illinois at Urbana-Champaign, Urbana, IL, U.S.A.
Räsänen, Sisko (*)	University of Joensuu, Finland
Sakai, Makoto	Hiroshima University, Japan
Sarvas, Jukka (*)	University of Helsinki, Finland
Schober, Glenn	Indiana University, Bloomington, IN, U.S.A.
Schwarz, Binyamin	Technion-Israel Institute of Technology, Haifa, Israel
Seppälä, Mika (*)	Helsinki School of Economics, Finland
Shea, Daniel	University of Wisconsin, Madison, WI, U.S.A.
Siciak, Józef	Uniwersytet Jagielloński, Kraków, Poland
Siddiqi, Jamil A.	Université Laval, Québec, Canada
Sontag, Alexia	Wellesley College, MA, U.S.A.
Sorvali, Esko (*)	Technical College of Joensuu, Finland
Sorvali, Tuomas (*)	University of Joensuu, Finland
Steinmetz, Norbert	Universität Karlsruhe (TH), Federal Republic of Germany
Stoica, Lucreţiu (*)	INCREST, Bucureşti, Romania
Strebel, Kurt	Universität Zürich, Switzerland
Suciu, Ion (*)	INCREST, Bucureşti, Romania

Sung, Chen-Han	Purdue University, West Lafayette, IN, U.S.A.
Tammi, Olli(*)	University of Helsinki, Finland
Tanaka, Hiroshi	Hokkaido University, Sapporo, Japan
Tietz, Horst	Technische Universität Hannover, Federal Republic of Germany
Vaaler, Jeffrey	University of Texas, Austin, TX, U.S.A.
Valuşescu, Ilie(*)	INCREST, Bucureşti, Romania
Vasilescu, Florin-Horia(*)	INCREST, Bucureşti, Romania
Voiculescu, Dan(*)	INCREST, Bucureşti, Romania
Vuorinen, Matti(*)	University of Helsinki, Finland
Wallin, Hans	Umeå Universitet, Sweden
Weitsman, Allen	Purdue University, West Lafayette, IN, U.S.A.
Winkler, Jörg	Technische Universität Berlin, Federal Republic of Germany

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AN EXTREMAL DISPLACEMENT MAPPING IN n-SPACE

G. D. Anderson and M. K. Vamanamurthy

1. Introduction.

1.1. Statement of problem. In this paper we solve the extremal problem of finding a self-homeomorphism F_n of the unit ball B^n in R^n satisfying the following conditions: For fixed r, $0 < r < 1$,

a) F_n keeps the boundary $\partial B^n = S^{n-1}$ pointwise fixed,

b) $F_n(0,0,\ldots,0) = (-r,0,\ldots,0)$,

c) F_n maps a 2-dimensional plane section $R^2 \cap B^n$ containing $B^1 = \{(x_1,0,\ldots,0)\} : |x_1| < 1\}$ onto another such,

d) F_n is quasiconformal with minimum linear dilatation

$$K(F_n) = \operatorname*{ess\,sup}_{x\in B^n} \frac{L_n(x)}{\ell_n(x)},$$

where

$$L_n(x) = \limsup_{y\to x} \frac{|F_n(y) - F_n(x)|}{|y - x|}, \quad \ell_n(x) = \liminf_{y\to x} \frac{|F_n(y) - F_n(x)|}{|y - x|}$$

denote the maximum and minimum stretchings at x, respectively. We shall call F_n an extremal displacement mapping in n-space.

1.2. Acknowledgement. The authors wish to thank Professor F. W. Gehring for conversations about this problem.

1.3. Description of the mapping. Since the extremal problem was solved by Teichmüller for $n = 2$ [4] and since we make use of his result, we begin with a brief description of the extremal displacement mapping F_2 of Teichmüller.

The plane mapping

$$x_1 + ix_2 = f_1(u_1 + iu_2) = r\,\mathrm{tn}^2\left(\frac{2K}{\pi}\sinh^{-1}\frac{u_1 + iu_2}{2}, r'\right)$$

maps the quarter ellipse $u_1^2/b^2 + u_2^2/a^2 < 1$, $u_1 > 0$, $u_2 > 0$ conformally onto the upper half disk $|x_1 + ix_2| < 1$, $x_2 > 0$. Here K and K' are the complete elliptic integrals of the first kind defined by

$$K = K(r) = \int_0^1 [(1 - t^2)(1 - r^2t^2)]^{-1/2}dt,$$

$$K' = K(r'),\ r' = (1 - r^2)^{1/2},$$

tn denotes the Jacobian elliptic tangent function, and

$$a = R + R^{-1},\ b = R - R^{-1},\ R = \exp \frac{\pi K'}{4K} . \qquad (1)$$

Likewise

$$y_1 + iy_2 = f_2(v_1 + iv_2) = r\ \mathrm{tn}^2\left(\frac{2K}{\pi} \cosh^{-1} \frac{v_1 + iv_2}{2}, r'\right)$$

maps the quarter ellipse $v_1^2/a^2 + v_2^2/b^2 < 1$, $v_1 > 0$, $v_2 > 0$ conformally onto the upper half disk $|y_1 + iy_2| < 1$, $y_2 > 0$. Then the extremal displacement mapping F_2 is given by

$$F_2 = f_2 \circ \varphi \circ f_1^{-1}$$

for $x_2 > 0$, where φ is the affine mapping

$$v_1 + iv_2 = \varphi(u_1 + iu_2) = \frac{a}{b}u_1 + i\,\frac{b}{a}u_2 .$$

Finally the mapping F_2 is extended by reflection in the x_1-axis to the unit x_1x_2-disk.

Now for each $n \geqq 3$ let F_n be the self-mapping of B^n obtained by rotating F_{n-1} about R^{n-2} in R^n (see §3 below). Then F_n has the above required properties, and we shall prove the following

<u>Theorem 1.</u> For $0 < r < 1$ and $n \geqq 2$ the mapping F_n described above is an extremal quasiconformal self-mapping of B^n with $K(F_n) = K(F_2) = \coth^2 \frac{\pi K'(r)}{4K(r)}$.

<u>Conjecture.</u> Condition c) in §1.1 above can be removed.

2. Proof of theorem for $n = 3$.

First take $n = 3$ and let P_1 be any point in B^3. By symmetry we may obviously assume that $P_1 = (x_1,x_2,0)$, where $x_2 \geqq 0$. Let $P_2 = (y_1,y_2,0)$ and $Q_1 = (u_1,u_2)$, $Q_2 = (v_1,v_2)$, where

$$y_1 + iy_2 = F_2(x_1 + ix_2)$$

$$u_1 + iu_2 = f_1^{-1}(x_1 + ix_2) ,$$

$$v_1 + iv_2 = f_2^{-1}(y_1 + iy_2) .$$

Then the maximum stretching $L_2(P_1) = L_2$ and the minimum stretching $\ell_2(P_1) = \ell_2$ of F_2 at P_1 are

$$L_2 = \frac{a|f_2'(Q_2)|}{b|f_1'(Q_1)|},\quad \ell_2 = \frac{b|f_2'(Q_2)|}{a|f_1'(Q_1)|}.$$