# NUCLEAR STRUCTURE

## DUBNA SYMPOSIUM 1968

PROCEEDINGS SERIES

# NUCLEAR STRUCTURE

## DUBNA SYMPOSIUM 1968

INVITED PAPERS FROM
THE INTERNATIONAL SYMPOSIUM ON NUCLEAR STRUCTURE
ORGANIZED BY THE
JOINT INSTITUTE FOR NUCLEAR RESEARCH, DUBNA,
AND HELD IN DUBNA, 4 - 11 JULY 1968

Supported by:
The International Union of Pure and Applied Physics
and The International Atomic Energy Agency

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 1968

NUCLEAR STRUCTURE: DUBNA SYMPOSIUM 1968
(Proceedings Series)

ABSTRACT. Invited papers of a Symposium organized by the Joint Institute for Nuclear Research, Dubna, supported by IUPAP and IAEA, and held in Dubna from 4 to 11 July 1968. The meeting was attended by about 450 scientists from 30 countries. The volume contains the invited papers, all by distinguished scientists, and the discussions and short contributions that followed the presentation of these papers.

Contents: I. Nuclear structure at low excitations (15 papers); II. Nuclear structure at high excitations (6 papers): III. Open problems in nuclear physics (3 papers); IV. Equilibrium deformations (6 papers); V. General properties of nuclei (6 papers); VI. Closing remarks; List of contributions; List of seminar papers; List of participants; Author index.

All papers, discussions and short contributions are in English; the abstracts are in English and Russian, which were the working languages of the Symposium.

(642 pp., 16 × 24 cm, paper-bound, 250 figures; 1968) Price: US $15.00; £6.5.0

The diagram on the cover shows the $0^+$ excited states in the pairing scheme of Bohr, and the corresponding two-nucleon transfer reactions in the mass region A = 50 - 58. For purposes of illustration, the energies of the states are not shown rigorously to scale.

NUCLEAR STRUCTURE:
DUBNA SYMPOSIUM 1968
IAEA, VIENNA, 1968
STI/PUB/189

Printed by the IAEA in Austria
December 1968

# CONTENTS

## I. NUCLEAR STRUCTURE AT LOW EXCITATIONS

## II. NUCLEAR STRUCTURE AT HIGH EXCITATIONS

## III. OPEN PROBLEMS IN NUCLEAR PHYSICS

## IV. EQUILIBRIUM DEFORMATIONS

V. GENERAL PROPERTIES OF NUCLEI

VI. CLOSING REMARKS

# I

# NUCLEAR STRUCTURE AT LOW EXCITATIONS

Chairmen

H. NIEWODNICZAŃSKI
J. SCHINTLMEISTER
R.F. BELL
B.S. DZHELEPOV
N. SODNOM
A.S. DAVYDOV
Zh. ZHELEV
G. RASMUSSEN

Secretaries

V.G. CHUMIN
R. ARLT
V.K. LUKYANOV
M. ADAM
V.A. KARNAUKHOV
B.N. KALINKIN
I.Z. PETKOV
S. REVAI

# ONE-PARTICLE MOTION IN SPHERICAL NUCLEI

B.L. COHEN
UNIVERSITY OF PITTSBURGH,
PITTSBURGH, PA., UNITED STATES OF AMERICA

### Abstract — Аннотация

ONE-PARTICLE MOTION IN SPHERICAL NUCLEI. New information on locations of single-particle states as determined by single-nucleon transfer reactions and elastic proton scattering through isobaric analogue states is summarized. The situation with neutron single particle states is reasonably satisfactory although there is considerable uncertainty about the particle states in N = 83 nuclei. Information on proton single-particle states has accumulated rapidly recently. The single-particle level structure is very similar for neutrons and protons in nuclei of the same mass number A, more so than for either one at very different A. The principal A-dependence is that high-j states move downward rapidly relative to low-j states with increasing A, for well understood reasons. Information on imperfect major shell closure at "magic numbers" is summarized, and it is pointed out that it is not unexpected. The only really good closed shell nuclei are those near $^{208}$Pb. The significance of deep-lying hole states found in (e, e' p) reactions is mentioned. Information on neutron single quasiparticle states in the 28-50 and 50-82 shells is reviewed. Experiments are now available on non-single-closed shell nuclei in the latter region; fragmentation of single quasiparticle states is somewhat larger than heretofore encountered, but there is generally one nuclear state that contains a large fraction of their strength. An important difficulty is encountered in the $g_{7/2}$ and $h_{11/2}$ states where results from stripping and pick-up reactions differ greatly in nuclei between Mo and In. Plots of the "degree of emptiness", $U_j^2$, and of single quasiparticle energies, $E_j$, are given for all single-particle states for nuclei with N = 28-82, and comparisons are made with pairing theory predictions. In general, the agreement is good although there are several difficulties that are discussed.

ОДНОЧАСТИЧНОЕ ПЕРЕДВИЖЕНИЕ В СФЕРИЧЕСКИХ ЯДРАХ. Дается обзор новых данных о положении одночастичных состояний, определяемых из однонуклонных реакций передачи и упругого рассеяния протонов на изобарических аналоговых состояниях. Положение с нейтронными одночастичными уровнями весьма удовлетворительно, хотя и имеются значительные неясности в вопросе одночастичных состояний в ядрах с N = 83. В последнее время быстро накапливается информация по одночастичным протонным состояниям. В ядрах с одинаковым массовым числом А структура одночастичных уровней протонов и нейтронов весьма схожа, и даже больше схожа, чем структура какой-либо одной системы для ядер с сильно отличающимися А. Главным в зависимости структуры одночастичных уровней от А является то, что с ростом А состояния с большими моментами понижаются быстрее, чем состояния с малыми моментами, по хорошо понятным причинам. Дается обзор данных о нарушении в заполнениях оболочек при "магических числах" и указывается, что это не является неожиданным. Единственными ядрами с хорошо замкнутыми оболочками являются ядра в районе $^{208}$Pb. Упоминается важность низколежащих (глубоких) дырочных состояний, обнаруженных в (е, е' р) реакциях. Дается обзор информации об одночастичных нейтронных состояниях в области оболочек 28 – 50 и 50 – 82. В настоящее время получены экспериментальные результаты в последней указанной выше области для ядер с неодночастичной замкнутой оболочкой. Размывание одноквазичастичных состояний несколько больше, чем до сих пор считалось, однако имеется, в основном, одно состояние с большим весом этого состояния. Возникло серьезное затруднение в объяснении $g_{7/2}$ и $h_{11/2}$ состояний, когда результаты, полученные от реакций стрипинга и подхвата, значительно отличаются по ядрам между Mo и In. Графики вероятностей "дырочных состояний" $U_j^2$ и одноквазичастичные энергии $E_j$ приведены для всех одночастичных состояний с N = 28 – 82, и проведено сравнение с предсказаниями теории парных корреляций. В основном, согласие хорошее, хотя имеются некоторые трудности, которые требуют объяснения.

## NEUTRON SINGLE PARTICLE STATES

The best method for determining locations of single particle states is by studies of single nucleon transfer reactions on closed shell nuclei (or their equivalent by elastic scattering through isobaric analogue states). By this method, one not only determines the angular momentum and parity of nuclear states, but also checks that the spectroscopic factors sum to the proper value which assures that all nuclear states containing appreciable fractions of the single particle state are included. The energy of the single particle state is then taken as the centre of gravity of the nuclear states, weighting each by its spectroscopic factor.

A few years ago, the information on neutron states seemed to be fairly complete [1] but there were several uncertainties and a few missing points. Work on the missing points has proceeded well in the past year. Yagi [2] has located the $g_{7/2}$ and $d_{5/2}$ hole states in the N = 81 nucleus $^{139}$Ce; he found the previous estimates [1] to be rather poor. Fou et al. [3] and Bassani et al. [4] have located the $p_{3/2}$ and $f_{5/2}$ hole states in N = 50 nuclei. The only single particle or single hole states in adjacent shells still not located in this way are the $i_{13/2}$ and $h_{11/2}$ single particle states in N = 82 and N = 50 nuclei respectively. We have recently searched for the latter again with the $^{92}$Mo (d, p) reaction [5], and have found an important component at 2.3 MeV, but its spectroscopic factor is only about $\frac{1}{3}$ of that expected for the single particle state.

With the recent availability of much improved accelerators and detectors, there has been a great deal of work in checking previous determination of energies of single particle states; in many cases the earlier experiments were crude and left reasonable doubts. Rather extensive work [6] has been done on locating the single particle states in N = 83 nuclei where previous studies were made before the availability of distorted wave Born approximation (DWBA). It has been found that sums of spectroscopic factors fall considerably short of unity for all but the $f_{7/2}$ and $h_{9/2}$ states. The locations of the $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ states must therefore be considered to be very questionable, and further studies of this would be highly desirable.

The most complete checking has been done in the shells adjacent to $^{208}$Pb. In Table I, the spectroscopic factors for the previously assigned single particle states in $^{209}$Pb are shown according to determinations with (d, p) reactions at several bombarding energies at Michigan [7], isobaric analogue studies at Texas [8], and (t, d) reactions at Los Alamos [9]. In all cases, the simple theory predicts unity. This is typical of the general type of agreement one usually gets. An optimist points to the fact that DWBA calculations are uncertain by about 20% at best and can sometimes be much worse, and is therefore satisfied with the results. Where discrepancies are large as for the $g_{7/2}$ in (d, p) and the $j_{15/2}$ in (t, d), he can point to the fact that other experiments give more favourable results. A pessimist can often find reasons to argue that the DWBA uncertainties are smaller in cases giving unfavourable results like the $j_{15/2}$ where the (t, d) measurement is remarkably intensitive to the choice of optical model parameters.

A similar disagreement exists over the $h_{9/2}$ hole state in $^{207}$Pb where (p, d) work [2] indicates that the 3.6 MeV level contains almost all of the $h_{9/2}$ strength while $^{3}$He, $\alpha$) studies [10] indicate that it contains only a little more than half. In both this case and the $j_{15/2}$ case discussed above, it is

TABLE I. SPECTROSCOPIC FACTORS FOR SINGLE PARTICLE STATES IN $^{209}$Pb BY VARIOUS METHODS

| S.P. State | (d, p) [7] | Analogue [8]<br>(p, p) | (t, d) [9] |
|---|---|---|---|
| $2g_{9/2}$ | 0.67 | 0.97 | 0.93 |
| $1i_{11/2}$ | 0.94 |  | 1.05 |
| $1j_{15/2}$ | 1.13 |  | 0.51 |
| $3d_{5/2}$ | 1.00 | 0.85 | 0.86 |
| $4s_{1/2}$ | 0.93 | 0.90 | 0.86 |
| $2g_{7/2}$ | 1.17 | 0.84 | 0.90 |
| $3d_{3/2}$ | 1.17 | 0.86 | 0.83 |

not difficult to explain fragmentation of the single particle state into several nuclear states, but no one has found any of the other fragments.

It has long been contended by nuclear structure theorists (see, for example, Ref. [11] ) that the shell model picture of single particle states is only about 85% correct. The sum of spectroscopic factors should therefore be only about 0.85 instead of 1.0; the remaining 15% of the strength should be scattered over tens of MeV. The uncertainty in DWBA analyses is easily large enough to adjust all previous data for this, and few would argue that there is experimental evidence to dispute the point. Interest in this matter has recently been revived by the new stripping theory of Butler, Hewitt, May and McKellar [12], which seems to be much less parameter-sensitive than DWBA in determinations of absolute spectroscopic factors. It consistently finds spectroscopic factors much less than unity for states that have long been considered to be good single particle states. While very few calculations have been made with this theory and there have been no checks on its consistency when applied to different reactions, it should be viewed as a hopeful development. The uncertainties in absolute determinations of spectroscopic factors are a much more serious problem than is widely realized, and any theory that would substantially reduce them would be extremely welcome.

In spite of the new developments, the previous analysis [13] of locations of neutron single particle states is not appreciably altered. These locations can be explained [13] by properly taking into account [14] special neutron-proton interactions when they are in states with strongly overlapping wave-functions, self-binding energy (i.e. a single particle state moves down in energy as it fills owing to extra strong interactions among the particles in it), a spin-orbit force of the form $r^{-1}$ (dV/dr), a velocity dependence corresponding to an effective mass of about 1.3 times the nucleon mass, and a symmetry energy of the type known from the optical model.

## PROTON SINGLE PARTICLE STATES

The most important recent development in the area under discussion has been the cataloguing of proton single particle states principally with ($^3$He, d) and (d, $^3$He) reactions. As a result of work from Oak Ridge [15], Aldermaston [16], Tokyo [17], Saclay [18], Moscow [19], Argonne [20], Pennsylvania [21], and Los Alamos [22], studies are now available on both particle and hole states at each closed shell. Unfortunately, however, there seems to be more fragmentation here [17, 22] than was present in the study of neutron states, so the accurate determination of single particle state locations is not always easy.

The best information is for the Z = 50-82 and the Z = 82-126 shells This is shown and compared with related neutron results in Fig. 1a. In that figure, the results are arranged in order of increasing A, without regard to whether they are neutron or proton single particle states. The neutron states in the Sn region deduced from studies of the fullness of single quasiparticle states [23] are included.

Nearly all the changes in relative energies in Fig. 1a are shifts downward with increasing A for high angular momentum (j) states by amounts which increase with j (or equivalently, upward shifts of low j states). This may be explained by realizing that high-j nucleons interact more strongly with other high-j nucleons that with low-j nucleons because of the better overlap in their wave-functions – they have the same n quantum number and are pushed outward by similar angular momentum barriers. Thus, as high-j states fill, the energy of other high-j states is lowered. This effect is larger for high-j states because of the large number of nucleons they contain.

An especially notable feature of Fig. 1a is the similarity between proton and neutron single particle states in the same shell and in the same nuclei. There are examples of this in Fig. 1a for the 82-126 states in the Pb region and for the 50-82 states in the Sn region. This similarity is far stronger than that between the same neutron states, or between the same proton states, at different values of A. This implies that single particle level structure is determined principally by what other states are occupied, and does not depend much on whether we are dealing with neutrons or protons.

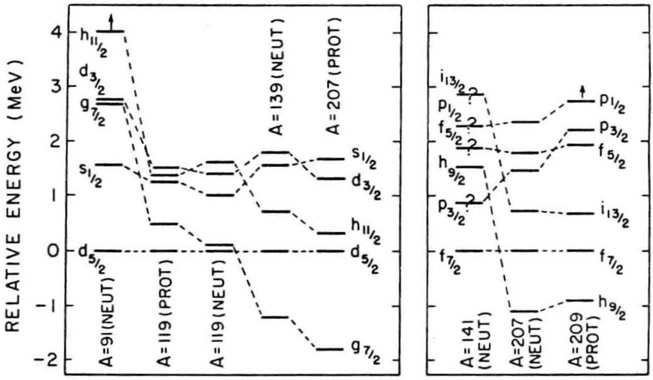

FIG. 1a. Energies of single particle states in the 50-82 and 82-126 shells.

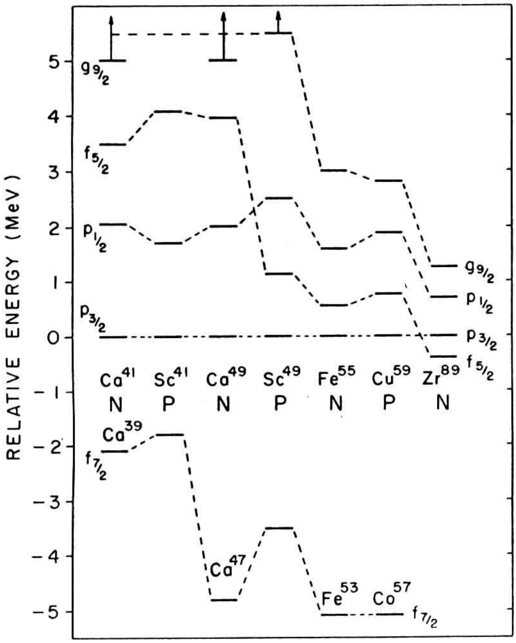

FIG. 1b. Energies of single particle states in the 20-50 shells.

The situation in the 20-50 shells is shown in Fig. 1b. For neighbouring nuclei $^{55}$Fe and $^{59}$Cu, and again for $^{41}$Ca and $^{41}$Sc, the similarity between proton and neutron single particle states in neighbouring nuclei persists. However in $^{49}$Ca and $^{49}$Sc, it fails rather badly. Still it seems significant that in four of the five cases where comparisons can be made, including the three of highest mass, there is much greater similarity between neutron and proton single particle states at the same mass than there is between neutron single particle states at different masses. The old custom of giving different single particle level diagrams for neutrons and protons is misleading; it would be much better to give such diagrams as a function of A, with the stipulation that it is only valid along the "line of beta stability."

## IMPERFECT MAJOR SHELL CLOSURE AT "MAGIC NUMBERS"

It has long been realized that major shell closure is not perfect in nuclei with "magic numbers" of neutrons or protons. Several years ago, Blair [22] found that well-known $7/2^-$ states in Cu isotopes were excited in ($^3$He, d) reactions on Ni with a spectroscopic factor indicating that the $f_{7/2}$ proton single particle state lacks being full by about 0.8 particle. Fulmer et al. [24] found by a combination of (d, p) and (d, t) reactions that the $f_{7/2}$ neutron single particle states lack being full in $^{58}$Ni and $^{60}$Ni by about 0.36 and 0.28 particles even though these nuclei contain 30 and 32 neutrons respectively. By similar methods, Jolly [25] found that there is already

0.2 $2f_{7/2}$ neutrons in the 78 neutron nucleus, $^{130}$Te. Lin [26] found similar evidence that there are about 0.6 neutrons in the $2d_{5/2}$ state in the 44, 46, and 48 neutron nuclei $^{78}$Se, $^{80}$Se, and $^{82}$Se. Evidence of this type has expanded rapidly in the past year. Peterson [27] reported that $^{48}$Ca has about 0.9 2p neutrons. Zeidman's [28] experiments showed that Fe nuclei, which have only 26 protons, already have 0.2-0.5 $p_{3/2}$ protons; this number increases with the neutron number. Heidelberg [28] and Argonne [20] work indicates imperfect shell closure at the "magic number" 20: $^{40}$Ca already has 0.8 neutrons and 0.8 protons in the $f_{7/2}$ single particle states. Purser et al. [29] found that the ground state of $^{16}$O may be written approximately as

$$\psi_{16} = \sqrt{0.68}\ p^{12} + \sqrt{0.23}\ p^{10}\ d^2 + \sqrt{0.09}\ p^{10}\ s^2$$

The 0.46 d-particles and 0.18 s-particles are mostly protons, but some are neutrons.

To the best of my knowledge there is no evidence for imperfect shell closure in $^{208}$Pb for either neutrons or protons. This region has been studied very thoroughly, and they seem to be the best closed shells we have.

There is, of course, nothing very surprising about imperfect shell closure. Separations between single particle energies in different major shells are not infinite, and the amount of mixing across shells is of about the magnitude expected from our knowledge of residual interactions.

## DEEP-LYING HOLE STATES

All the previous discussion has dealt with states near the top of the "Fermi Sea". Single particle states deep down in the Fermi Sea — two or three shells below the top — have been studied with (p, 2p) reactions, and more recently and successfully, with (e, e'p) reactions [30]. The binding energies of these states greatly exceed 50 meV, the approximate depth of a static shell theory potential; this requires a velocity dependence corresponding to an effective mass considerably less than the nucleon mass in agreement with nuclear matter theory but in contrast to the value above the nucleon mass encountered near the top of the Fermi Sea. This velocity dependence was pointed out by G. E. Brown several years ago, but its experimental confirmation is welcome.

## SINGLE QUASIPARTICLE STATES

Studies of single quasiparticle states — including determinations of their energy $E_j$ and the ratio of their particle and hole natures, usually expressed as the fullness of the single particle states, $V_j^2$ — have been going on for many years using single nucleon transfer reactions. In heavy nuclei, these studies were formerly limited to nuclei near closed shells, but since higher energy precision accelerators with attached spectrographs have become available in the past few years, virtually any nucleus in the periodic table is now open to such studies. Much of this work has been done on non-spherical nuclei and will be discussed in other papers at this meeting.

At our laboratory we have been concentrating rather on spherical nuclei in an effort to follow single quasiparticle states through entire major shells and observe how they gradually shift from purely particle to purely hole states. Fig. 2 shows an example [31] of the type of data obtained; it is from the $^{108}$Pd (d, p) reaction. Vertical lines show the locations of nuclear levels which contain components of the neutron single quasiparticle state in whose row they are shown, and the heights of these lines are the spectroscopic factors which measure the fraction of their wave-function that is the single quasiparticle state. The sum of spectroscopic factors is $U_j^2$, the degree to which the single particle state is empty, and the centre of gravity of the components, shown by the ⊗, is $E_j$, the energy of the single quasiparticle state.

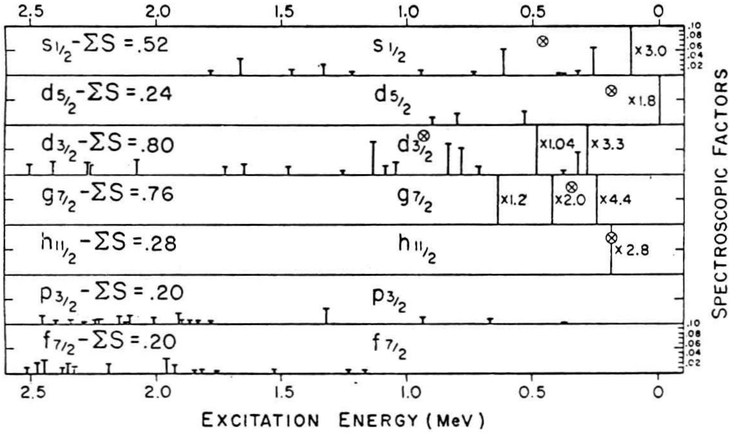

FIG.2. Energies, $I^{\pi}$ assignments, and spectroscopic factors of nuclear states excited in $^{108}$Pd (d, p) $^{109}$Pd reactions. Also shown are summed spectroscopic factors for each single particle state, which are $U_j^2$ in Fig.3, and the "centres of gravity" of states for each j, which are $E_j$ in Fig.4.

Fig. 3 shows the value of $U_j^2$ for all nuclei studied in the N = 50-82 shell. It includes some data from Argonne on Ru [32] as well as experiments on Zr, Nb, Mo, Pd, Cd, In, Sn, Te, Ba and Ce from our laboratory [33]. The two curves show the predictions of simple pairing theory for the single particle energies at the beginning and at the end of the shell. For the $g_{7/2}$ and $h_{11/2}$ states in Pd and Cd, two sets of points are shown, one for determinations from (d, p), and the other for determinations from (d, t) reactions. The lack of agreement between these is the strangest phenomenon the author has ever encountered in many years of studies of this type; he believes it is due to a break-down in basic assumptions of stripping reaction theory [34].

An optimist can find much reason to be satisfied by the results of Fig. 3, especially if he ignores the $g_{7/2}$ and $h_{11/2}$ results from Pd, Cd, and Ru (d, p) reactions (the lower points for $h_{11/2}$ and the higher points for $g_{7/2}$). Near the ends of the shell, the results are close to the prediction lines, and in the middle of the shell they lie between the predictions. This is as expected from the monotonic and relatively smooth shifting of levels with A seen in Fig. 1. On the other hand, there are several disturbing features. The low values for the $s_{1/2}$ states in Ru and Pd, the high values

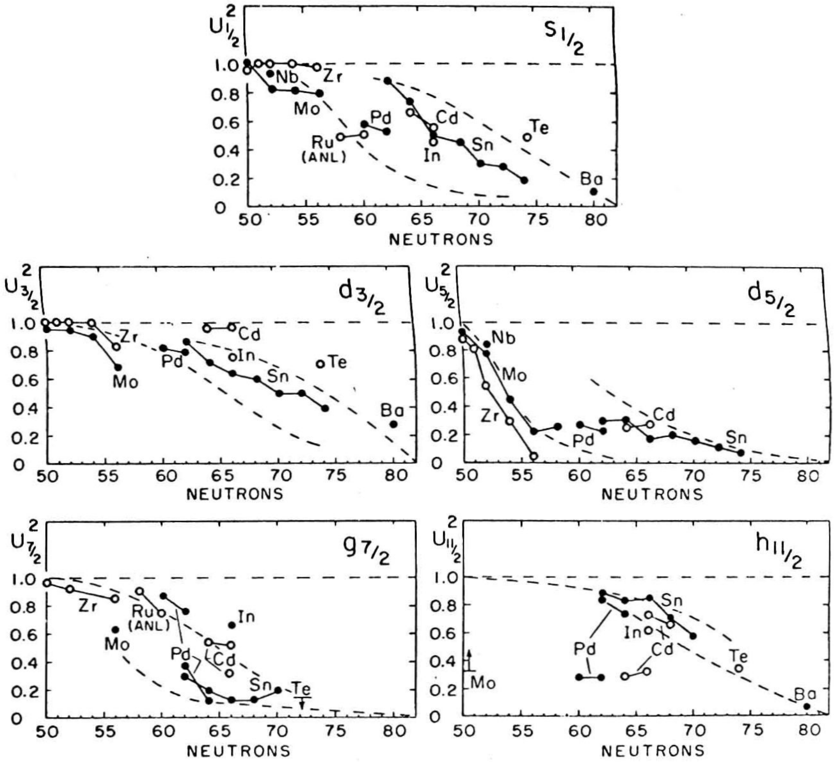

FIG .3. "Emptiness", $U_j^2$, for each single particle state versus neutron number in the 50-82 shell. The dashed lines are predictions of pairing theory. Those starting at the left and right are based on single particle energies in 51 and 81 neutron nuclei respectively.

for the $d_{3/2}$ states in Te and Ba, and the generally erratic behaviour of the $g_{7/2}$ states are examples. These problems can generally be eliminated by judicious assumptions about energies of single particle states if one does not require their variation with A to be smooth. However, the author feels that uncertainties due to difficulties with reaction theories should first be reduced before jumping to such conclusions. We are therefore in the process of making further studies with (d, t) reactions.

The single quasiparticle energies $E_j$ found in these studies are shown in Fig. 4, along with pairing theory predictions on the same basis as those in Fig. 3. Here again there is a considerable area of agreement with theory, but there are still some discrepancies. In the case of $E_j$, there is little chance of error in the analysis of experiments, but pairing theory predictions are not reliable. For example, the quadrupole force has long been known to produce large shifts in energies of nuclear states.

Figs 5 and 6 show similar but less elaborate studies from our laboratory on the N = 28-50 shell [35]. The data are for Ni, Ge and Se, and the pairing theory calculations are based on the single particle energies in $^{55}$Fe and $^{89}$Zr. The latter are shown at the right in Fig. 6. The single

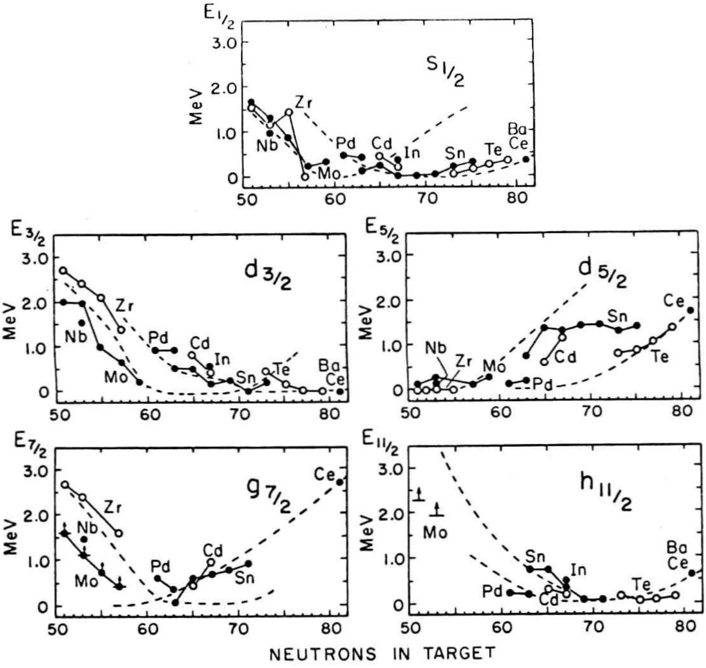

FIG.4. Energy of single quasiparticle states versus neutron number in the 50-82 shell. Dashed lines are pairing theory predictions as described in caption for Fig.3.

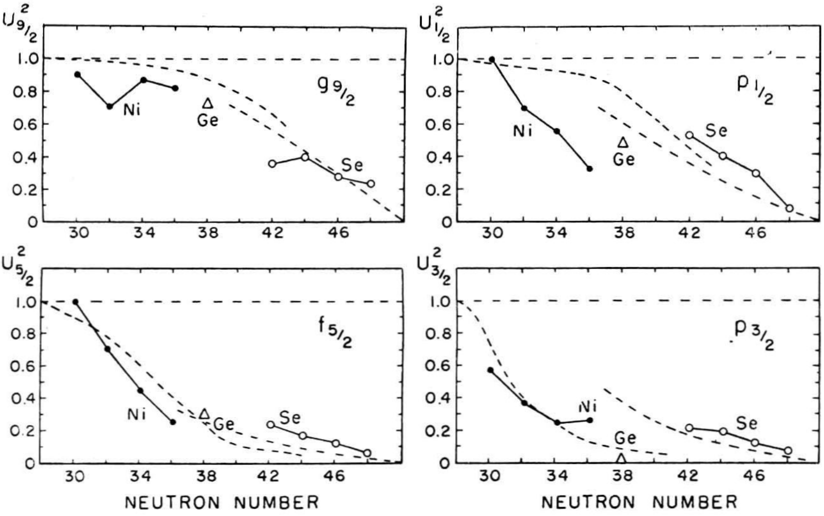

FIG.5. "Emptiness", $U_j^2$, for each single particle state versus neutron number in the 28-50 shell. The dashed lines are predictions of pairing theory. Those starting at left and right are based on single particle energies in 29 and 49 neutron nuclei respectively.