

# An Introduction to 3D Computer Vision Techniques and Algorithms



Bogusław Cyganek  
J. Paul Siebert

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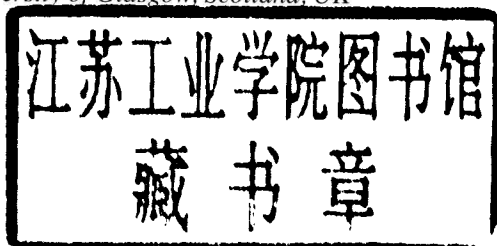
# AN INTRODUCTION TO 3D COMPUTER VISION TECHNIQUES AND ALGORITHMS

**Bogusław Cyganek**

*Department of Electronics, AGH University of Science and Technology, Poland*

**J. Paul Siebert**

*Department of Computing Science, University of Glasgow, Scotland, UK*



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# **AN INTRODUCTION TO 3D COMPUTER VISION TECHNIQUES AND ALGORITHMS**

To Magda, Nadia and Kamil  
From Bogusław

To Sabina, Konrad and Gustav  
From Paul

# Preface

Recent decades have seen rapidly growing research in many areas of computer science, including computer vision. This comes from the natural interest of researchers as well as demands from industry and society for qualitatively new features to be afforded by computers. One especially desirable capability would be automatic reconstruction and analysis of the surrounding 3D environment and recognition of objects in that space. Effective 3D computer vision methods and implementations would open new possibilities such as automatic navigation of robots and vehicles, scene surveillance and monitoring (which allows automatic recognition of unexpected behaviour of people or other objects, such as cars in everyday traffic), medical reasoning, remote surgery and many, many more.

This book is a result of our long fascination with computers and vision algorithms. It started many years ago as a set of short notes with the only purpose ‘to remember this or that’ or to have a kind of ‘short reference’ just for ourselves. However, as this diary grew with the years we decided to make it available to other people. We hope that it was a good decision! It is our hope that this book facilitates access to this enthralling area, especially for students and young researchers. Our intention is to provide a very concise, though as far as possible complete, overview of the basic concepts of 2D and 3D computer vision. However, the best way to get into the field is to try it oneself! Therefore, in parallel with explaining basic concepts, we provide also a basic programming framework with the hope of making this process easier. We greatly encourage the reader to take the next step and try the techniques in practice.

Bogusław Cyganek, Kraków, Poland  
J. Paul Siebert, Glasgow, UK

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# Notation and Abbreviations

$I_k(x, y)$	Intensity value of a $k$ -th image at a point with local image coordinates $(x, y)$
$\overline{I_k(x, y)}$	Average intensity value of a $k$ -th image at a point with local image coordinates $(x, y)$
<b>I</b>	Identity matrix; image treated as a matrix
<b>P</b>	A vector (a point), matrix, tensor, etc.
$T[\mathbf{I}, \mathbf{P}]$	The Census transformation $T$ for a pixel $\mathbf{P}$ in the image <b>I</b>
$i, j$	Free coordinates
$d_x, d_y$	Displacements (offset) in the $x$ and $y$ directions
$D(\mathbf{p}_l, \mathbf{p}_r)$	Disparity between points $\mathbf{p}_l$ and $\mathbf{p}_r$
<b>D</b>	Disparity map (a matrix)
$U(x, y)$	Local neighbourhood of pixels around a point $(x, y)$
$\mathbf{O}_c$	Optical centre point
$\mathbf{P}_c = [X_c, Y_c, Z_c]^T$	Coordinates of a 3D point in the camera coordinate system
$\Pi$	Camera plane; a projective plane
$\mathbf{o} = (o_x, o_y)$	Central point of a camera plane
$f$	Focus length of a camera
$b$	Base line in a stereo system (a distance between cameras)
$h_x, h_y$	Physical horizontal and vertical dimensions of a pixel
$\mathbf{P} = [X, Y, Z]^T$	3D point and its coordinates
$\mathcal{S}^n$	N-dimensional projective space
$\mathbf{P} = [X_h, Y_h, Z_h, 1]^T$	Homogenous coordinates of a point
<b>M</b>	Camera matrix
$\mathbf{M}_i$	Intrinsic parameters of a camera
$\mathbf{M}_e$	Extrinsic parameters of a camera
<b>E</b>	Essential matrix.
<b>F</b>	Fundamental matrix.
$\mathbf{e}_i$	Epipole in an $i$ -th image
SAD	Sum of absolute differences
SSD	Sum of squared differences
ZSAD	Zero-mean sum of absolute differences
ZSSD	Zero-mean sum of squared differences
ZSSD-N	Zero-mean sum of squared differences, normalized

SCP	Sum of cross products
SCP-N	Sum of cross products, normalized
RMS	Root mean square
RMSE	Root mean square error
$\langle L_{xx}, L_{yy} \rangle$	Code lines from a line $L_{xx}$ to $L_{yy}$
HVS	Human Visual System
SDK	Software Development Kit
$\wedge$	logical ‘and’
$\vee$	logical ‘or’
LRC	Left-right checking (cross-checking)
OCC	Occlusion constraint
ORD	Point ordering constraint
BMD	Bimodality rule
MGJ	Match goodness jumps
NM	Null method
GT RMS	Ground-truth RMS
WTA	Winner-takes-all
*	Convolution operator

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