Multidimensional Systems Theory

Progress, Directions and Open Problems in Multidimensional Systems

Edited by

N. K. Bose

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With contributions by
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Editor's Preface

> Approach your problem from the right end and begin with the answers.
> Then one day, perhaps you will find the final question.
>
> The Hermit Clad in Crane Feathers in R. van Gulik's The Chinese Maze Murders.

> It isn't that they can't see the solution.
> It is that they can't see the problem.
>
> G. K. Chesterton. The Scandal of Father Brown The point of a Pin.

Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related.

Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and covering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces. And in addition to this there are such new emerging subdisciplines as "experimental mathematical", "CFD", "completely integrable systems", "chaos, synergetics and large-scale order", which are almost impossible to fit into the existing classification schemes. They draw upon widely different sections of mathematics. This programme, Mathematics and Its Applications, is devoted to new emerging (sub)disciplines and to such (new) interrelations as exempla gratia:

- a central concept which plays an important role in several different mathematical and/or scientific specialized areas;
- new applications of the results and ideas from one area of scientific endeavour into another;
- influences which the results, problems and concepts of one field of enquiry have and have had on the development of another.

The Mathematics and Its Applications programme tries to make available a careful selection of books which fit the philosophy outlined above. With such books, which are stimulating rather than definitive, intriguing rather than encyclopaedic, we hope to contribute something towards better communication among the practitioners in diversified fields.

As mathematical disciplines go, system and control theory is a young one. Definitely post World War II though, of course, its roots go back further. It has already established itself as a determined user of the most sophisticated mathematical tools available such as algebraic geometric intersection theory and cohomology, all kinds of spectral factorizations, various chapters in interpolation theory, H^2 and H^∞ spaces, ... It also, concomitantly, seems to be an inexhaustible source of new and interesting problems.

Some of these problems have to do with different aspects (than usually considered) of well-established areas of concern.

Examples are: is there an effective (constructive) proof of the Quillen–Suslin theorem on algebraic vector bundles, and what happens to the idea of a conservation law if imputs are allowed?

The lines just written are true even for the usual kind of systems, or 1-D systems, and in fact even for linear systems which at first sight seem so simple (mathematically speaking) that it is hard to believe that sophisticated tools will be needed to deal with them.

Modern technological demands such as for image processing and the remarkable successes of 1-D theory (i.e. the usual kind of mathematical system theory) caused the emergence of 2-D (and *n*-D) theory. This is a field in which things are in rapid flux calling for an occasional taking stock of the situation. That is exactly what this book aims to do.

The unreasonable effectiveness of mathematics in science . . .

Eugene Wigner

Well, if you know of a better 'ole, go to it.

Bruce Bairnsfather

What is now proved was once only imagined.

William Blake.

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited.
But when these sciences joined company they drew from each other fresh vitality and thenceforward marched on at a rapid pace towards perfection.

Joseph Louis Lagrange.

Bussum, March 1985

MICHIEL HAZEWINKEL

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Preface

Towards the end of 1981, Professor M. Hazewinkel invited me to write a book on multidimensional systems theory in his series devoted to mathematics and its applications. At that time, however, I had already completed the manuscript for the text entitled, 'Applied Multidimensional Systems Theory', which was published by Van Nostrand Reinhold Co. during the first quarter of 1982. Though, at that time, the need for a textbook on the subject was realized, it was felt that the area of multidimensional systems theory was expanding at a prolific rate from the standpoints of both theoretical and applied research. This was brought about, sometimes, by independent groups of researchers with different academic backgrounds who worked without the benefit of interaction between groups. Since the subject has a rich mathematical flavor, any attempt to promote dialogue between engineers emphasizing theory and mathematicians willing to gear their research towards applications, is, unquestionably, very beneficial. This book, then, gradually evolved out of the realization of the necessity of documenting the significant progress and novel directions of research in the arena of multidimensional systems theory, since the publication of my textbook, with the ultimate objective of alerting the reader about the existence of an apparently inexhaustible source of open problems, only some of which could be explicitly stated in this book.

The first chapter contains a description of very recent results in topics of fundamental importance in multidimensional systems theory. Perusal of this chapter is likely to convince the reader of the broadening scopes for applications of either some key theoretical results or limitations to such applications caused by problems, which still defy complete, satisfactory, solutions. Since, approximation theory is a very vast topic, only multivariate rational approximants of the Padé-type has been briefly surveyed in Chapter 2, because this type of rational approximation technique and its variants are popular in various problems of multidimensional systems theory. A rigorous analysis of a type of 2–D feedback system is provided in Chapter 3, where the advantages of incorporating the weakly causal feature in the design of stabilizing compensators is highlighted. Multidimensional feedback systems have been proposed in several applications, and the future role of feedback in the design of multidimensional structures should not be underestimated. In Chapter 4, the problems of existence and construction of feedback compensators for spatio-temporal systems, whose inputs and outputs are functions of a temporal and a spatial variable, are studied. The approach in Chapter 3 is a transform domain approach based on tools from algebra while that of Chapter 4 is based on techniques from the theory of linear systems with coefficients in a commutative ring and the criteria for stabilizability are specified in terms of a state-space representation (and also a transfer matrix representation). Chapter 5 introduces the reader to the modeling, analysis, and applications of linear, shift-variant multidimensional systems while Chapter 6 describes the theory of Gröbner bases which are known to have proven as well as potential applications in multidimensional systems theory problems. Chapter 7 considers conditions for the solution of a system of equations over the ring $C[z, w]$, motivated by the availability of some recent results in the mathematical literature and the applicability of the solution to the problem considered to one formulated in Chapter 3.

This book is aimed towards fulfilling the needs of those mathematicians who want to learn about current applications in an area requiring a wide variety of mathematical resources and also towards scientists, engineers, and researchers in industries and universities who want to keep abreast of latest developments in multidimensional systems theory, the directions along which this subject is expanding and some of the problems that are, currently, open to investigation. The book could also be, effectively, used in advanced seminars, selected continuing education courses and, possibly, as a supplement to a text adopted for courses in multidimensional systems theory or spatial and temporal signal processing.

The book could not be written without the cooperation of the contributors, some of whom, in addition to writing their respective portions, also made available some open problems for the benefit of the readers. Special thanks go to Dr. J. P. Guiver, who stimulated many interesting discussions and provided useful insights to some research problems during his stay, here, at the University of Pittsburgh. I have greatly benefited from interaction with colleagues at various universities, who made available to me many of their research results before those formally appeared in journals. The brief survey in Chapter 2 was partially influenced by the doctoral dissertation of A. Cuyt. The motivation and help provided by other colleagues is partly reflected in the contents of this book and I shall not attempt to acknowledge everyone here because it is very easy to, inadvertently, omit a name that would be expected to appear. I wish to express my sincere gratitude to the Air Force Office of Scientific Research, where I have been fortunate to work in Dr. Joseph Bram's program in the Directorate of Mathematical and Information Sciences, and to the National Science Foundation for continuing support of the research I have been conducting with my group of researchers. Some results of that research are included in this book. I thank Professor M. Hazewinkel for extending to me his kind invitation to contribute in his useful series and to the D. Reidel Publishing Company for their cooperation in the successful completion of this project.

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Introducing the Contributors

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Chapter 1

N. K. Bose

Trends in Multidimensional Systems Theory

1.1. INTRODUCTION

The theories of functions and polynomials of several complex and/or real variables along with their numerous applications in several areas of systems theory primarily concerned with the topics of multidimensional digital filter, stability, stabilization and design, multivariate network realizability theory, digital array processing in the general framework of multidimensional signal processing techniques needed to process signals carried by propagating wave phenomena, in addition to problems occurring in control theory concerned with 2–D state-space models, notions of controllability, observability, minimality, feedback and pole-placement as well as stabilization via output feedback provided the subject matter of a Special Issue devoted to Multidimensional Systems [1.1]. Though the vast majority of the papers in [1.1] were concerned with multidimensional deterministic systems, a paper concerned with methods for inference about models of random processes on multidimensional Euclidean space from observed data was also included especially because the mathematical tools employed partially fitted those widely used in the Issue and also because it was felt at that time that the future scopes for development of multidimensional systems theory should not be restricted to deterministic systems only. In fact, broadly speaking, multidimensional systems theory spans deterministic and statistical approaches to the modeling, analysis and design of spatio-temporal continuous and discrete systems. A collection of reprints geared particularly to the developments in the area of multidimensional signal processing (deterministic as well as statistical) till about 1977 has been compiled in [1.2]. As a result of increasing research activity in the area and the realization of the necessity to encourage interaction between computer scientists, engineers, and mathematicians, a collection of reprints which characterized the significant developments that took place in the domain of multidimensional systems theory till about 1979, was published in [1.3], where an introductory survey article and a list of open problems were also included. The importance of documenting the scattered research results in an unified

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form so that the theoretical fundamentals, even though based on relatively advanced and broad range of mathematical topics, could be presented to advanced graduate students and research scientists in a classroom or seminar types of settings, the need was felt to select and expound fundamental results of proven and potential significance in the area of mathematical multidimensional systems theory. This was especially so because though a sizeable number of books had appeared of an applied nature, especially in the area of image processing, books devoted exclusively to the more mathematical aspects that support such applications were non-existent at that time. One complete chapter in [1.4] was devoted to multiparameter systems including two-dimensional filters, distributed processes and statistical and probabilistic models for random fields. More recently, a comprehensive account of the developments in the theory of deterministic multidimensional systems along with an exposition of the supporting mathematical tools required in selected branches of study where such tools are used, was given in [1.5].

The objective here will be to document the progress in multidimensional systems theory since [1.5] was published so that the reader is alerted to the flurry of activities generated by certain fundamental results, some of which are finding applications in more than one area of research. In the process it will become apparent that the topic of multidimensional systems continues to provide challenging theoretical problems which arise in the continuously increasing domains of applications. Efforts will be made to relate briefly but succinctly past efforts, present status and future trends so that the reader following his perusal of the contents of this chapter will not only be able to appreciate better the contents of the succeeding chapters but also will become cognizant of the resources available to tackle the open problems, implicitly as well as explicitly mentioned in this book. Though it is recognized that a fundamental concept, result, or theoretical limitation, usually affects more than one area of application and attempts to segment the range of applications to distinct domains is sometimes futile, it is felt that when, for the sake of clarity in exposition, it becomes necessary to consider one particular topic in one particular section then the relevant links with other topics or applications need to be cited and attended to. It is, therefore, hoped that the topics selected for discussion in the following sections will represent and reveal the existence of strong coupling between theoretical fundamentals and applications that fall under the umbrella of multidimensional systems theory.

1.2. MULTIDIMENSIONAL SYSTEMS STABILITY

The concept of stability plays an important role in various areas that fall under the jurisdiction of multidimensional systems theory. Akin to the occurrence of several fundamental stability criteria in other fields of science (for example, stability of solutions of differential equations in nonlinear systems theory could be investigated under the definitions of Liapunov stability, asymptotic stability, conditional stability, orbital stability, stability under perturbation, etc.), the topic of multidimensional systems also uses the concept of stability in various context-dependent forms. An exposition of the state of this topic until about the end of 1981 is given in a recent book by Bose, [1.5], whose coverage of stability is geared towards linear shift-invariant multidimensional discrete systems, delay-differential systems, lumped-distributed and variable-parameter networks and to a lesser extent, stiff differential systems occuring in discussions of stability questions for numerical methods. Here, the important characteristics and developments of stability theory in these and related areas will be highlighted and attention will be directed to some questions that need to be satisfactorily resolved.

1.2.1. *Multidimensional Digital Filters*

The most commonly used criterion for assessing the stability of linear shift-invariant (LSI) multidimensional digital filters is the *bounded input/ bounded output* (BIBO) criterion, which is well known to be equivalent to the requirement of absolute summability of the impulse response sequence characterizing the filter. A LSI n-dimensional recursive filter is also characterizable by a transfer function,

$$H(z_1, z_2, \ldots, z_n) = \frac{A(z_1, z_2, \ldots, z_n)}{B(z_1, z_2, \ldots, z_n)} \tag{1.1}$$

where $H(z_1, z_2, \ldots, z_n)$ is viewed as a rational function of $z_1, z_2, \ldots, z_n$, $z_1^{-1}, z_2^{-1}, \ldots, z_n^{-1}$ (recursible filters include causal and weakly causal filters). For notational convenience, the z-transform $H(z_1, z_2, \ldots, z_n)$ of an n–D sequence, $\{h(k_1, k_2, \ldots, k_n)\}$, will be defined as a power series involving the superposition of the products of monomials of the type $z_1^{k_1} z_2^{k_2} \ldots z_n^{k_n}$ and the generic element, $h(k_1, k_2, \ldots, k_n)$, of the sequence. Physically, the indeterminates $z_1, z_2, \ldots, z_n$ are the respective delay variables along the spatial or temporal directions of sampling during the analog to digital conversion of a multidimensional spatio-temporal signal. In the case of first quadrant quarter-plane filters, $A(z_1, z_2, \ldots, z_n)$ and $B(z_1, z_2, \ldots, z_n)$ are polynomials. Since a ring isomorphism (see Chapters 3 and 5) maps a weakly causal filter onto a first quadrant quarter-plane filter, $A(z_1, z_2, \ldots, z_n)$, $B(z_1, z_2, \ldots, z_n)$, will be understood to be relatively prime *polynomials* in the delay variables $z_1, z_2, \ldots, z_n$ unless mentioned otherwise.

For a first quadrant quarter-plane digital filter, characterized by $H(z_1, z_2, \ldots, z_n)$, which is assumed to be holomorphic around the origin (this is assured by assuming $B(0, 0, \ldots, 0) \neq 0$), thereby permitting a Taylor series expansion,

$$H(z_1, z_2, \ldots, z_n) = \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} h(k_1, \ldots, k_n) z_1^{k_1} \cdots z_n^{k_n} \tag{1.2}$$

the investigation into BIBO stability reduces to the determining of conditions under which the sum

$$\sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} |h(k_1, \ldots, k_n)| \tag{1.3}$$

converges.† Denoting by $\bar{U}^n$, U^n, and T^n the closed unit polydisc ($|z_i| \leq 1$, $i = 1, \ldots, n$), the open unit polydisc ($|z_i| < 1$, $i = 1, \ldots, n$), and the distinguished boundary ($|z_i| = 1$, $i = 1, \ldots, n$), it is well-known that convergence of (1.3) implies uniform convergence in $\bar{U}^n$ of (1.2), which in turn implies that $H(z_1, \ldots, z_n)$ is holomorphic in U^n and continuous on $\bar{U}^n$. Also, if $H(z_1, \ldots, z_n)$ is holomorphic in a neighbourhood of $\bar{U}^n$, then (1.3) converges. In the $n = 1$ case, for a rational function, $H(z_1) = [A(z_1)]/[B(z_1)]$, it is simple to establish that (1.3) with $n = 1$ is absolutely summable if and only if the polynomial $B(z_1) \neq 0$, $|z_1| \leq 1$. This fact does not generalize in the $n > 1$ case and $B(z_1, \ldots, z_n) \neq 0$ in $\bar{U}^n$ is only a sufficient condition for BIBO stability of $H(z_1, \ldots, z_n)$ in (1.1). Though, in this context a set of necessary and sufficient conditions have not yet been obtained, some recent progress made towards the attainment of that goal is worth recording. The notations in [1.6] will be adopted. Let m_n be the Lebesgue measure divided by $(2\pi)^n$ that is carried with the compact

†Absolute convergence in (1.3) implies that $h(k_1, \ldots, k_n)$'s are uniformly bounded [1.66, p. 102], i.e. there exists a constant K such that, $|h(k_1, \ldots, k_n)| \leq K$, $k_1, \ldots, k_n = 0, 1, 2, \ldots$.

Abelian group (with componentwise multiplication as group operation) T^n, so that $m_n(T^n) = 1$. Also, for $0 < p < \infty$, let $H^p(U^n)$ be the class of all holomorphic functions, $F(z_1, \ldots, z_n)$ in U^n, for which,

$$\|F\|_p \triangleq \sup_{0 \le r < 1} \left\{ \int_{T^n} |F(r\mathbf{z})|^p \, dm_n \right\}^{1/p} < \infty. \tag{1.4}$$

The following results, due to Dautov, will be stated and proved because of its unavailability in the western literature.

THEOREM 1.1. (*Dautov* [1.7]). *For the convergence of the summation in* (1.3), *it is sufficient that the function*

$$G(z_1, \ldots, z_n) \triangleq \frac{\partial^n [H(z_1, \ldots, z_n) z_1 \ldots z_n]}{\partial z_1 \partial z_2 \ldots \partial z_n}$$

belongs to $H^1(U^n)$.

Proof. By differentiation,

$$\frac{\partial^n \left[H(z_1, \ldots, z_n) \prod_{i=1}^{n} z_i \right]}{\partial z_1 \ldots \partial z_n} = \sum_{k_1=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \tag{1.5}$$

$$(k_1 + 1) \ldots (k_n + 1) h(k_1, \ldots, k_n) z_1^{k_1} \ldots z_n^{k_n}.$$

Denote $g(k_1, \ldots, k_n) \triangleq (k_1 + 1) \ldots (k_n + 1) h(k_1, \ldots, k_n)$.
The Hardy–Littlewood inequality generalized to the n–D case yields

$$\sum_{k_1=0}^{\infty} \ldots \sum_{k_n=0}^{\infty} \frac{|g(k_1, \ldots, k_n)|}{(k_1 + 1) \ldots (k_n + 1)} \le \pi^n \|G\|_1. \tag{1.6}$$

From (1.5) and (1.6), the proof of the theorem is easily completed.

The following definition must be given before stating the next result of Dautov, which is useful to check whether a function belongs to $H^1(U^n)$.

DEFINITION 1.1. If $F(z_1, \ldots, z_n)$ is any function in U^n, we define $\hat{F}(z_1, \ldots, z_n)$ by (note that $\mathbf{z} = (z_1, \ldots, z_n)$),

$$\hat{F}(\mathbf{z}) = \lim_{r \to 1} F(r\mathbf{z})$$

at every $\mathbf{z} \in T^n$ at which this radial limit exists.

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