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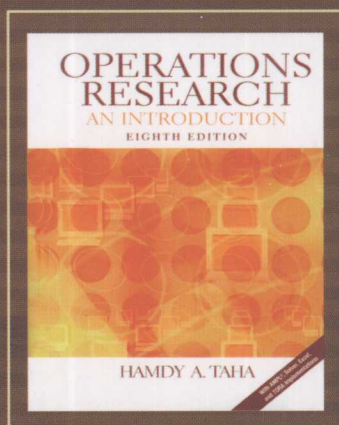
Operations Research
An Introduction

运筹学导论

高级篇

(英文版 · 第8版)

[美] Hamdy A. Taha 著



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内 容 提 要

本书是运筹学方面的经典著作之一, 为全球众多高校采用. 高级篇共 12 章, 内容包括高级线性规划、概率论基础复习、概率库存模型、模拟模型、马尔可夫链、经典最优化理论、非线性规划算法、网络和线性规划算法进阶、预测模型、概率动态规划、马尔可夫决策过程、案例分析等, 并附有统计表、部分习题的解答、向量和矩阵复习及案例研究.

本书可供经管类专业和数学专业研究生以及 MBA 作为教材或者参考书, 也可供相关研究人员参考.

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前 言

本书第8版对内容作了很多的修订，在教材的编排上突出反映运筹学中的应用问题和计算方法。

- 第2章通过城市规划、货币套利交易、投资、生产计划、混合配比、排序以及下料等实际问题的应用，主要介绍了线性规划的建模。新增加的习题也涉及从水质管理和交通控制到军事领域等多个运筹问题。

- 第3章以一种简单和直接的方式介绍了一般性的线性规划灵敏度分析，包括对偶价格和简约费用 (reduced cost)，作为单纯形表计算部分的直接扩充。

- 本版的第4章主要讲述基于对偶性进行线性规划最优化后的分析。

- 针对旅行商问题 (Traveling Salesperson Problem)，介绍了一个基于Excel的组合式最近邻点反向启发式算法。

- 新增的第17章扩充了马尔可夫链的处理方法。

- 在全新的第24章里，详细介绍了15个实际应用案例。对这些案例的分析通常涉及多种OR技术 (例如启发式算法和线性规划，或者线性整数规划和排队论)，用来进行建模、数据采集以及问题的求解计算等。这些应用问题在相关的各章都有引用，使读者能够充分了解实际生活中如何运用运筹学技术。

- 新的附录E收录了按照章节排列的约50个小型实用问题的例子。

- 本书还包含了1000多个节后习题。

- 每章开始都有本章导读，以帮助读者了解教材内容和有效利用附带的软件程序。

- 把教材与软件相结合可以让读者对需要深入介绍的概念进行实际检验：

1. 全书都用到了Excel程序，包括动态规划、旅行商问题、库存问题、层次分析法、贝叶斯概率、“电子化”统计表、排队问题、模拟、马尔可夫链和非线性规划等。一些程序中的交互式用户输入功能有助于对相应方法的更好理解。

2. 对Excel Solver程序的使用扩展到了全书，特别用在线性规划、网络规划、整数规划和非线性规划问题。

3. AMPL[®]是一种强大的商业化建模语言，本书将AMPL结合在大量的例题中，这些例子涉及线性、网络、整数和非线性规划问题。附录A给出了AMPL的语句规则以及本书例题中所引用的语言素材。

4. 本书中，TORA仍然充当教学软件的重要角色。

- 所有与计算机相关的材料都相对独立，有的作为单独的章节，有的按照标题 *AMPL/Excel/Solver/TORA* 程序作为一小节，以尽量不影响本书的主要内容的介绍。

为了限制本书的页数，我们把一些小节、一部分整章以及两个附录都放在了附

带的CD里，作者把运筹学导论课程中不太经常用到的内容截选下来，也放在CD里¹。

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CHAPTER 13

Advanced Linear Programming

Chapter Guide. This chapter presents the mathematical foundation of linear programming and duality theory. The presentation allows the development of a number of computationally efficient algorithms, including the revised simplex method, bounded variables, and parametric programming. Chapter 20 on the CD presents two additional algorithms that deal with large-scale LPs: decomposition and the Karmarkar interior-point algorithm.

The material in this chapter relies heavily on the use of matrix algebra. Appendix D on the CD provides a review of matrices.

The three topics that should receive special attention in this chapter are the revised simplex method, the bounded-variables algorithm, and parametric programming. The use of matrix manipulations in the revised simplex method allows a better control over machine roundoff error, an ever-present problem in the row operations method of Chapter 3. The bounded variables algorithm is used prominently with the integer programming branch-and-bound algorithm (Chapter 8). Parametric programming adds a dynamic dimension to the LP model that allows the determination of the changes in the optimum solution resulting from making continuous changes in the parameters of the model.

The task of understanding the details of the revised simplex method, bounded variables, decomposition, and parametric programming is improved by summarizing the results of matrix manipulations in the easy-to-read simplex tableau format of Chapter 3. Although matrix manipulations make the algorithms appear different, the theory is exactly the same as in Chapter 3.

This chapter includes 1 real-life application, 8 solved examples, 58 end-of-section problems, and 4 end-of-chapter comprehensive problems. The comprehensive problems are in Appendix E on the CD. The AMPL/Excel/Solver/TORA programs are in folder ch13Files.

Real-Life Application—Optimal Ship Routing and Personnel Assignment for Naval Recruitment in Thailand

Thailand Navy recruits are drafted four times a year. A draftee reports to one of 34 local centers and is then transported by bus to one of four navy branch bases. From there, recruits are transported to the main naval base by ship. The docking facilities at the branch bases may restrict the type of ship that can visit each base. Branch bases have limited capacities but, as a whole, the four bases have sufficient capacity to accommodate all the draftees. During the summer of 1983, a total of 2929 draftees were transported from the drafting centers to the four branch bases and eventually to the main base. The problem deals with determining the optimal schedule for transporting the draftees, first from the drafting centers to the branch bases and then from the branch bases to the main base. The study uses a combination of linear and integer programming. The details are given in Case 5, Chapter 24 on the CD.

13.1 SIMPLEX METHOD FUNDAMENTALS

In linear programming, the feasible solution space is said to form a **convex set** if the line segment joining any two *distinct* feasible points also falls in the set. An **extreme point** of the convex set is a feasible point that cannot lie on a line segment joining any two *distinct* feasible points in the set. Actually, extreme points are the same as corner point, the more apt name used in Chapters 2, 3, and 4.

Figure 13.1 illustrates two sets. Set (a), which is typical of the solution space of a linear program, is convex (with six extreme points), whereas set (b) is nonconvex.

In the graphical LP solution given in Section 2.3, we demonstrated that the optimum solution can always be associated with a feasible extreme (corner) point of the solution space. This result makes sense intuitively, because in the LP solution space every feasible point can be determined as a function of its feasible extreme points. For example, in convex set (a) of Figure 13.1, a feasible point \mathbf{X} can be expressed as a **convex combination** of its extreme points $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_5,$ and \mathbf{X}_6 using

$$\mathbf{X} = \alpha_1\mathbf{X}_1 + \alpha_2\mathbf{X}_2 + \alpha_3\mathbf{X}_3 + \alpha_4\mathbf{X}_4 + \alpha_5\mathbf{X}_5 + \alpha_6\mathbf{X}_6$$

where

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 = 1$$

$$\alpha_i \geq 0, i = 1, 2, \dots, 6$$

This observation shows that extreme points provide all that is needed to define the solution space completely.

FIGURE 13.1
Examples of a convex and a nonconvex set

