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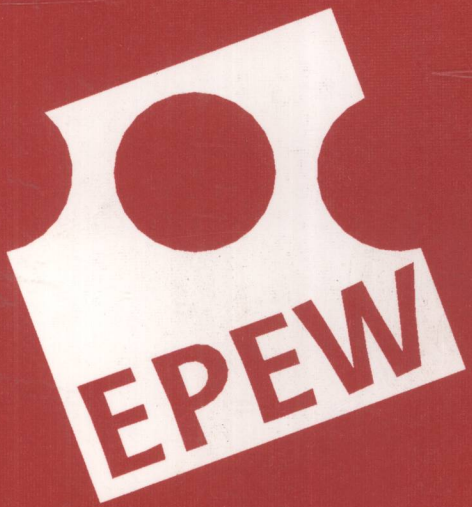

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Volume Editor

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Preface

This volume contains the papers presented at the 4th European Performance Engineering Workshop held during September 27–28 in Berlin.

There were 53 submissions. Each submission was reviewed by at least three Programme Committee members. From these, the committee decided to accept 20 papers.

We were very happy to have Isi Mitrani from Newcastle University give a keynote lecture on his recent work and future challenges in applied queueing theory.

The submitted papers cover all areas of performance engineering. We were able to compose an interesting program in six sessions, including sessions on theoretical work in performance engineering techniques as well as sessions presenting applications of performance engineering techniques. The final workshop program, as well as this volume, comprises the thematic sessions:

- Markov Chains
- Process Algebra
- Wireless Networks
- Queueing Theory and Applications of Queueing
- Benchmarking and Bounding
- Grid and Peer-to-Peer Systems

The volume includes very theoretical papers on topics such as bounds in stochastic ordering, canonical representation of phase-type-distributions and algorithms to solve closed queueing networks. Some papers study properties of numerical solution algorithms, other contributions evaluate hardware or software design and propose benchmarks. On the application side there are, furthermore, evaluations of wireless protocols, simulation studies of distributed systems and performance evaluation of system monitoring tools. We hope that this volume will provide a reference for fundamental work in performance engineering.

The success of the workshop is due to many helping hands. First of all, the members of the Program Committee were very cooperative, spent much time on reading and evaluating the submitted papers and gave advice where needed. Luckily, Miklos Telek passed on his experience after organizing last year's workshop.

Thanks to Levente Bodrog from Budapest the workshop had a professionally designed Web site. The EasyChair conference management software eased the administration of the PC meeting and composition of this volume. We thank the publisher for his support and continuity.

Last, but not least, I would like to thank the local organizers Johannes Zapotoczky and Steffen Tschirpke. Philipp Reinecke derserves special thanks for his help at all times.

July 2007 Katinka Wolter

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Optimization Problems in Service Provisioning Systems

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A service provisioning system typically contains a number of servers which may be distributed, heterogeneous and intermittently unavailable. They are used by the host in order to offer different services to a community of users. There may or may not be Service-Level Agreements involving Quality of Service constraints. In this context, there are several areas where dynamic optimisation problems arise quite naturally. These are (a) Routing and load-balancing: Where should an incoming request be sent for execution? If some queues grow large while others are short, can something be gained by transferring jobs among them? (b) Resource allocation: If different servers are dedicated to different types of service, how many should be assigned to each? When should a server be switched from one type of service to another? (c) Revenue maximisation: How are resource allocation and job admission policies affected by economic considerations? In particular, if service-level agreements specify payments for serving jobs and penalties for failing to provide a given quality of service, how many servers should be assigned to each type of service and when should jobs of that type be accepted?

The talk will describe models that address the above problems and will discuss routing, allocation and admission policies that may be adopted in practical systems.

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Untold Horrors About Steady-State Probabilities: What Reward-Based Measures Won't Tell About the Equilibrium Distribution*

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Abstract. These days, parallel and distributed state-space generation algorithms allow us to generate Markov chains with hundreds of millions of states. In order to solve such Markov chains for their steady-state behaviour, we typically use iterative algorithms, either on a single machine, or on a cluster of workstations. When dealing with such huge Markov chains, the accuracy of the computed probability vectors becomes a critical issue.

In this paper we report on experimental studies of, among others, the impact of different iterative solution techniques, erratic and stagnating convergence, the impact of the state-space ordering, the influence of the processor architecture chosen and the accuracy of the measure of interest, in relation to the accuracy of the individual state probabilities.

To say the least, the paper shows that the results from analysing extremely large Markov chains should be "appreciated with care", and, in fact, questions the feasibility of the ambitious "5 nines programs" that some companies have recently started.

1 Introduction

With the advent of high-level description languages for Markovian models, such as those based on stochastic Petri nets or stochastic process algebras, it has become easy to specify extremely large Markovian models. Also, the deployment of structured and symbolic approaches towards state space generation, such as those using Kronecker algebra and those based on, for instance, multi-terminal binary decision diagrams, has made Markovian models with thousands of millions of states a reality. However, describing and generating state spaces is one thing, solving the Markov chains associated with these enormous state spaces is another issue. The largest Markovian models we are aware of that have been solved numerically have close to a billion states [2] (using an explicit state space representation and a disk-based parallel solver). Clearly, currently the solution step is lagging behind.

* The title of this paper has been inspired by [11].

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In this paper we address the question of 1 how much confidence one actually can have in performance and dependability measures derived from numerical steady-state solutions of such extremely large Markov chains. What can we actually say about the accuracy of the computed probabilities? When we have so many states, can we still compute the state probabilities accurately enough? And how do the numerical algorithms "react" on such very small probabilities? Furthermore, if we employ parallel algorithms for the solution of the steady-state probabilities, does the way in which we distribute the state space over the nodes or the timing of information-exchange between the nodes (non-determinism) affect the accuracy of the measures we compute?

In order to illustrate our thoughts with experimental data, we present results for a generalised stochastic Petri net (GSPN) that has been used by many researchers in the past, the Flexible Manufacturing System (FMS) model [3]. This choice also gives us the ability to compare results computed at four different sites, i.e., at the RWTH Aachen, at the College of William and Mary, at Imperial College, and, most recently, at the University of Twente.

The result of our paper is not so much a recipe for obtaining steady-state probabilities that are always accurate enough. Instead, the aim of the paper merely is to show how difficult it is to actually obtain accurate results, and shows pitfalls and problems that will be all around. In doing so, it actually shows that determining very accurate performance and dependability measures, like needed in the "5 nines programs" of some industrial research laboratories (implying to determine, in a model-based fashion, that the system long-term availability is at least equal to 0.99999, which coincides with a downtime of, roughly, only 5 minutes per year), is far from trivial. In fact, the practical feasibility of such endeavours must be seriously questioned.

The rest of this paper is organised as follows. Section 2 addresses specific issues related to the employed numerical algorithms, and in Section 3 we present experimental results based on our computations and compare them to other published results. Finally, Section 4 concludes the paper with a summary and outlook.

2 Iterative Solvers for Markov Chains

For the solution of very large Markov chains, only iterative solutions can be employed; their background is rehearsed in Section 2.1 (for more details on iterative methods for Markov chains, see [10]). Since these iterative methods only produce approximations of the solution, we discuss in Section 2.2 how to make sure that a certain accuracy has been achieved. Section 2.3 gives background on the usually employed floating point number representation. Because the computed state probabilities often differ by several orders of magnitude, we address the problem caused by summing large numbers of such values in Section 2.4.

2.1 Background

During the computation of the steady-state distribution for a CTMC with generator matrix Q iterative linear equation solvers compute a sequence of

Untold Horrors About Steady-State Probabilities: What Reward-Based Measures Won't Tell About the Equilibrium Distribution*

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Abstract. These days, parallel and distributed state-space generation algorithms allow us to generate Markov chains with hundreds of millions of states. In order to solve such Markov chains for their steady-state behaviour, we typically use iterative algorithms, either on a single machine, or on a cluster of workstations. When dealing with such huge Markov chains, the accuracy of the computed probability vectors becomes a critical issue.

In this paper we report on experimental studies of, among others, the impact of different iterative solution techniques, erratic and stagnating convergence, the impact of the state-space ordering, the influence of the processor architecture chosen and the accuracy of the measure of interest, in relation to the accuracy of the individual state probabilities.

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of $\pi^{(k)}$ equals 1 or by using the criterion $e^{(k)} = \max_i \left(\frac{|\pi_i^{(k)} - \pi_i^{(k-1)}|}{|\pi_i^{(k)}|} \right) \leq \delta$, which computes the relative error between two successive approximations.

Secondly, it may falsely detect convergence if the iteration process converges very slowly, hence, the difference between two successive approximations is smaller then δ, even though an appropriate solution would require far more iterations. Stewart [10] suggests to check the differences of non-successive approximations resulting in a stopping criterion $e^{(k)} = \max_i \left(\frac{|\pi_i^{(k)} - \pi_i^{(k-m)}|}{|\pi_i^{(k)}|} \right) \leq \delta$, where approximations lying m iterations apart are compared. Note that m is not required to be constant, but may be chosen as a function of the convergence rate or the iteration count. An obvious disadvantage of this criterion is the fact that an additional old approximation has to be stored whereas the comparison of two successive approximations can be done on-the-fly even for a Gauss-Seidel iteration using only a single vector.

The stopping criteria discussed above can only be used if the successive approximations get better during each iteration step. If the method exhibits so-called erratic convergence (see the example in Section 3.3 for the CGS method [9]), then no conclusions about the achieved accuracy can be drawn from the comparison of two successive (or m-step apart) approximations. Hence, stopping criteria based on the residual $r^{(k)} = \pi^{(k)}Q$ should be used in conjunction with the CGS method [2,6]. Of course, these can also be applied in combination with the methods of Jacobi and Gauss-Seidel. The quality of an approximation is better the closer the residual is to zero. Note that the standard definition of the residual of a linear system $Ax = b$ is $r = Ax^{(k)} - b$. As before, the absolute magnitude of the entries in the residual vector can only be interpreted meaningfully if we compare them to the magnitude of the (sought for) elements in the solution vector. Hence, the most common stopping criterion based on the residual is $e^{(k)} = \frac{||r^{(k)}||}{||\pi^{(k)}||} \leq \delta$. Again, any norm will do, but the maximum norm is the most common choice. If we use it and rewrite the stopping criterion as $||r^{(k)}||_\infty \leq \delta ||\pi^{(k)}||_\infty$ we see that the largest entry in the residual, which should be as close to zero as possible, is at most δ times the largest entry in the approximation vector $\pi^{(k)}$. For the rest of this paper, if not mentioned otherwise, we will use this relative residual criterion as the stopping criterion. Note that the residual can be computed at no additional cost during Jacobi and CGS iterations [7].

A stopping criterion not based on the achieved accuracy but on the speed of convergence can be applied to methods like Jacobi and Gauss-Seidel that typically exhibit nearly monotone linear convergence up to a certain accuracy. If this accuracy is achieved, often no further progress will be made. One can observe this point by analysing the fraction of two successive error approximations $\frac{e^{(k-1)}}{e^{(k)}}$. An example where this point is reached will be given in Section 3.2. Note that this criterion can also be used for approximations that are more than just one iteration apart without the need to store the iteration vectors.

approximations $\pi^{(0)}, \pi^{(1)}, \pi^{(2)}, \cdots$ for the solution vector π of the linear system $0 = \pi Q$ (which we may rewrite as $Q^{\mathrm{T}}\pi^{\mathrm{T}} = 0$ to correspond to the more general representation of linear systems $Ax = b$). Any iterative solver computes the next approximation $\pi^{(i+1)}$ by the iteration $\pi^{(i+1)} = H \cdot \pi^{(i)} + c$, where H is called the iteration matrix. Clearly, one iteration step "costs" one matrix-vector product (MVP). The number of iterations k required for an accuracy ϵ can be approximated from the spectral radius ρ of the iteration matrix H as $k = \frac{\log \epsilon}{\log \rho}$ (see: [1,10]). Instead of the spectral radius the magnitude of the sub-dominant eigenvalue can be used. Although this result looks very attractive it is of little use in practice as the computation of the eigenvalues of H requires approximately the same effort as the computation of the steady-state solution. Hence, other methods to detect convergence and hence to limit the number of iterations k have to be used, as will be discussed below.

An important issue to address is the number of solution vectors that needs to be stored at any point in time during the iterative solution process. Using double precision floating point numbers, a single solution vector (which is non-sparse) costs 8 megabyte per 1 million states. On a machine with 1 gigabyte of main memory, roughly speaking, the solution vector for a Markov chain with 100 million states can be stored, provided only a single iteration vector is required, such as is the case for Gauss-Seidel. For the Jacobi method, already two vectors are required, thus limiting the number of states to roughly 50 million. For Conjugate Gradient Squared (CGS), even more vectors are required. In all these cases, it is assumed that the matrix Q is either stored very compactly, recomputed on the fly, or stored on disk. We note that whereas for the serial solution of the steady-state probabilities methods like Gauss-Seidel, SOR, Jacobi and the CGS can be employed, parallel implementations tend to use only the Jacobi and CGS method as they can be parallelised more easily and efficiently.

2.2 Stopping Criteria

The simplest properties that can be used as stopping criteria are either to limit the maximum number of iterations or the time spent computing them. This surely limits the iteration count k but can not guarantee that the remaining error $e^{(k)} = \pi^{(k)} - \pi$ is smaller than some chosen limit. Better, but still traditional stopping criteria are based on the norm of the difference of successive iterates $e^{(k)} = ||\pi^{(k)} - \pi^{(k-1)}||$, cf. [10], where the iteration is stopped if this norm falls below $\delta > 0$. Although any norm will do, the most popular choice is the maximum norm $||x||_\infty = \max_i |x_i|$, as it requires the fewest floating point operations to perform and no underflows or overflows can occur with it. This consideration applies to all norms we will use in this section. This approach has several problems, though. First of all, it does not take into account the magnitude of the (largest) elements of the solution vector, which, indeed, may all be very small if the probability vectors consist of several hundreds of millions of entries. This problem can be overcome by either scaling $\pi^{(k)}$ in a way that the largest element