

**INTERNATIONAL TABLES
FOR
X-RAY CRYSTALLOGRAPHY
VOLUME II
MATHEMATICAL TABLES**

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Volume II
MATHEMATICAL TABLES

Edited by
JOHN S. KASPER and KATHLEEN LONSDALE (General Editor)

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Volume II. ERRATA

- p. 46. Last sentence: for *the last two cases* read *cases (c) and (d)*.
- p. 186. The diagram is inverted. Correction needed in the legend: for *bottom* read *top*.

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