Optimal Control: Theory, Algorithms, & Applications

Optimal Control

Theory, Algorithms, and Applications

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KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

Library of Congress Cataloging-in-Publication Data

ISBN 0-7923-5067-7

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Sold and distributed in North, Central and South America
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

Printed in the Netherlands

Preface

February 27 – March 1, 1997, the conference Optimal Control: Theory, Algorithms, and Applications took place at the University of Florida, hosted by the Center for Applied Optimization. The conference brought together researchers from universities, industry, and government laboratories in the United States, Germany, Italy, France, Canada, and Sweden. There were forty-five invited talks, including seven talks by students. The conference was sponsored by the National Science Foundation and endorsed by the SIAM Activity Group on Control and Systems Theory, the Mathematical Programming Society, the International Federation for Information Processing (IFIP), and the International Association for Mathematics and Computers in Simulation (IMACS).

Since its inception in the 1940s and 1950s, Optimal Control has been closely connected to industrial applications, starting with aerospace. The program for the Gainesville conference, which reflected the rich cross-disciplinary flavor of the field, included aerospace applications as well as both novel and emerging applications to superconductors, diffractive optics, nonlinear optics, structural analysis, bioreactors, corrosion detection, acoustic flow, process design in chemical engineering, hydroelectric power plants, sterilization of canned foods, robotics, and thermoelastic plates and shells.

The three days of the conference were organized around the three conference themes, theory, algorithms, and applications. This book is a collection of the papers presented at the Gainesville conference. We would like to take this opportunity to thank the sponsors and participants of the conference, the authors, the referees, and the publisher for making this volume possible. In addition, we thank Nancy Moore for her patience and persistence with the final typesetting.

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Uniform Decays in Nonlinear Thermoelastic Systems *

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Received May 1, 1997; Accepted in revised form December 29, 1997

Abstract

The uniform stability of a nonlinear thermoelastic plate model is investigated, where the abstract nonlinearity here satisfies assumptions which allow the specification of the von Kármán nonlinearity, among other physically relevant examples. Linear analogs of this work were considered in [1] and [2]. Even in the absence of inserted dissipative feedbacks on the boundary, this system is shown to be stable with exponential decay rates which are uniform with respect to the "finite energy" of the given initial data (uniform stability of a linear thermoelastic plate with added boundary dissipation was shown in [8], as was that of the analytic case in [14]). The proof of this result involves a multiplier method, but with the particular multiplier invoked being of a rather nonstandard (operator theoretic) nature. In addition, the

*The research of G. Avalos is partially supported by the NSF Grant DMS-9710981. The research of I. Lasiecka is partially supported by the NSF Grant DMS-9504822 and by the Army Research Office Grant DAAH04-96-1-0059.

"free" boundary conditions in place for the plate component give rise to higher order terms which pollute the decay estimates, and to deal with these a new result for boundary traces of the wave equation must be employed.

1 Introduction

1.1 Statement of the Problem

Let Ω be a bounded open subset of $\mathbb{R}^2$ with sufficiently smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_0 and Γ_1 both nonempty and $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \varnothing$. We consider here the following thermoelastic system (the linear model was introduced and studied in the monograph [8] of J. Lagnese):

$$\begin{cases} \begin{cases} \omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta + F(\omega) = 0 \\ \beta\theta_t - \eta\Delta\theta + \sigma\theta - \alpha\Delta\omega_t = 0 \end{cases} & \text{on } (0,\infty)\times\Omega; \\ \omega = \dfrac{\partial\omega}{\partial\nu} = 0 & \text{on } (0,\infty)\times\Gamma_0; \\ \begin{cases} \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = 0 \\ \dfrac{\partial\Delta\omega}{\partial\nu} + (1-\mu)\,\dfrac{\partial B_2\omega}{\partial\nu} - \gamma\,\dfrac{\partial\omega_{tt}}{\partial\nu} + \alpha\,\dfrac{\partial\theta}{\partial\nu} = 0 \end{cases} & \text{on } (0,\infty)\times\Gamma_1; \\ \dfrac{\partial\theta}{\partial\nu} + \lambda\theta = 0 & \text{on } (0,\infty)\times\Gamma,\ \lambda \ge 0; \\ \omega(t=0) = \omega_0,\ \omega_t(t=0) = \omega_1,\ \theta(t=0) = \theta_0 & \text{on } \Omega. \end{cases} \tag{1}$$

Here, the parameters α, β and η are strictly positive constants; positive constant γ is proportional to the thickness of the plate and assumed to be small with $0 < \gamma \le C_\gamma$; the constant $\sigma \ge 0$ and the boundary operators B_1 and B_2 are given by

$$\begin{aligned} B_1\omega &\equiv 2\nu_1\nu_2\frac{\partial^2\omega}{\partial x\partial y} - \nu_1^2\frac{\partial^2\omega}{\partial y^2} - \nu_2^2\frac{\partial^2\omega}{\partial x^2}; \\ B_2\omega &\equiv \left(\nu_1^2 - \nu_2^2\right)\frac{\partial^2\omega}{\partial x\partial y} + \nu_1\nu_2\left(\frac{\partial^2\omega}{\partial y^2} - \frac{\partial^2\omega}{\partial x^2}\right). \end{aligned} \tag{2}$$

The constant μ is the familiar Poisson's ratio $\in (0, \frac{1}{2})$. In addition, we impose that the nonlinearity $F(\cdot)$ which is present in the plate component of the coupled system satisfy the following conditions:

(Fi) The mapping $F : H^2(\Omega) \to H^{-1}_{\Gamma_0}(\Omega)$ is locally Lipschitz continuous, where $H^{-1}_{\Gamma_0}(\Omega)$ is denoted to be the topological dual of $H^1_{\Gamma_0}(\Omega) \equiv \{\phi \in H^1(\Omega) : \phi|_{\Gamma_0} = 0\}$. That is to say, for every ω_1, $\omega_2 \in H^2(\Omega)$ there exists a constant $C\left(\|\omega_1\|_{H^2(\Omega)}, \|\omega_2\|_{H^2(\Omega)}\right)$ such that

$$\begin{aligned} &\|F(\omega_1) - F(\omega_2)\|_{H^{-1}_{\Gamma_0}(\Omega)} \\ &\leq C\left(\|\omega_1\|_{H^2(\Omega)}, \|\omega_2\|_{H^2(\Omega)}\right) \|\omega_1 - \omega_2\|_{H^2(\Omega)} \end{aligned} \quad (3)$$

(where here and in what follows below $C(\cdot)$ denotes a function which is bounded for bounded values of its argument).

(Fii) The mapping F further satisfies the relation

$$\int_\Omega F(\omega)\omega_t d\Omega = \frac{d}{dt} E_F(\omega(t)), \quad (4)$$

where $E_F : H^2(\Omega) \to \mathbb{R}$ is a functional which obeys the inequality

$$0 \leq E_F(\omega) \leq C\left(\|\omega\|_{H^2(\Omega)}\right). \quad (5)$$

(Fiii) The mapping $F : H^2(\Omega) \to H^{-1}_{\Gamma_0}(\Omega)$ satisfies the following norm bound for every $\omega \in H^2(\Omega)$ and some $\epsilon > 0$:

$$\|F(\omega)\|_{H^{-1}_{\Gamma_0}(\Omega)} \leq C\left(\|\omega\|_{H^2(\Omega)}\right) \|\omega\|_{H^{2-\epsilon}(\Omega)}. \quad (6)$$

The given model mathematically describes a Kirchoff plate of which the displacement is represented by the function ω; the plate is subjected to a thermal damping which quantified by the function θ. It has been recently been shown in [2] that solutions to the linear version of (1) ($F \equiv 0$) decay uniformly. We will be concerned here with obtaining an analogous stability result for solutions $[\omega, \omega_t, \theta]$ to (1) with the nonlinearity F in place. It can be shown directly (as it is so done in [3]) that examples of nonlinearities which meet the abstract assumptions (Fi)–(Fiii) above include the following: