State-Space Models with Regime Switching

Classical and Gibbs-Sampling Approaches with Applications

Chang-Jin Kim and Charles R. Nelson

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To Young-Ho and Kate

Preface and Acknowledgments

State-space models and Markov-switching models have both been highly productive paths for research in econometrics because they address primary issues in our attempts to understand the economy. Unobserved variables are important actors in our stories about consumption behavior, unemployment, inflation dynamics, indices of economic activity, monetary policy, and financial markets. In these situations the state-space framework, made operational by the Kalman filter, is the only one we have for making statistical inference in the time series context. There is also compelling empirical evidence that economic systems exhibit occasional jumps from one regime to another. When such a switch occurs the distribution of the data seems to change. For example, the macroeconomy periodically switches from boom to recession and back again, and dynamics differ between these two regimes. Financial markets periodically switch from a low-volatility regime to a high-volatility regime, and then back again. It is attractive to model such transitions as a Markov process.

The problem of marrying state-space modeling, employed to make inferences about unobserved variables, with a Markov process, employed to make inferences about the timing and nature of switches in regime, is not a trivial one. This becomes clear when we think about the fact that the timing of switches between regimes is unknown, and inference is conditional, in principle, upon all possible combinations of such dates. Recently, it has yielded to two lines of attack. One is from the classical perspective of maximum likelihood estimation, in which an approximation is used to reduce the dimensionality of the problem to manageable proportions. The second approach casts the problem in the Bayesian framework and uses Gibbs-sampling methodology to break the problem down into a sequence of steps that involve only Monte Carlo simulation. An advantage of the latter is that it permits exact inference in finite samples, demonstrating that the computer is not just a tool but is fundamentally changing how we do inference in econometrics. Subsequently, the range of applications for these methods has expanded much faster than we have been able to investigate them, and we felt that it was clearly time to gather results in this area together, along with computer programs, to make these tools generally available to research workers.

This book grew out of lecture notes developed for courses in econometrics taught by Chang-Jin Kim at Korea University and at the University of Washington, Seattle. The students in those courses were the source of numerous helpful comments, questions, and corrections. We have debts to more individuals than we can acknowledge here, but we will try to mention a few. Andrew Harvey is responsible for introducing state-space models to econometrics and

developing inference for them. James Hamilton convinced the profession that Markov switching captured important features of the dynamics of the economy and showed how to do inference. We were also influenced particularly by the work of Siddhartha Chibb, Francis X. Diebold, James Stock, and Mark Watson. We have a special debt to Arnold Zellner, who suggested the Gibbs-sampling approach in a letter to Chang-Jin Kim and has been an invaluable source of wisdom and encouragement to Charles Nelson throughout his career. We are also grateful to our coauthors on papers that form the background for this book: Myung J. Kim, Richard Startz, and Charles Engel. James Morley, Chris Murray, and Young-Sook Lee read drafts of the manuscript carefully and provided corrections and suggestions. Finally, we are grateful to Terry Vaughn of The MIT Press for giving us the freedom to prepare the manuscript of this book in accordance with our instincts.

One of the guiding principles of this book is operationality, and we felt that it would be incomplete without ready access to the computer programs we used in its preparation. These are available from a web site at the University of Washington with the address

http://weber.u.washington.edu/~cnelson/SSMARKOV.html

If that address ceases to be available, please contact one of the authors by email at cjkim@kuccnx.korea.ac.kr or cnelson@u.washington.edu. Use of the programs in published work should be acknowledged by citing this book.

State-Space Models with Regime Switching

Contents

1 Introduction

This book aims to introduce to a wider audience of researchers in economics and finance recent advances in the estimation of state-space models in which switching between regimes occurs stochastically according to a Markov process. These are systems in which there is an unobserved or state variable that we would like to estimate through time, but we also want to allow for periodic shifts in the parameters that describe the system's dynamics or volatility. For example, we may have a vector of economic indicators measured monthly and would like to extract an index or common factor reflecting the state of the economy today. At the same time, we have reason to believe that dynamics and volatility differ between recessions and expansions, so we would like to allow for switching in parameters between these two regimes. Until recently, it was feasible to address either the state variable problem or the regime-switching problem, but not both at the same time. With the methods presented in this book, this and many other previously intractable models in economics and finance become operational.

Part I of the book discusses the classical approach to the estimation of state-space models with Markov switching in the maximum likelihood framework, then part II presents, in parallel, the Bayesian approach using Gibbs-sampling, a computation-intensive methodology that simulates joint posterior densities of parameters, state variables, and regimes. The methodology is illustrated at each step by examples, including time-varying regression coefficients with heteroskedasticity, fads and time-varying volatility in financial markets, a model of the business cycle that allows for asymmetry between expansions and contractions, extraction of an unobserved index of economic activity from a set of indicator variables with regime switching at turning points, decomposition of heteroskedastic exchange rates into permanent and transitory components, and testing for mean reversion in asset prices when returns are heteroskedastic. Actual output from programs and data sets available to the reader accompany each example.

Although more familiar to economists, the classical approach poses special difficulties in this context because of the potentially very large number of evaluations of the likelihood function required. Fortunately, the approximation method due to Kim (1994) makes maximum likelihood estimation feasible. However, two shortcomings of the classical approach motivate interest in the Bayesian alternative. First, the degree of approximation in any paticular case is unknown, though comparison with exact results in examples of interest suggest it may be small. Second, and of perhaps greater practical importance, in the classical approach estimation of state variables is conditional on maximum likelihood estimates of the parameters. In constrast, treatment of state variables, parameters, and regimes as jointly distributed random variables means that with the Bayesian approach, estimates of each appropriately reflect uncertainly about the others.

1.1 State-Space Models and Markov Switching in Econometrics: A Brief History

The state-space model and Markov switching are not new in the statistics and econometrics literatures. But a growing number of published papers that employ them demonstrates their usefulness and widening application. In essence, a state-space model is one in which an observed variable is the sum of a linear function of the state variable plus an error. The state variable, in turn, evolves according to a stochastic difference equation that depends on parameters that in economic applications are generally unknown. Thus, both the path of the state variable through time and the parameters—describing the dynamics of the state variable, its relationship to the data, and the covariance structure of stochastic disturbances—are to be inferred from the data. Harvey, in his influential 1981 book and a series of early papers, introduced to economists the use of the Kalman (1960) filter for obtaining maximum likelihood estimates of parameters through prediction error decomposition was introduced. It became clear from Harvey's work and others' that a surprising range of econometric models, including regression models with time-varying coefficients, autoregressive moving average (ARMA) models, and unobserved-components time series models, could be cast in state-space form and thus be rendered amenable to the Kalman machinery for parameter estimation and extraction of estimates of state variables.

Economists have long recognized the possibility that parameters may not be constant through time but rather that structural shifts may occur, dividing the period into distinct regimes with different parameter values. In the regression model context, Quandt (1972) studied the case of independent switches in regime, then Goldfeld and Quandt (1973) extended the analysis to regime-dependent-switching probabilities according to a Markov chain. Dynamic models with Markov switching between regimes are relatively new. However, since James Hamilton introduced them in his seminal 1989 paper as a tool for dealing with endogenous structural breaks, the number of published papers that apply Markov-switching models has been enormous. An important appeal of these models is their ability to account for the accumulating evidence that business cycles are asymmetric, as discussed in a literature in which the work of Neftci (1984) is particularly influential. Furthermore, they suggest an alternative to the autoregressive conditional heteroskedasticity (ARCH) model of Engle (1982) and its extensions for modeling conditional heteroskedasticity, an important feature of asset returns.

Though models that incorporate both state variables and regime switching seem a natural extension of these literatures and have many obvious potential applications, their estimation posed serious computational barriers. However, with the algorithm for approximate maximum likelihood estimation developed by Kim (1993a, 1993b, 1994), a broad class of models becomes operational that could not be handled before. Applications discussed in this book include a time-varying parameter model of monetary uncertainty with heteroskedasticity, transient fads and crashes, Friedman's (1964, 1993) "plucking model," which accounts for a number of features of business cycle asymmetry, and construction of an index of coincident indicators where Markov switching occurs between business cycle regimes. Thus, the state-space model with Markov switching may be considered a general approach to dealing with endogenous structural breaks.

As mentioned above, the necessity of approximation for obtaining a computationally feasible algorithm for estimation and the treatment of parameter estimates as fixed when estimating state variables in the classical framework lead us to consider the Bayesian approach as an alternative. To make the latter point clear, an index of coincident indicators for the economy is obtained in two steps: In the first step, the parameters of the state-space model are estimated via approximate maximum likelihood; then, in the second step, we condition on those estimates so we can run the Kalman filter to extract the estimate of the index from the observed indicators. Similarly, inferences about which regime, recession or expansion, the economy is in at each point in time condition on the maximum likelihood estimates of the parameters.

Bayesian methods have a long history in econometrics; the classical reference work is Zellner 1971. Gibbs-sampling methods, originally introduced by Geman and Geman (1984), turn out to be the key to feasible estimation of the state-space model with Markov switching in the Bayesian framework. Albert and Chib (1993) introduced Gibbs-sampling in the context of Markov switching, and Carlin, Polson, and Stoffer (1992) and Carter and Kohn (1994) introduced Gibbs-sampling in the context of state-space models. Kim and Nelson (1998) make Gibbs-sampling operational for the state-space model with

Markov switching and construct an experimental index of coincident indicators that encompasses both comovement among economic variables and nonlinearity in the evolution of the business cycle. Gibbs-sampling exploits these models' conditioning features to break the problem down into feasible steps. The additional empirical examples presented in this book demonstrate both the operationality of the Gibbs-sampling approach in a wide range of applications and the empirical relevance of treating state variables, regime switches, and parameters as jointly distributed random variables. Thus, in the example of an index of coincident indicators, uncertainty about whether the economy is in recession or expansion at a particular date is appropriately reflected in the estimate of the state variable, the index of coincident indicators, for that date.

Looking to the future, we think that the most exciting prospect that this methodology holds is for dealing with evolution and change in economic systems. The empirical literature from many areas seems to convey the same message: Relationships between variables as well as dynamics evolve over time or shift abruptly, and disturbances are heteroskedastic. We find that predictability in asset returns varies across subsamples, as does volatility; money demand functions are notoriously unstable and heteroskedastic; and the dynamics of recessions differs greatly from that of expansions. The case for modeling change is compelling, and we hope that its feasibility becomes more apparent with this book.

1.2 Computer Programs and Data

All computations presented in this book can be replicated using programs written in Gauss © by C.-J. Kim and data sets that may be downloaded from the web site

http://weber.u.washington.edu/~cnelson/SSMARKOV.html

Readers who use the programs in published work are asked to acknowledge their use and cite this book as the reference.

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I THE CLASSICAL APPROACH