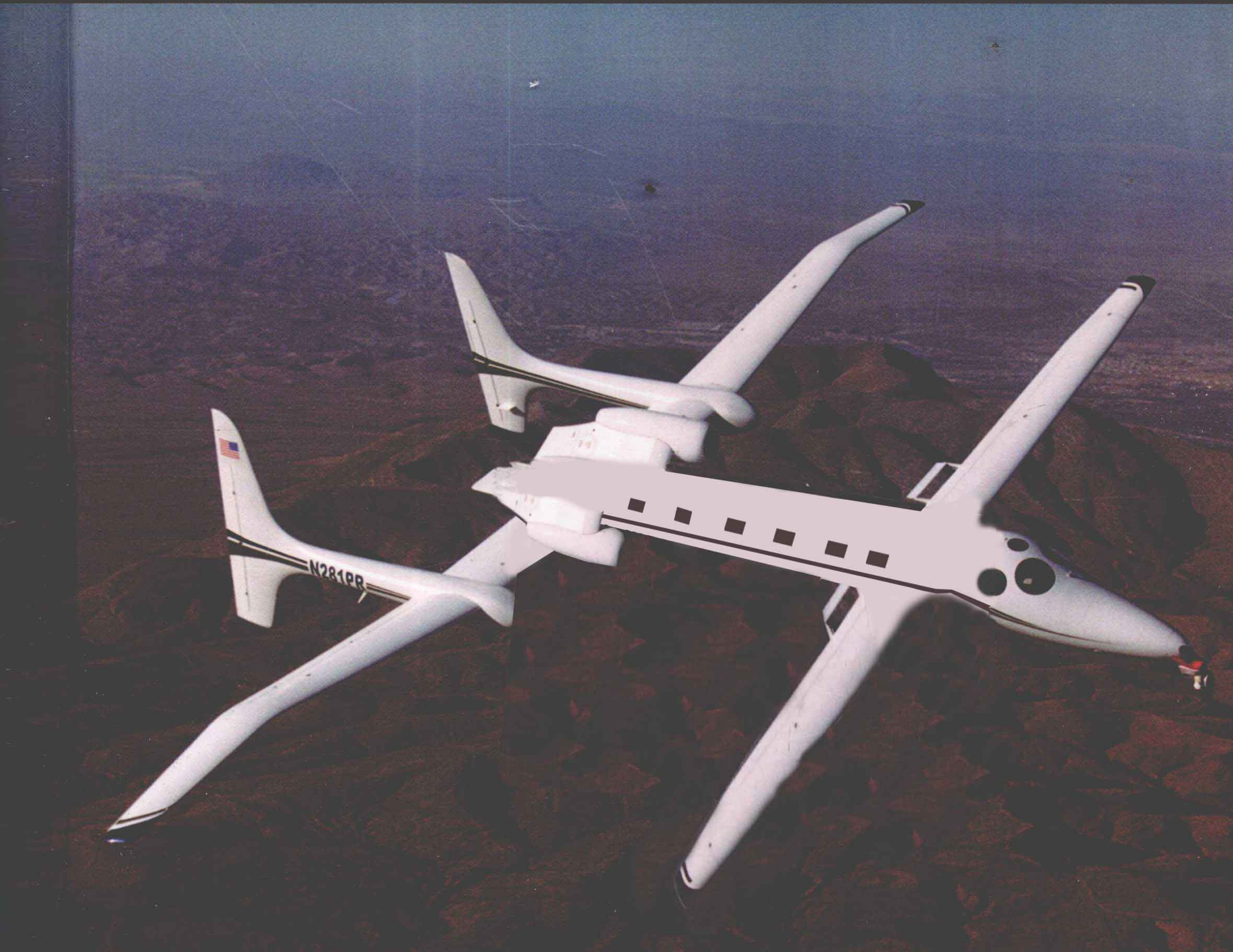
Ferdinand P. Beer ▪ E. Russell Johnston, Jr. ▪ William E. Clausen

VECTOR MECHANICS *for* ENGINEERS

Dynamics

Seventh Edition



Seventh Edition

VECTOR MECHANICS FOR ENGINEERS

Dynamics

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Cover Photograph: Courtesy of NASA. The Proteus aircraft was designed and built by Burt Rutan, president of Scaled Composites, Inc., to carry an 18-foot diameter telecommunications antenna system for relay of broadband data over major cities. The design allows for Proteus to be reconfigured at will for a variety of other missions such as atmospheric research, reconnaissance, commercial imaging, and launch of small space satellites. In October of 2000, Proteus set three world records for performance in its weight class, including a maximum altitude record of 62,786 feet. The flights were conducted under the sponsorship of the NASA Office of Earth Science with funding provided by the NOAA/DoD/NASA Integrated Program Office. This sponsorship includes evaluation of the Proteus aircraft as an atmospheric science and remote sensing airborne platform. The aircraft consists of an all composite airframe with graphite-epoxy sandwich construction. It has a wingspan of 77 feet 7 inches, expandable to 92 feet with removable wingtips installed. It is 56.3 feet long and 17.6 feet high and weighs 5,900 pounds, empty. Proteus is powered by two Williams-Rolls FJ44-2 turbofan engines developing 2,300 pounds of thrust each.

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About the Authors

As one of the most successful author teams in engineering education, Ferd Beer and Russ Johnston are often asked how, with one author at Lehigh and the other at the University of Connecticut, they happened to write their books together and how they managed to keep collaborating on their many books with many successive revisions.

The answer to this question is simple. Russ Johnston's first teaching appointment was in the Department of Civil Engineering and Mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in mechanics.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the mechanics courses. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented to them in a graphic way. Together they wrote lecture notes in statics and dynamics, to which they later added problems they felt would appeal to future engineers, and soon they produced the manuscript of the first edition of *Mechanics for Engineers*.

The second edition of *Mechanics for Engineers* and the first edition of *Vector Mechanics for Engineers* found Russ Johnston at Worcester Polytechnic Institute and the next editions at the University of Connecticut. In the meantime, both Ferd and Russ assumed administrative responsibilities in their departments, and both were involved in research, consulting, and supervising graduate students—Ferd in the area of stochastic processes and random vibrations and Russ in the area of elastic stability and structural analysis design. However, their interest in improving the teaching of the basic mechanics courses had not subsided, and they both taught sections of these courses as they kept revising their texts and began writing the manuscript of the first edition of their *Mechanics of Materials* text.

Their collaboration has spanned many years and many successful revisions of all of their textbooks, and Ferd and Russ's contributions to engineering education have earned them a number of honors and awards. They were presented with the Western Electric Fund Award for excellence in the instruction of engineering students by their respective regional sections of the American Society for Engineering Education, and they both received the Distinguished Educator Award from the Mechanics Division of the same society. Starting in 2001, the New Mechanics Educator Award of the Mechanics Division has been named in honor of the Beer and Johnston author team.

Ferd and Russ are delighted to bring a new co-author and collaborator to the Beer and Johnston team for the seventh edition of *Vector Mechanics for Engineers: Dynamics.* William Clausen, who studied with and was inspired by Ferd Beer, joins the team as co-author for *Dynamics*. George Staab developed the interactive tutorial to accompany the sixth and seventh editions, and now he joins the seventh edition team as a collaborator on both the *Statics* and *Dynamics* texts.

Ferdinand P. Beer. Born in France and educated in France and Switzerland, Ferd holds an M.S. degree from the Sorbonne and an Sc.D. degree in theoretical mechanics from the University of Geneva. He came to the United States after serving in the French army during the early part of World War II and taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Following his service at Williams College, Ferd joined the faculty of Lehigh University where he taught for thirty-seven years. He held several positions, including University Distinguished Professor and Chairman of the Department of Mechanical Engineering and Mechanics, and in 1995 Ferd was awarded an honorary Doctor of Engineering degree by Lehigh University.

E. Russell Johnston, Jr. Born in Philadelphia, Russ holds a B.S. degree in civil engineering from the University of Delaware and an Sc.D. degree in the field of structural engineering from the Massachusetts Institute of Technology. He taught at Lehigh University and Worcester Polytechnic Institute before joining the faculty of the University of Connecticut where he held the position of chairman of the Civil Engineering Department and taught for twenty-six years. In 1991 Russ received the Outstanding Civil Engineer Award from the Connecticut Section of the American Society of Civil Engineers.

William E. Clausen. Bill holds a B.S. degree in engineering mechanics from Lehigh University and M.S. and Ph.D. degrees in engineering mechanics from the Ohio State University. Bill is a registered professional engineer specializing in structural dynamics and vibration measurements. He taught for thirty years and served as vice chairman in the Department of Engineering Mechanics at the Ohio State University and has also taught in the Department of Mechanical Engineering.

George H. Staab. George holds B.S., M.S., and Ph.D. degrees in aeronautical engineering from Purdue University. He worked at Sikorsky Aircraft for three years and is currently an Associate Professor in Mechanical Engineering at the Ohio State University where he has taught for twenty-two years. His scholarly activities are focused on teaching and service and he has received two college-wide awards: the Charles E. MacQuigg Outstanding Teaching Award in 1998 and the Boyer Teaching Award in 1999. George serves as the campus representative for the American Society for Engineering Education, and is the faculty advisor for the student chapter of the American Society of Mechanical Engineering and several student project teams.

Preface

OBJECTIVES

The main objective of a first course in mechanics should be to develop in the engineering student the ability to analyze any problem in a simple and logical manner and to apply to its solution a few, well-understood, basic principles. It is hoped that this text, as well as the preceding volume, *Vector Mechanics for Engineers: Statics*, will help the instructor achieve this goal.†

GENERAL APPROACH

Vector algebra was introduced at the beginning of the first volume and is used in the presentation of the basic principles of statics, as well as in the solution of many problems, particularly three-dimensional problems. Similarly, the concept of vector differentiation will be introduced early in this volume, and vector analysis will be used throughout the presentation of dynamics. This approach leads to more concise derivations of the fundamental principles of mechanics. It also makes it possible to analyze many problems in kinematics and kinetics which could not be solved by scalar methods. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.‡

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies*. This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics*, the statics of particles is treated first, and the principle of equilibrium of a particle was immediately applied to practical

†Both texts also are available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics*, seventh edition.

‡In a parallel text, *Mechanics for Engineers: Dynamics*, fourth edition, the use of vector algebra is limited to the addition and subtraction of vectors, and vector differentiation is omitted.

situations involving only concurrent forces. The statics of rigid bodies is considered later, at which time the vector and scalar products of two vectors were introduced and used to define the moment of a force about a point and about an axis.

- In *Dynamics,* the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

New Concepts Are Introduced in Simple Terms. Since this text is designed for the first course in dynamics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach has been achieved. For example, the concept of potential energy is discussed in the general case of a conservative force. Also, the study of the plane motion of rigid bodies is designed to lead naturally to the study of their general motion in space. This is true in kinematics as well as in kinetics, where the principle of equivalence of external and effective forces is applied directly to the analysis of plane motion, thus facilitating the transition to the study of three-dimensional motion.

Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive,* simple applications are considered first. For example:

- The kinematics of particles (Chap. 11) precedes the kinematics of rigid bodies (Chap. 15).
- The fundamental principles of the kinetics of rigid bodies are first applied to the solution of two-dimensional problems (Chaps. 16 and 17), which can be more easily visualized by the student, while three-dimensional problems are postponed until Chap. 18.

The Presentation of the Principles of Kinetics Is Unified. The seventh edition of *Vector Mechanics for Engineers* retains the unified presentation of the principles of kinetics which characterized the previous six editions. The concepts of linear and angular momentum are introduced in Chap. 12 so that Newton's second law of motion can be presented not only in its conventional form $\mathbf{F} = m\mathbf{a}$, but also as a law relating, respectively, the sum of the forces acting on a particle and the sum of their moments to the rates of change of the linear and angular momentum of the particle. This makes possible an earlier introduction of the principle of conservation of angular momentum and a more meaningful discussion of the motion of a particle under a central force (Sec. 12.9). More importantly, this approach can be readily extended to the study of the motion of a system of particles (Chap. 14) and leads to a more concise and unified treatment of the kinetics of rigid bodies in two and three dimensions (Chaps. 16 through 18).

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems. Free-body diagrams were introduced early in statics, and their importance was emphasized throughout. They were used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on "free-body-diagram equations" rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers,* has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Four-Color Presentation Uses Color to Distinguish Vectors. Color is used, not only to enhance the quality of the illustrations, but also to help students distinguish among the various types of vectors they will encounter. While there is no intention to "color code" this text, the same color is used in any given chapter to represent vectors of the same type. Throughout *Statics,* for example, red is used exclusively to represent forces and couples, while position vectors are shown in blue and dimensions in black. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text. In *Dynamics,* for the chapters on kinetics, red is used again for forces and couples, as well as for effective forces. Red is also used to represent impulses and momenta in free-body-diagram equations, while green is used for velocities, and blue for accelerations. In the two chapters on kinematics, which do not involve any forces, blue, green, and red are used, respectively, for displacements, velocities, and accelerations.

A Careful Balance between SI and U.S. Customary Units Is Consistently Maintained. Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder are in U.S. customary units. The authors believe that this approach will best serve the need of the students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or $lb \cdot s^2/ft$). The authors, therefore, believe that problem assignments should include both systems of units.

The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

Optional Sections Offer Advanced or Specialty Topics. A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic dynamics course. They can be omitted without prejudice to the understanding of the rest of the text.

The topics covered in the optional sections include graphical methods for the solution of rectilinear-motion problems, the trajectory of a particle under a central force, the deflection of fluid streams, problems involving jet and rocket propulsion, the kinematics and kinetics of rigid bodies in three dimensions, damped mechanical vibrations, and electrical analogues. These topics will be found of particular interest when dynamics is taught in the junior year.

The material presented in the text and most of the problems requires no previous mathematical knowledge beyond algebra, trigonometry, elementary calculus, and the elements of vector algebra presented in Chaps. 2 and 3 of the volume on statics.† However, special problems are included, which make use of a more advanced knowledge of calculus, and certain sections, such as Secs. 19.8 and 19.9 on damped vibrations, should be assigned only if students possess the proper mathematical background. In portions of the text using elementary calculus, a greater emphasis is placed on the correct understanding and application of the concepts of differentiation and integration, than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of moment of area firmly before introducing the use of integration.

†Some useful definitions and properties of vector algebra have been summarized in Appendix A at the end of this volume for the convenience of the reader. Also, Secs. 9.11 through 9.18 of the volume on statics, which deal with the moments of inertia of masses, have been reproduced in Appendix B.

NEW TO THIS EDITION

While retaining the well-received approach and organization of previous editions, the *seventh edition* offers the following new features and improvements:

- Ninety percent of the homework problems for the seventh edition are new or revised. The emphasis on industry-related and discipline-specific questions of the new edition problems provides motivation for today's students.
- The computer problems have been revised to be used with popular computational software, and the number of problems has been increased. The computer problems, many of which are relevant to the design process, are included in a special section at the end of each chapter. The problems focus on symbolic manipulation and plotting, as opposed to the programming-based computer problems in previous editions of the text.
- Numerous in-chapter photographs have been added to help students better visualize important concepts.
- Chapter outlines have been added to the introduction of each chapter to provide a preview of topics that will be covered in the chapter.
- A Fundamentals of Engineering Examination Appendix has been added for use when students prepare for the FE exam.

CHAPTER ORGANIZATION AND PEDAGOGICAL FEATURES

Chapter Introduction. Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems. New chapter outlines provide students with a preview of chapter topics.

Chapter Lessons. The body of the text is divided into units, each consisting of one or several theory sections, one or several sample problems, and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic.

Sample Problems. The sample problems are set up in much the same form that students will use when solving the assigned problems. They thus serve the double purpose of amplifying the text and demonstrating the type of neat, orderly work that students should cultivate in their own solutions.

Solving Problems on Your Own. A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the homework problems. Also included in these sections are specific suggestions and strategies that will enable the students to more efficiently attack any assigned problems.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Computer Problems. Each chapter includes a set of problems designed to be solved with computational software. Many of these problems provide an introduction to the design process. For example, they may involve the determination of the motion of a particle under initial conditions, the kinematic or kinetic analysis of mechanisms in successive positions, or the numerical integration of various equations of motion. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.

SUPPLEMENTS

An extensive supplements package for both instructors and students is available with the text. Instructor resources include: an instructor's solutions manual with complete solutions to all text problems; image sets with electronic files of all text art and photo images; PowerPoint lecture presentations for all text chapters; transparencies of additional solved problems; scripts in various computational software formats for all text computer problems; access to course management systems to accommodate your online course needs; and various other presentation and course organization resources.

Students have access to S.M.A.R.T. (Self-paced, Mechanics, Algorithmic, Review, and Tutorial), an online interactive tutorial with algorithmic quizzing which can also be used as a classroom presentation tool. Other student resources include FE Exam-style multiple-choice quizzes with feedback; a guide for using computational software packages in mechanics courses; and many more internet-based content and learning tools.

Please visit our *Vector Mechanics for Engineers: Statics and Dynamics* seventh edition Online Learning Center (OLC) at www.mhhe.com/beerjohnston7 for more information on the supplements available with this text.

ACKNOWLEDGMENTS

The authors wish to acknowledge the collaboration of George Staab to this seventh edition of *Vector Mechanics for Engineers* and thank him especially for his crucial role in making the extensive problem set revision possible.

A special thanks go to our colleagues who thoroughly checked the solutions and answers of all problems in this edition and then prepared the solutions for the accompanying *Instructor's and Solution Manual:* Dean Updike of Lehigh University, Richard H. Lance of Cornell University, Petru Petrina of Cornell University, and Gerald Rehkugler of Cornell University.

The authors thank the many companies that provided photographs for this edition. We also wish to recognize the determined efforts and patience of our photo researcher Sabina Dowell.

The authors also thank the members of the staff at McGraw-Hill for their support and dedication during the preparation of this new edition. We particularly wish to acknowledge the contributions of Sponsoring Editor Debra Matteson and Project Manager Jane Mohr.

The authors gratefully acknowledge the many helpful comments and suggestions offered by users of the previous editions of *Vector Mechanics for Engineers*.

Ferdinand P. Beer
E. Russell Johnston, Jr.
William E. Clausen

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The McGraw-Hill engineering mechanics editorial and marketing team would like to join the authors in gratefully acknowledging the individuals listed in the authors' preface for their contributions to the seventh editions of the texts and solutions manuals. In addition, we would also like to thank the following reviewers, focus group attendees, and symposium attendees whose comments and suggestions over the past few years have not only helped us to develop the supplemental and media resources for the seventh editions of *Vector Mechanics for Engineers* but have also helped us to establish standards and goals for developing other texts, supplements, and new media resources for statics and dynamics courses.

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List of Symbols

$\mathbf{a}, a$	Acceleration
a	Constant; radius; distance; semimajor axis of ellipse
$\overline{\mathbf{a}}, \overline{a}$	Acceleration of mass center
$\mathbf{a}_{B/A}$	Acceleration of B relative to frame in translation with A
$\mathbf{a}_{P/\mathcal{F}}$	Acceleration of P relative to rotating frame $\mathcal{F}$
$\mathbf{a}_c$	Coriolis acceleration
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Reactions at supports and connections
$A, B, C, \ldots$	Points
A	Area
b	Width; distance; semiminor axis of ellipse
c	Constant; coefficient of viscous damping
C	Centroid; instantaneous center of rotation; capacitance
d	Distance
$\mathbf{e}_n, \mathbf{e}_t$	Unit vectors along normal and tangent
$\mathbf{e}_r, \mathbf{e}_\theta$	Unit vectors in radial and transverse directions
e	Coefficient of restitution; base of natural logarithms
E	Total mechanical energy; voltage
f	Scalar function
f_f	Frequency of forced vibration
f_n	Natural frequency
$\mathbf{F}$	Force; friction force
g	Acceleration of gravity
G	Center of gravity; mass center; constant of gravitation
h	Angular momentum per unit mass
$\mathbf{H}_O$	Angular momentum about point O
$\dot{\mathbf{H}}_G$	Rate of change of angular momentum $\mathbf{H}_G$ with respect to frame of fixed orientation
$(\dot{\mathbf{H}}_G)_{Gxyz}$	Rate of change of angular momentum $\mathbf{H}_G$ with respect to rotating frame $Gxyz$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors along coordinate axes
i	Current
$I, I_x, \ldots$	Moments of inertia
$\overline{I}$	Centroidal moment of inertia
$I_{xy}, \ldots$	Products of inertia
J	Polar moment of inertia
k	Spring constant
k_x, k_y, k_O	Radii of gyration
$\overline{k}$	Centroidal radius of gyration
l	Length
$\mathbf{L}$	Linear momentum
L	Length; inductance
m	Mass
m'	Mass per unit length
$\mathbf{M}$	Couple; moment
$\mathbf{M}_O$	Moment about point O
$\mathbf{M}_O^R$	Moment resultant about point O
M	Magnitude of couple or moment; mass of earth
M_{OL}	Moment about axis OL
n	Normal direction

$\mathbf{N}$	Normal component of reaction
O	Origin of coordinates
$\mathbf{P}$	Force; vector
$\dot{\mathbf{P}}$	Rate of change of vector **P** with respect to frame of fixed orientation
q	Mass rate of flow; electric charge
$\mathbf{Q}$	Force; vector
$\dot{\mathbf{Q}}$	Rate of change of vector **Q** with respect to frame of fixed orientation
$(\dot{\mathbf{Q}})_{Oxyz}$	Rate of change of vector **Q** with respect to frame *Oxyz*
$\mathbf{r}$	Position vector
$\mathbf{r}_{B/A}$	Position vector of *B* relative to *A*
r	Radius; distance; polar coordinate
$\mathbf{R}$	Resultant force; resultant vector; reaction
R	Radius of earth; resistance
$\mathbf{s}$	Position vector
s	Length of arc
t	Time; thickness; tangential direction
$\mathbf{T}$	Force
T	Tension; kinetic energy
$\mathbf{u}$	Velocity
u	Variable
U	Work
$\mathbf{v}, v$	Velocity
v	Speed
$\overline{\mathbf{v}}, \overline{v}$	Velocity of mass center
$\mathbf{v}_{B/A}$	Velocity of *B* relative to frame in translation with *A*
$\mathbf{v}_{P/\mathcal{F}}$	Velocity of *P* relative to rotating frame $\mathcal{F}$
$\mathbf{V}$	Vector product
V	Volume; potential energy
w	Load per unit length
$\mathbf{W}, W$	Weight; load
x, y, z	Rectangular coordinates; distances
$\dot{x}, \dot{y}, \dot{z}$	Time derivatives of coordinates x, y, z
$\overline{x}, \overline{y}, \overline{z}$	Rectangular coordinates of centroid, center of gravity, or mass center
$\boldsymbol{\alpha}, \alpha$	Angular acceleration
α, β, γ	Angles
γ	Specific weight
δ	Elongation
ε	Eccentricity of conic section or of orbit
$\boldsymbol{\lambda}$	Unit vector along a line
η	Efficiency
θ	Angular coordinate; Eulerian angle; angle; polar coordinate
μ	Coefficient of friction
ρ	Density; radius of curvature
τ	Periodic time
τ_n	Period of free vibration
ϕ	Angle of friction; Eulerian angle; phase angle; angle
φ	Phase difference
ψ	Eulerian angle
$\boldsymbol{\omega}, \omega$	Angular velocity
ω_f	Circular frequency of forced vibration
ω_n	Natural circular frequency
$\boldsymbol{\Omega}$	Angular velocity of frame of reference