LINEAR CONTROL SYSTEM ANALYSIS AND DESIGN:
Conventional and Modern

Third Edition

John J. D'Azzo
Constantine H. Houpis

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PREFACE

This textbook is intended to provide a clear, understandable, and motivated account of the subject which spans both conventional and modern control theory. The authors have tried to exert meticulous care with explanations, diagrams, calculations, tables, and symbols. They have tried to ensure that the student is made aware that rigor is necessary for advanced control work. Also stressed is the importance of clearly understanding the concepts which provide the rigorous foundations of modern control theory. The text provides a strong, comprehensive, and illuminating account of those elements of conventional control theory which have relevance in the design and analysis of control systems. The presentation of a variety of different techniques contributes to the development of the student's working understanding of what A. T. Fuller has called "the enigmatic control system." To provide a coherent development of the subject, an attempt is made to eschew formal proofs and lemmas with an organization that draws the perceptive student steadily and surely onto the demanding theory of multivariable control systems. It is the opinion of the authors that a student who has reached this point is fully equipped to undertake with confidence the challenges presented by more advanced control theories as typified by Chapters 18 through 22. The importance and necessity of making extensive use of computers is emphasized by references to comprehensive computer-aided-design (CAD) programs.

The establishment of appropriate differential equations to describe the performance of physical systems, networks, and devices is set forth in Chapter 2, which also introduces some elementary matrix algebra, the block diagram, and the transfer function. The essential concept of modern control theory, the state space, is dealt with also. The approach used is the simultaneous derivation of the state-vector differential equation with the single-input single-output differential equation for a chosen physical system. The relationship of the transfer function to the state equation of the system is deferred until Chapter 4. The derivation of a mathematical description of a physical system by using Lagrange equations is also given.

The first half of Chapter 3 deals with the classical method of solving differential equations and with the nature of the resulting response. This serves the individual who needs to learn this material or needs a reference. Once the state-variable equation has been introduced, careful account is given of its solution. The central importance of the state transition matrix is brought out, and the state transition equation is derived. The idea of an eigenvalue is next explained and this theory is used with the Cayley-Hamilton and Sylvester theorems to evaluate the state transition matrix.

The early part of Chapter 4 presents a comprehensive account of Laplace transform methods and pole-zero maps. Some further aspects of matrix algebra are dealt with before dealing with the solution of the state equation by the use of Laplace transforms. Finally the evaluation of transfer matrices is clearly explained.

Chapter 5 begins with system representation by the conventional block-diagram approach. It is followed by a straightforward account of simulation diagrams and the determination of the state transition equation by the use of signal flow graphs. By deriving parallel state diagrams from system transfer functions, the advantages of having the state equation in uncoupled form are established. This is followed by the methods of diagonalizing the system matrix. A feature of this chapter is the clear treatment of how to transform an A matrix which has complex eigenvalues into a suitable alternative block diagonal form and the transformation to companion form.

In Chapter 6 the system characteristics are introduced. This includes the relationship between system type and the ability of the system to follow or track polynomial inputs.

Chapters 7 to 11 are updated versions of the same chapters in the previous edition and present substantially the same material, with a greater emphasis on CAD packages. In Chapter 7 the details of the root-locus method of analysis are presented. Then the frequency-response method of analysis is given in Chapters 8 and 9, using both the log and the polar plots. These chapters include the following topics: Nyquist stability criterion; correlation between the s plane, frequency domain, and time domain; and gain setting to achieve a desired output response peak value. Chapters 10 and 11 describe the possible improvements in system performance, along with examples of the technique for applying cascade and feedback compensators. Both the root-locus and frequency-response methods of designing compensators are covered.

The concept of modeling a desired control ratio which has figures of merit that satisfy the system performance specifications is developed in Chapter 12. The system inputs generally fall into two categories: (1) a desired input which the system output is to track (a tracking system) and (2) a disturbance input for which the system output is to be minimal (a disturbance-rejection system). Desired control ratios for both types of systems are synthesized by the proper placement of its poles and inclusion of zeros if required. Chapter 12 also includes the Guillemin-Truxal design procedure for designing a tracking control system and a design procedure for a disturbance-rejection control system.

The technique of achieving desired system characteristics by using complete state-variable feedback are developed thoroughly and carefully in Chapter 13. The very important concepts of modern control theory—controllability and observability—are treated in a simple, straightforward, and correct manner. Although the treatment is brief, it provides sufficient coverage of these topics for the requirements of the remainder of the book. This provides a useful foundation for the work of Chapters 16 to 22.

Chapter 14 includes a presentation of the sensitivity concepts of Bode for the variation of system parameters. Also included is the method of using feedback transfer functions to form estimates of inaccessible states for use in state feedback.

Additional matrix algebra is presented in Chapter 15 with particular emphasis on quadratic forms. This material is used in the presentation of a short account of some of the important aspects of stability considered from the Liapunov point of view. An account of trajectories in the state space and some associated phase-plane techniques is also given. A feature of this approach is that the arguments are extended to nonlinear systems. The treatment of such systems by linearization is presented. The use of a Liapunov function is presented for the determination of system stability and instability. It serves as an introduction to its use in establishing a performance index, as shown in Chapter 16.

Chapter 16 starts with a careful and comprehensive treatment of the use of performance indexes for the parameter-optimization methods for single-input single-output systems. The chapter goes on to deal with the nature of the problem of optimal control. The solution to the infinite-time linear quadratic problem is deal with *in extenso* in terms of the algebraic Riccati equation, but the results are derived using the Liapunov function approach rather than the calculus of variations of the method proposed by Pontryagin et al. The heuristic advantages of this approach for beginning students are very great. A treatment of the same problem from the standpoints of the Bode diagram and root-square locus underscores the essential unity of the subject and the mutuality of the modern and conventional control theories.

Chapter 17 presents some methods of optimal linear system design. The relationship of these methods to the conventional methods is stressed and evaluated. Although the account is limited, for heuristic convenience, to single-input systems, the subsequent correlation provides the student with the opportunity to develop an invaluable insight into the nature of the linear quadratic optimal control problem.

Chapters 18 and 19 provide a thorough presentation of the principles and techniques of entire eigenstructure assignment for multiple-input multiple-output systems by means of state feedback. This extends the use of eigenvalue assignment to include the simultaneous assignment of the associated eigenvectors and provides the means for shaping the output response to meet design specifications.

There are many worthwhile control-system design techniques available in the technical literature which are based on both modern and conventional control theory. Each technique may have limited applicability to certain classes of design problems. The control engineer must have a sufficiently broad perspective to be able to apply the right technique to the right design problem. For some techniques the designer is assisted by available computer-aided-design (CAD) packages. Design techniques like LQR, LQG, LTR, etc. are thoroughly covered in the literature.

Two additional design techniques have definite design characteristics that warrant their inclusion in a textbook for both the beginning and the practicing engineer. Thus, this third edition has been expanded to include, in Chapter 20, an output feedback state-space technique based on singular perturbation theory and, in Chapter 21, a conventional control technique based on quantitative feedback theory (QFT). The authors feel that these methods have proven their applicability to the design of practical multiple-input multiple-output control systems. These chapters are intended to further strengthen the fundamentals presented earlier in the text and to "whet the appetite" of the budding control engineer. Application of the singular perturbation theory is applied to achieve robust output feedback high-gain systems in Chapter 20. This leads to the design principles developed by Professor Brian Porter which incorporate proportional plus integral controllers. Chapter 21 presents an introduction to and lays the foundation for the quantitative feedback theory (QFT) developed by Professor Isaac M. Horowitz. This technique incorporates the concept of designing a robust control system that maintains the desired system performance over a prescribed region of plant parameter uncertainty.

Chapter 22 presents an introduction to digital control systems. The advances in digital computers and microprocessors have made their use very attractive as components in control systems. The effectiveness of digital compensation is clearly demonstrated. The concept of a pseudo-continuous-time (PCT) model of a digital system permits the use of continuous-time methods for the design of digital control systems.

The authors have tried to provide students of control engineering with a clear, unambiguous, and relevant account of appropriate, contemporary, and state-of-the-art control theory. It is suitable as an introductory and bridging text for undergraduate and graduate students.

The text is arranged so that it can be used for self-study by the engineer in practice. Included are as many examples of feedback control systems in various areas of practice (electrical, aeronautical, mechanical, etc.) as space permits while maintaining a strong basic feedback control text that can be used for study in any of the various branches of engineering. To make the text meaningful and valuable to all engineers, the authors have attempted to unify the treatment of physical control systems through use of mathematical and block-diagram models common to all. The text has been class-tested, thus enhancing its value for classroom and self-study use. There are many computer-aided-design (CAD) packages available to assist a control engineer in the analysis, design, and simulation of control systems. Some of these are listed in Appendix B.

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CONTENTS