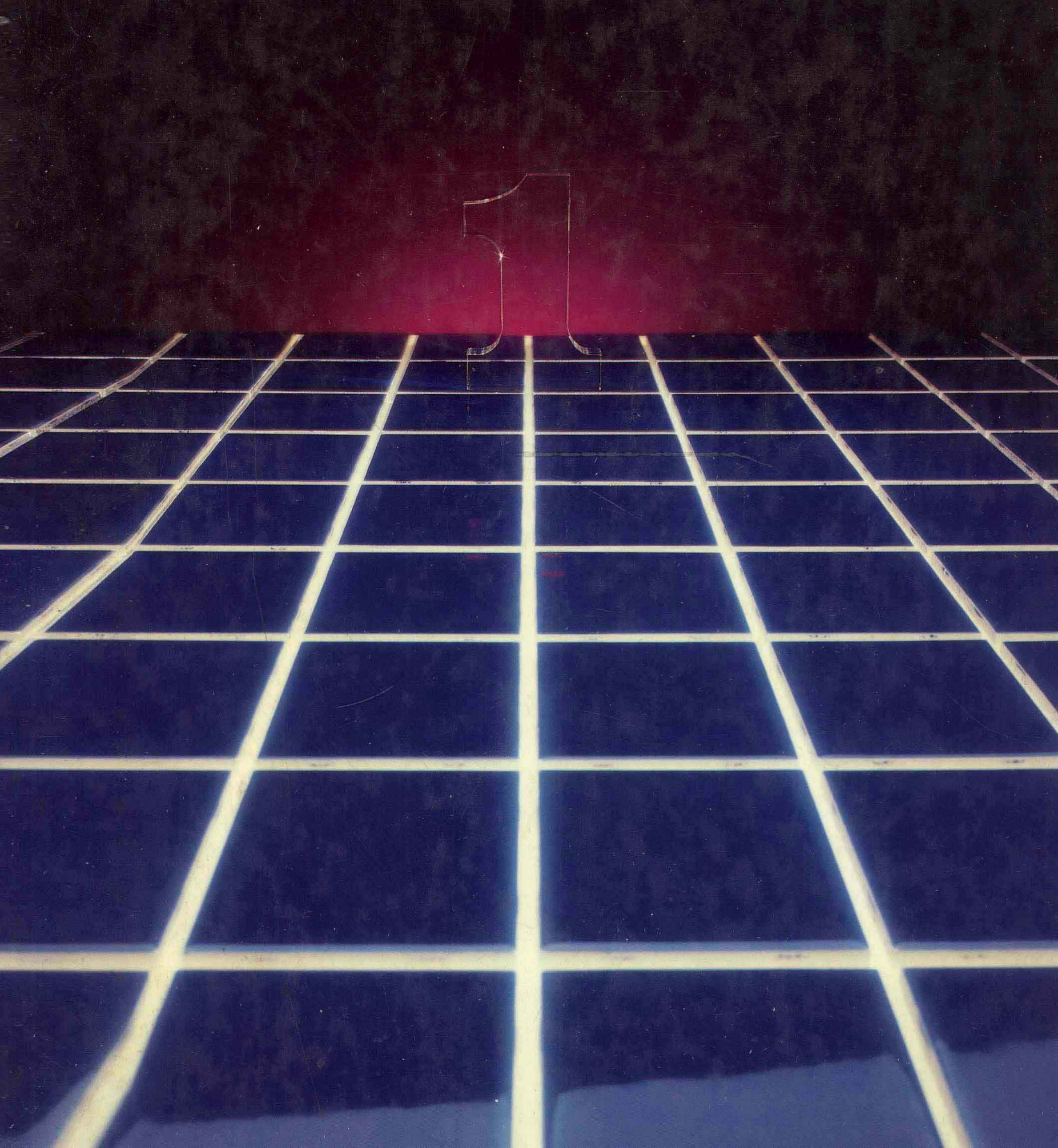
Algebra and Trigonometry

RONALD J. HARSHBARGER JAMES J. REYNOLDS



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Brooks/Cole Publishing Company
Monterey, California

BROOKS/COLE PUBLISHING COMPANY
A Division of Wadsworth, Inc.

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging-in-Publication Data

Harshbarger, Ronald J.,

Algebra and trigonometry.
Includes index.
1. Algebra. 2. Trigonometry. I. Reynolds, James.
II. Title.
QA154.2.H375 1987 512'.13 85-26910

ISBN 0-534-06288-1

Sponsoring Editor: Jeremy Hayhurst
Editorial Assistant: Amy Mayfield
Production Management: Miller/Scheier Associates, Palo Alto
Manuscript Editor: Carol Dondrea
Permissions Editor: Carline Haga
Art Coordinator: Miller/Scheier Associates, Palo Alto
Interior Illustrations: John Foster
Cover Photo: West Light © Walter Urie
Typesetting: Syntax International
Printing and Binding: R. R. Donnelley & Sons Co.

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Preface

The goal of this text is to present the algebra and trigonometry skills and concepts necessary for the successful study of calculus. Knowledge of the concepts of algebra and trigonometry is crucial to the study of calculus as evidenced by the fact that most colleges and universities measure readiness for calculus by testing only algebra and trigonometry skills.

Although we assume that students have had two years of algebra, we included all the basics. The text takes each topic from an introductory level to a level necessary for success in calculus, using interesting and meaningful applications to introduce, motivate, and reinforce the concepts and skills. Completely worked out examples and clear readable exposition help students learn without dependency on their instructor.

The text contains:

Procedure–example tables, which present the step-by-step procedure for solving a problem with an example worked step by step beside it.

Exercises specifically designed to prepare students for calculus.

Integration of the use of the calculator into algebra and trigonometry, with keystrokes included when new functions or techniques are introduced.

Subdivision of each section into subsections to help instructors organize their presentation and assignments. Exercise sets are keyed to these subsections.

Extensive coverage of analytic geometry topics.

Numerous examples that illustrate the concepts and develop the skills of algebra and trigonometry.

A wide variety of exercises, including drill problems and applications, analytic geometry problems, and preparing for calculus problems.

A large number of relevant applications from the physical, life, management, and social sciences.

High-quality review exercises at the end of each chapter keyed to chapter sections.

Cumulative review after Chapter 3 to help instructors determine what material from Chapters 1 to 3 needs to be studied.

Detailed coverage of graphing.

Helpful summary tables for topics such as conic sections, trigonometric equations, and inverse trigonometric functions.

Since the goal of the text is to have students able to perform the algebraic and trigonometric skills necessary for success in calculus, advanced level exercises deal with algebraic and trigonometric applications to calculus. "Preparing for Calculus" exercises ask algebraic and trigonometric questions that arise in working calculus problems. These exercises are not intended to teach calculus, but to motivate the study of algebra and trigonometry.

The "Preparing for Calculus" exercises show how different but related algebra skills are needed for calculus.

Examples are:

a. Writing $\dfrac{1}{x(x-1)}$ as $\dfrac{-1}{x} + \dfrac{1}{x-1}$ for integration.

b. Using $\dfrac{x+1}{x}$ in the form $1 + \dfrac{1}{x}$ for integration.

c. Writing $\dfrac{1}{x^2}$ as x^{-2} for differentiation.

d. Writing $\ln \dfrac{x}{x+1}$ as $\ln x - \ln(x+1)$ for differentiation.

We assume that each student has a scientific calculator available for use with the text. We refer to the following calculator functions and their inverses:

$$\ln, \log, \sin, \cos, \tan, y^x, 1/x, e^x$$

The use of calculator technology lessens our dependence on tables for evaluating trigonometric and logarithmic equations. The calculator makes it unnecessary to devote sections to table reading and interpolation with trigonometric functions and logarithms, providing time to cover important topics more thoroughly and to add important topics to the course. Tables are included and referred to, but not emphasized.

Because calculators radically simplify the solution of trigonometric problems, we introduce trigonometry using right triangles, and then move on to a more extensive and exhaustive discussion of trigonometric functions. Circular functions are introduced via the unit circle and are related to trigonometric functions; then they are used to graph the sine function.

The text has the flexibility to handle students of widely different mathematical backgrounds. It is complete enough and readable enough that a weak student can learn from it, yet it provides concepts such as piecewise–defined functions, halves of conic sections and preparing for calculus problems that challenge the more sophisticated student.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to the following educators whose reviews of their work provided encouragement and constructive criticism. They are: Donald Bennett, Murray State University; Ethel Dursht, Norwalk Community College; Harvey Elder, Murray State University; Ann Fitzmaurice, University of Hartford; Elton Fors, Northern State College; Philip A. Glynn, Mattatuck Community College; Alan Holz, California Polytechnic State University; Georgiana Klein, Grand Valley State College; Wally Kodis, Cabrillo College; Stephen Kuhn, University of Tennessee; Pauline Lowman, Western Kentucky University; Sandra McLaurin, University of North Carolina; Lynda Morton, University of Missouri, Emilio Roxin, University of Rhode Island; Gary Simundza, Wentworth Institute; John Tobey, North Shore Community College; John Vangor, Housatonic Community College.

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Algebra and Trigonometry

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The Real Numbers 1

1.1 REAL NUMBERS

A. PROPERTIES OF REAL NUMBERS

This text is designed to review and extend the concepts and skills of algebra and trigonometry necessary for success in studying calculus. In addition, the text provides skills and applications that are useful in many fields of study, from accounting to zoology.

Elementary calculus has its setting in the real numbers, so we deal with the real numbers throughout much of this text. Occasionally, we deal with an extension of the real numbers called the complex numbers. Examples of real numbers are: $-3, 7, \frac{1}{3}, 0.46, \pi, \sqrt[3]{15}, 0.666\ldots$ The equation $a = b$ (a equals b) means that the symbols a and b represent the same number. Some of the properties of equality follow.

Properties of Equality

For all real numbers a, b, c, d:

If $a = b$, then $b = a$.

If $a = b$, then $a + c = b + c$.

If $a = b$, then $ac = bc$.

If $a = b$ and $c = d$, then $a + c = b + d$ and $ac = bd$.

If $a = b$ and $b = c$, then $a = c$.

Real numbers are said to be **closed under addition.** This means that the **sum** of two real numbers, denoted $a + b$, is a real number. They are also **closed under multiplication,** because the **product** of two real numbers, denoted $a \cdot b$, $a \times b$, $a(b)$, $(a)(b)$, or ab, is a real number. In this case, a and b are called **factors** of the product ab. The properties of the real numbers, which follow, are fundamental to the study of algebra.

Properties of Real Numbers

For all real numbers a, b, and c, the following properties hold:

I. Commutative Property	
a. Addition	$a + b = b + a$
b. Multiplication	$ab = ba$
II. Associative Property	
a. Addition	$a + (b + c) = (a + b) + c$
b. Multiplication	$a(bc) = (ab)c$
III. Identities	
a. Additive identity is 0.	$a + 0 = 0 + a = a$
b. Multiplicative identity is 1.	$a \cdot 1 = 1 \cdot a = a$
IV. Inverses	
a. Each a has a unique **additive inverse,** denoted $(-a)$.	$a + (-a) = (-a) + a = 0$
b. Each a ($a \neq 0$) has a unique **multiplicative inverse,** denoted a^{-1} or $\frac{1}{a}$. Thus $a^{-1} = \frac{1}{a} = 1/a$.	$(a)(1/a) = (1/a)(a) = 1$ $aa^{-1} = a^{-1}a = 1$
V. Distributive Law of Multiplication over Addition	$a(b + c) = ab + ac$ $(b + c)a = ba + ca$

Some comments regarding these properties are in order.

Property IV for additive inverses gives us the means to subtract—namely, by adding the additive inverse:

Subtraction

$$a - x = a + (-x)$$

For example,

$$-12 - 9 = -12 + (-9) = -21$$
$$6 - 10 = 6 + (-10) = -4$$

From Property IV for additive inverses, it also follows that a is the additive inverse of $-a$, which we can denote as $-(-a)$. Thus,

$$a = -(-a)$$

For example, $-(-12) = 12$ and $9 - (-12) = 9 + 12 = 21$

Notice that there is a difference between the negative of a number (its additive inverse) and a negative number.

Property IV for multiplicative inverses provides a means to divide by nonzero numbers:

Division

$$\frac{a}{x} = a \div x = a \cdot \frac{1}{x} = ax^{-1} \qquad \text{if } x \neq 0$$

For example, $\frac{-12}{3} = -12 \cdot \frac{1}{3} = -4$

The number 0 has no inverse, so division by 0 is undefined. Thus, $\frac{5}{0}$ and $\frac{0}{0}$ are not defined.

From Property IV for multiplicative inverses, it also follows that a is the multiplicative inverse* of $1/a$, which we can denote as $1/(1/a)$. Thus

$$\left(\frac{1}{a}\right)^{-1} = \frac{1}{(1/a)} = a$$

For example, $\frac{1}{(1/2)} = 2$ and $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

Property V expresses a relation that exists between multiplication and addition. The distributive law will be useful in multiplying and factoring algebraic expressions. For example,

$$2(l + w) = 2l + 2w \qquad \text{and} \qquad 2a + 3a = (2 + 3)a = 5a$$

Example 1 State which of the properties of real numbers is used in each of the following:

a. $4 + 3 = 3 + 4$
b. $-3 \cdot 1 = -3$
c. $3 + [4 + (-7)] = 7 + (-7)$
d. $3c + 4c = 7c$
e. $(2/3)(3/2) = 1$
f. $7 - 4 = 7 + (-4)$

* The multiplicative inverse of a, $1/a$, is also called the reciprocal of a.

Solution

a.	Property I(a)	Commutative property of addition
b.	Property III(b)	Multiplicative identity
c.	Property II(a)	Associative property of addition
d.	Property V	Distributive law
e.	Property IV(b)	Multiplicative inverse
f.	Property IV(a)	Additive inverse ■

Some familiar and important consequences of the basic properties of real numbers are:

1. $a \cdot 0 = 0$ for all real numbers a
2. $ab = 0$ if and only if $a = 0$ or $b = 0$ or both
3. $-a(b) = -(ab) = a(-b)$
4. $(-a)(-b) = ab$

B. SUBSETS OF REAL NUMBERS

We frequently refer to the real numbers as the *set* of real numbers. In general, any well-defined collection of objects may be called a **set.** The objects in the set are called **elements** or **members** of the set. If every member of one set, A, is also a member of a second set, B, then we say A is a **subset** of B, denoted $A \subset B$.

Within the set of real numbers, there are special subsets of interest to us.

Subsets of Real Numbers	*Descriptions*
Counting Numbers	1, 2, 3, . . .*
Integers	Counting numbers, zero, and the additive inverses of counting numbers, . . . , −3, −2, −1, 0, 1, 2, 3, . . .
Rational Numbers	All real numbers that can be written as p/q, with p and q integers and $q \neq 0$; or, equivalently, all real numbers that can be written as a decimal that repeats or terminates.
Irrational Numbers	All real numbers that are not rational; or, equivalently, all decimals that neither repeat nor terminate.

* The three dots (. . .) indicate that the numbers continue with the same pattern.

Figure 1.1 shows the relationship among these subsets.

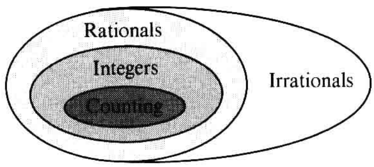

Figure 1.1 Subsets of the real numbers

Example 2 From the set of real numbers containing 1, $\sqrt{2}$, $\frac{1}{3}$, 2, 0.666 . . . , $\frac{28}{4}$, π, -6, $7\frac{1}{4}$, $-\sqrt{3}$, 5.2, identify:

a. The counting numbers
b. The integers
c. The rational numbers
d. The irrational numbers

Solution

a. The counting numbers are 1, 2, $\frac{28}{4}$ ($\frac{28}{4} = 7$).
b. The integers are 1, 2, $\frac{28}{4}$, -6.
c. The rational numbers are 1, $\frac{1}{3}$, 2, 0.666 . . . , $\frac{28}{4}$, -6, $7\frac{1}{4}$, 5.2.
d. The irrational numbers are $\sqrt{2}$, π, $-\sqrt{3}$. (These numbers can be approximated by terminating decimals but do not have decimal representations that terminate or repeat.) ■

We can also represent the real numbers on a **real number line.** Exactly one real number is associated with each point on the line, so we say there is a *one-to-one correspondence* between the real numbers and the points on the line. Thus the real number line is a graph of the real numbers (see Figure 1.2).

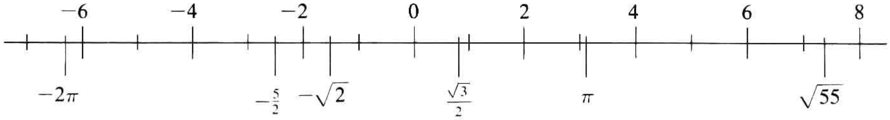

Figure 1.2

C. PROPERTIES OF QUOTIENTS

As mentioned previously, rational numbers are the subset of the reals consisting of quotients of integers, where the denominator is nonzero. In addition to rational numbers having fractional form, numbers such as $\pi/2$ and $\sqrt{2}/5$ are real numbers that are quotients of real numbers. Equality for quotients is defined by

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc \qquad \text{where } b \neq 0 \text{ and } d \neq 0.$$

Using this definition, the following properties may be established:

Properties of Quotients

For real numbers a, b, c, and d, and nonzero denominators:

I. $\dfrac{ac}{bc} = \dfrac{a}{b}$

II. $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

III. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b}\left(\dfrac{c}{d}\right)^{-1} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

IV. $\dfrac{a}{b} + \dfrac{c}{b} = \dfrac{a + c}{b}$

V. $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

VI. $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$

D. CALCULATORS

Calculator types vary widely, from those that perform only the four arithmetic operations to those that perform many operations automatically and those that can be programmed.

Two types of internal logic (methods of computation) are used in calculators: *algebraic* and *reverse Polish.* Reverse Polish logic is highly efficient and does not require parentheses; many people feel it is better for complex problems. Algebraic logic operates in a manner consistent with the order of operations (discussed in Section 1.2) and is easy to use for most elementary problems.

The text assumes that you have a scientific calculator with algebraic logic that can perform the four arithmetic operations, evaluate exponential and logarithmic expressions, and evaluate trigonometric and inverse trigonometric functions. However, since many differences exist among different models of calculators—even those that use algebraic logic—it is important that you refer to your manual.

When new types of calculations are introduced in the text, the calculator keystrokes will be indicated. The keystrokes given represent one way (but not the