LYNN H. LOOMIS

CALCULUS

THIRD EDITION

CALCULUS

LYNN H. LOOMIS
Harvard University

THIRD EDITION

ADDISON-WESLEY PUBLISHING COMPANY
Reading, Massachusetts ▪ Menlo Park, California
London ▪ Amsterdam ▪ Don Mills, Ontario ▪ Sydney

SPONSORING EDITOR: *Stephen H. Quigley*
PRODUCTION EDITOR: *Marion E. Howe*
DESIGNER: *Vanessa Piñeiro*
ILLUSTRATOR: *B. J. and F. W. Taylor*
COVER DESIGN: *Richard W. Hannus*

Library of Congress Cataloging in Publication Data
Loomis, Lynn H., 1915–
Calculus.

Includes index.
1. Calculus. I. Title.
QA303.L872 1982 515 81-14937
ISBN 0-201-05045-5 AACR2

ISBN 0-201-05045-5
ABCDEFGHIJ-DO-898765432

PREFACE

This book is intended as an intuitive, but mathematically sound, treatment of the standard calculus sequence. Its approach to the basic concepts—limit, continuity, derivative, integral—is mainly geometrical. Variables and "y is a function of x" are used extensively but carefully; Leibniz notation and function notation receive about equal time.

There is some emphasis on approximation and computation (though this material can be considered to be optional). For the most part this appears as estimation, directed to the question "How good an answer do I have?" However, Appendix 4 goes on to the natural follow-up, "What must I do to get the accuracy I want?," and in this computational context there is an introduction to the ϵ, δ theory of limits.

Most of the traditional topics from analytic geometry are covered in Chapter 1 and Appendix 3. Chapter 1 treats lines, circles, translation of axes, and completing the square to simplify quadratic equations, while Appendix 3 develops the conic sections from their standard locus definitions. Some related material involving polar coordinates and parametric equations will be found in Chapter 10.

The changes for this edition are extensive.

The development has been reorganized, for greater smoothness and flexibility. It will fit courses given in three terms and also courses given in four quarters. See below.

There is much new writing. The discussions of limits, continuity, and the definite integral are wholly new, as are sections on l'Hôpital's rule, improper integrals, certain integral applications, and the natural logarithm function. Many other sections have been almost completely rewritten.

The exposition has been simplified. Duplicated discussions have been eliminated. Some of the more complicated topics have been postponed (e.g., centers of mass, curvature) and several difficult arguments have been dropped. The integral is discussed entirely in terms of sequential convergence.

The problem collection has been reorganized and strengthened. The symbol ▦ has been introduced to suggest the use of a hand calculator.

The organization now provides convenient break points for courses given either in three terms or in four quarters. At six chapters per term, the book divides as follows:

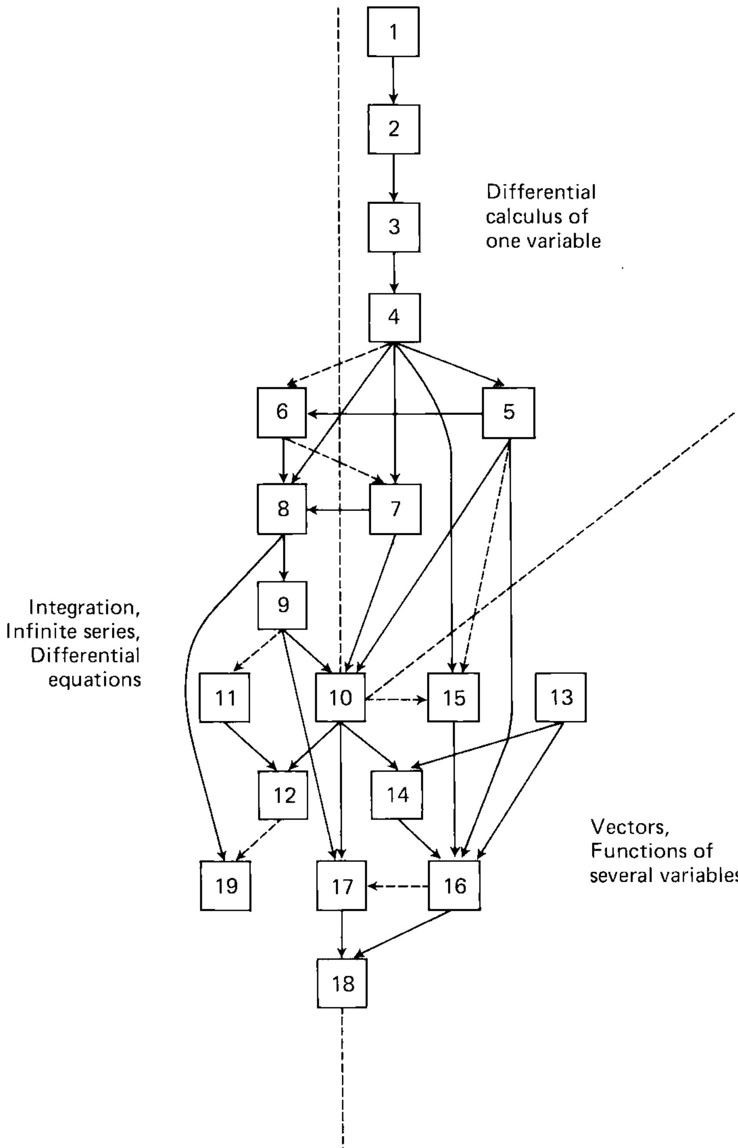
Term I (Chapters 1–6).† Differential calculus; introduction to the integral.
II (7–12). Integral calculus; infinite series.
III (13–17; 18 and/or 19). Vectors; functions of several variables.

For a course in four quarters (at four or five chapters per quarter) the breaks occur naturally at subject changes:

Quarter I (Chapters 1–5). Differential calculus.
II (6–10). Integral calculus.
III (10–14). Infinite series; vectors and vector functions of one variable.
IV (15–17; 18 and/or 19). Functions of several variables; (differential equations).

Chapter 10 is mentioned twice: it can go into either quarter or it can be split between them.

† Strong classes would presumably cover Chapter 1 only lightly. Classes that are less well prepared could omit some or all of the sections on estimation and theory (5.7–5.10, 6.9).


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Differential calculus of one variable
Integration, Infinite series, Differential equations
Vectors, Functions of several variables

Integration can be reached in the first quarter by putting off the trigonometric functions and some applications of the derivative, as follows:

Quarter I (Chapters 1–3, 4.1, 4.2, 5.1–5.5, 6). Differential calculus of algebraic functions; introduction to the integral.
II (4.3–4.5, 5.6 (5.7–5.10 optional), 7–10). Transcendental functions; further applications of the derivative; integral calculus.
III and IV as before.

For other possible re-alignments see the flow chart on page v. It shows the major dependencies between chapters, and any reordering of material that is consistent with the chart should involve only minor problems of accommodation. A dotted arrow indicates a single point of contact.

I would like to express my sincere appreciation and gratitude to my colleagues at Addison-Wesley for their constant help and encouragement.

Concord, Massachusetts L. H. L.
January 1982

INTRODUCTION

NEWTON, LEIBNIZ, AND THE CALCULUS

The invention of calculus is attributed to two geniuses of the 17th century, Isaac Newton in England, and, independently, Gottfried Leibniz in Germany. Earlier mathematicians had uncovered bits and pieces of the subject. Newton and Leibniz discovered its *pattern*. They thereby created an algorithmic discipline of enormous power, applicable to all sorts of fundamental questions about the nature of the world.

Although the new calculus obviously worked, Newton and Leibniz did not have a clear idea of *why* it worked. They tried to explain its successes by geometric reasoning, since at that time all mathematical phenomena were viewed in terms of geometry, but their explanations were unsatisfactory. In fact, the logical foundations of calculus remained a mystery for another century and a half. Some fragmentary progress occurred, and a new point of view gradually emerged, based on numbers, variables, and functional relationships between variables. Then, around 1820, the French mathematician, Augustin Cauchy, settled the matter by showing that calculus rests on the properties of the limit operation. This was still not what we today call rigor, and it took another fifty years of deeper probing to reach the bedrock of ϵ, δ reasoning and the completeness of the real-number system.

The chronological development of calculus was thus marked at several points by leaps in precision and sophistication. Now, three hundred years after Newton and Leibniz, we can start our study of the subject at practically any level we wish. Since there seems to be little point in repeating the confusions of the first one hundred fifty years, we shall approach calculus at about the level of Cauchy, which is still very intuitive. We can then increase our precision in a natural manner as the subject unfolds.

contents

CHAPTER 7 THE NATURAL LOGARITHM AND EXPONENTIAL FUNCTIONS; INVERSE FUNCTIONS

CHAPTER 8 TECHNIQUE OF INTEGRATION

CHAPTER 9 THE DEFINITE INTEGRAL: FURTHER DEVELOPMENT AND APPLICATIONS

CHAPTER 17 DOUBLE AND TRIPLE INTEGRALS

CHAPTER 18 GREEN'S THEOREM

CHAPTER 19 FIRST AND SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

CHAPTER 1

GRAPHS AND FUNCTIONS (PRECALCULUS REVIEW)

Calculus is about functions, and it is important before starting calculus to have a reasonably good understanding of what a function is and what its graph can be like. This preliminary chapter reviews the necessary background material about graphs and functions.

1 COORDINATES AND GRAPHS

Figure 1 illustrates a coordinate system on a line l.

First we have chosen on l an origin point O and a unit point E distinct from O. The origin O divides l into two half-lines, or *rays*. The half-line containing E is called *positive*, and the other one *negative*. The segment OE is taken as the unit of length. Then each point P on l is assigned a number x, called the coordinate of P, as follows:

> If P is on the positive side of O, then x is the length of the segment OP (in terms of the unit OE).

Figure 1