

REVISED EDITION | CALAHAN

# network Design

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# COMPUTER-AIDED NETWORK DESIGN

Revised Edition

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# COMPUTER-AIDED NETWORK DESIGN

**to Martha  
for her wonderful patience**

# Preface

The spectrum of interests represented in the field termed "Computer-Aided Network Design" is characterized by the following two rather extreme viewpoints:

- (1) the design viewpoint, where the motivation is to achieve a prescribed circuit function and computer assistance is sought only when intuitive design methods fail to yield an acceptable design;
- (2) the algorithmic viewpoint, where equation formulation and numerical solution methods are studied for their potential application to broad classes of simulation and design problems.

With the power and user-orientation of problem-oriented languages now being developed for circuit analysis, most design needs can be accommodated without serious concern for the computing algorithms involved. Indeed, if we consider that an increasing share of sophisticated circuit design is being performed by integrated circuit manufacturers, the need for formal instruction in circuit analysis and design algorithms to satisfy the design viewpoint (1) above is open to serious question.

As a source of model problems for learning digital-computer-based problem solving, however, circuit design offers the following advantages.

- (1) Elementary circuit analysis - on which most of this text is based - is a familiar or easily learned discipline, not requiring special

mathematics which can mask principles of equation formulation and numerical solution.

- (2) Most important numerical methods can be related to practical circuit design problems. These include solution of simultaneous linear and nonlinear algebraic equations, solution of ordinary differential equations, function minimization, and approximation techniques. The major exception is the numerical solution of partial differential equations, which is at present a strongly problem-dependent process.
- (3) The electrical network is an excellent analog of many physical, biological, and natural (ecological) systems. The "through" and "across" variables - current and voltage - appear in easily identifiable forms in structures, continuum mechanics, resource and transportation distribution systems, and the communication system of the human body, to name a few. The equation formulation procedures for such systems is remarkably similar.
- (4) The study of the digital simulation and automatic design of large static and dynamic systems, such as networks, is a natural complement to the systems theoretic design of small, approximate models of these same systems. The dimensionality and ill-conditioning problems which accompany analysis of larger systems are foreign to these traditional disciplines.
- (5) Most of the solution methods used in network simulation are based on general numerical analysis methods. However, efficient implementation of these methods for problem solving often requires data handling techniques which are not usually taught in numerical analysis courses. Also, formal numerical analysis procedures can often be explained in terms of network models, thus exploiting the insight and natural interests of the reader.

From an instructional viewpoint, a major attraction of computer-aided network design is the ease with which the student can combine elementary formulation and solution methods to produce his own analysis and design programs. These programs can be made quite user-oriented, similar to the popular problem-oriented languages which the student may have used in circuit design courses.

To exploit this motivational opportunity, the material of this text is divided into two parts:

- (1) Part I, Chapters 2-7, which
  - (a) introduce certain basic numerical methods such as solution of simultaneous linear equations;
  - (b) interpret other numerical processes - such as linearization and discretization - in terms of network models;
  - (c) examine certain forms of ill-conditioning and their solution;
  - (d) present complete computer programs which solve specific model problems;
  - (e) present major portions of programs intended to solve general classes of network problems;
  - (f) introduce the student to the rudiments of automatic design.
- (2) Part II, Chapters 8-12, which
  - (a) generalize and extend key algorithms of Part I using relatively more sophisticated mathematical analysis,
  - (b) examine techniques of data handling which considerably enhance solution speed.

This division may be considered a natural one for undergraduate/graduate instruction. Alternatively, it provides an opportunity for the undergraduate interested in an application other than electronic circuit design to advance selectively in one of the topics of Part II.



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The author is indebted jointly to the Air Force Office of Scientific Research (Applied Mathematics Division) for their continuing research support in the subject area of this text, and to the IBM Thomas J. Watson Research Center (Yorktown Heights, New York) for the opportunity to work with the staff of their Mathematical Sciences group. In particular, Drs. Robert Brayton, Fred Gustavson, Gary Hachtel, and Ralph Willoughby have been most challenging and cooperative in developing concepts and algorithms represented in this text. Locally, the assistance of Mr. Thomas Grapes has been of continuing value in the area of sparse matrix methods. Finally, the most important man-machine interface was provided by the secretaries of the Systems Engineering Laboratory. Many thanks to them.

Donald A. Calahan

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# Network Design by Computer

## 1.1 INTRODUCTION

As the size and sophistication of physical man-made systems has increased, a corresponding need has developed for thorough study and optimal design of these systems prior to construction. The design of electrical circuits is one of the best examples of this escalation, where 20 - 50 component discrete circuits have been replaced by 300 - 3000 component integrated circuits. Further, these circuits are usually produced in large quantities, so that significant effort can be expended in design of a prototype circuit.

A model for a modern computer-aided design (CAD) process is shown in Figure 1.1. Any block or partition of this flow chart can represent an investment in time from minutes to years, depending on the application. Network design fits this model well; moreover, for certain network design problems the model-design-build (B-A-B) loop can be compactly represented on a table-top. For example, the power of the computer can be brought into the laboratory, as indicated by the juxtaposition of graphic terminal and oscilloscope waveform displays in Figure 1.2. Any discrepancy between measured, computed, and ideal responses can be immediately detected and either the computer model or the physical circuit adjusted accordingly.



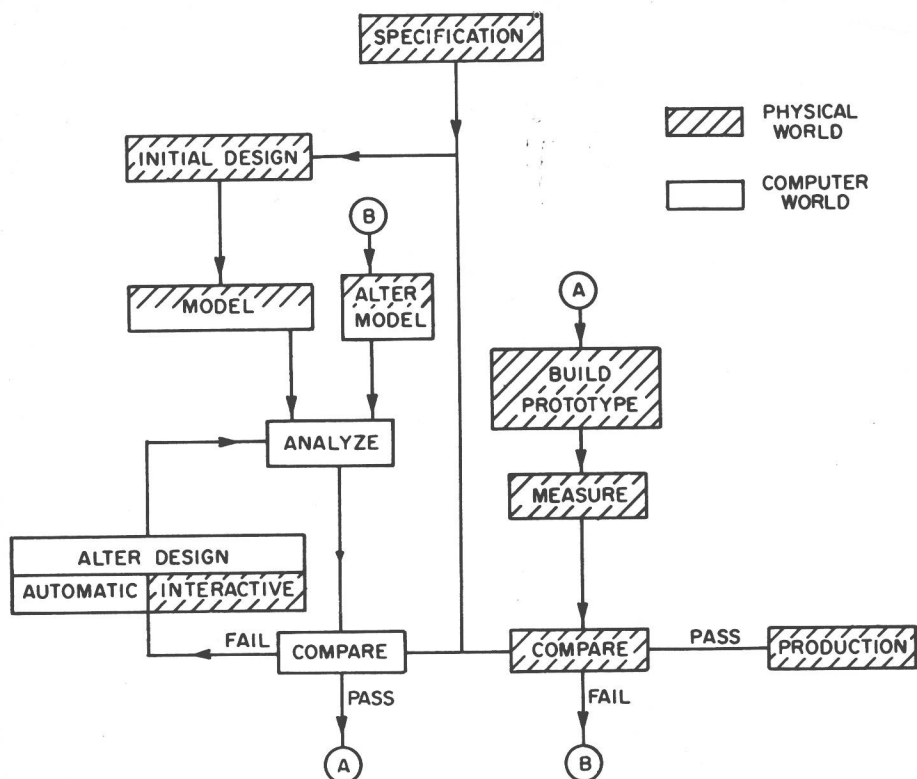


FIG. 1.1 Computer-Aided Design Model

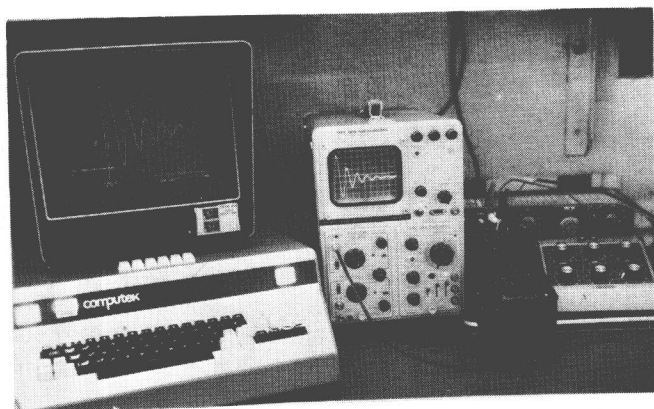


FIG. 1.2 Graphics in the Laboratory