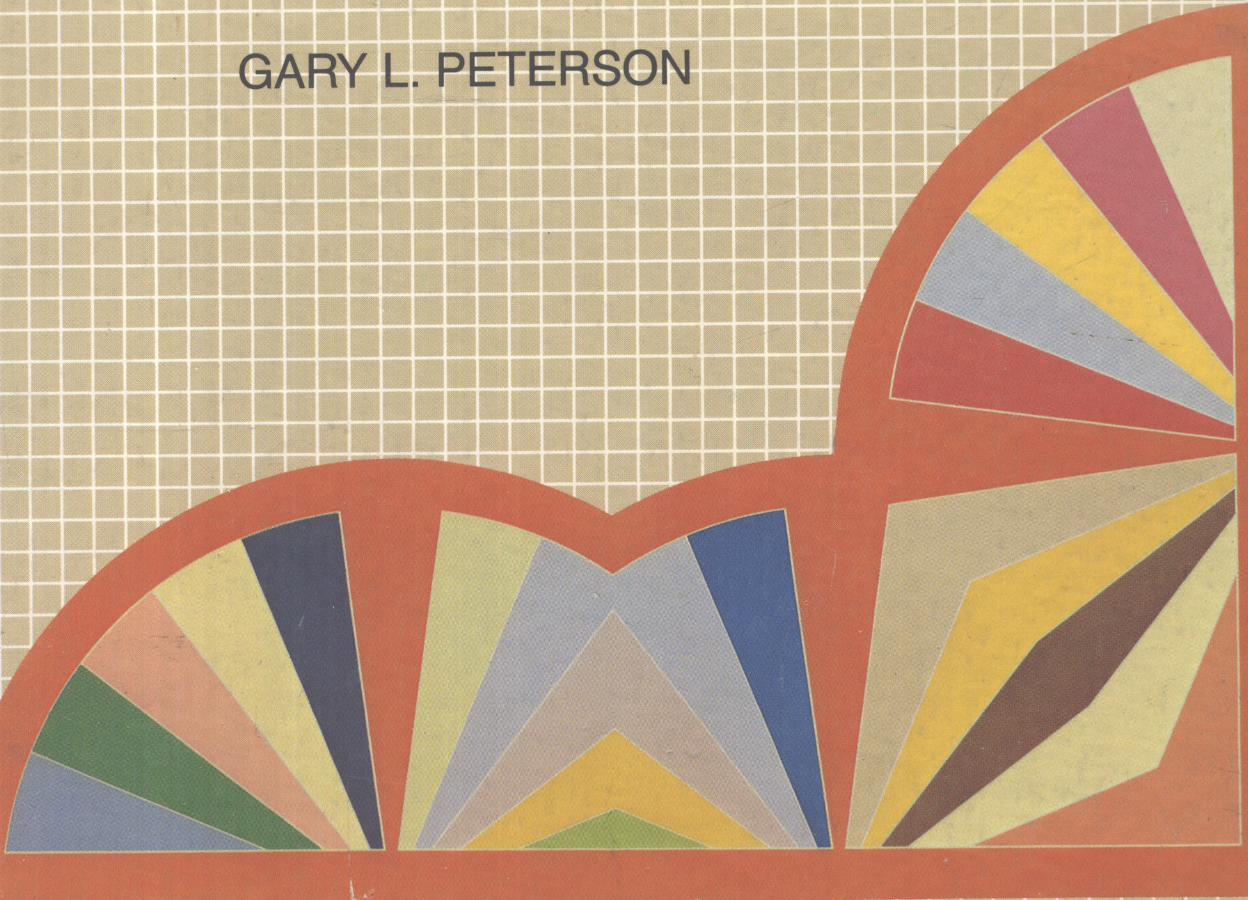
Algebra and Trigonometry

GARY L. PETERSON



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Preface

This text is designed to give a sound and comprehensive treatment of algebra and trigonometry. In writing this book, I have kept the student in mind and have striven to present the material in a succinct, informal manner that moves smoothly from one topic to the next. The text includes many detailed examples and substantial exercise sets with problems ranging from the routine to the complex.

Some noteworthy features of *Algebra and Trigonometry* are the following:

Algebra Review I have included a thorough, but not overly drawn out, review of basic algebra. Much of Chapter 1 deals with fundamental manipulative skills of algebra, and a good portion of Chapter 2 is devoted to reviewing elementary skills for solving equations and inequalities. Review of other basic topics is integrated into the text whenever necessary.

Complex Numbers Complex numbers are introduced in Chapter 1 so that the student is completely prepared for the study of quadratic equations in Chapter 2.

Graphing I have placed a strong emphasis on techniques for graphing functions and equations. One noteworthy feature is the inclusion of a section on graphing aids (Section 3-5) that covers translations, stretching and shrinking, reflections, and symmetries. These notions are applied often in the remainder of the text. I have also included a separate section dealing with functions involving absolute values and piecing (Section 4-3) and a brief section on parametric equations (Section 4-4).

Applications In recent years a spate of college algebra and trigonometry textbooks has stressed applications. All too often, however, the authors of these texts went too far. Having taught from one of these books myself, I am highly conscious of the fact that, for the benefit of the student, the majority of the applications must be easily accessible and understandable. Any additional applications should be strategically placed so that they do not interrupt the presentation of the material that most instructors have time to cover. Thus, I have included a large number of applications throughout the text, beginning with word problems in Section 2-4, but I have also been careful to place those extra applications so that they may be easily omitted.

Calculators I have tried in this book to be as flexible as possible regarding the use of calculators. For the instructor who wishes to emphasize them, discussions and problems involving the use of calculators are integrated into the text beginning in Section 1-9. At the same time, I have also included traditional problems that use common logarithm and trigonometric tables for the instructor who wishes to minimize the use of calculators or to discuss both kinds of approaches.

Trigonometry There is a great deal of division today about whether the trigonometric functions should be introduced using right triangles or as circular functions. My approach is to introduce them as circular functions in Sections 8-2 and 8-3 and then move quickly to the right triangle approach in Section 8-4. Thus the circular point of view is emphasized in such a way that the student is unlikely to forget it, but, at the same time, the right triangle approach is not unduly delayed.

Find-the-Error Problems To help the student learn to avoid common pitfalls, in many of the exercise sets I have included problems containing solutions illustrating common errors. The student is asked to find the error and then work the problem correctly (for example, see Exercises 61 to 66 on pages 38 and 39).

Chapter Reviews In addition to the list of key terms and review exercises included in most texts at the end of each chapter, I have also included a list of review questions to help the student gain an overview of the material in the chapter.

Answers to the odd-numbered exercises and all the review exercises appear at the back of this book. Frank Gunnip of Oakland Community College has prepared a solutions manual containing worked solutions to the even-numbered problems. This manual is available to instructors and, with departmental consent, may also be made available to students.

There are many people whom I wish to thank for their help in the preparation of this text. The staff at Wadsworth—especially Pete Fairchild, who guided this project through much of its initial development, and Jim Harrison, who saw it to completion—have been very helpful in bringing it to fruition. Michael Rideout, a most capable student, checked examples and answers to exercises. The following reviewers read all or parts of various drafts of the manuscript and offered many useful suggestions: Richard Anastasio; John Brevit, Western Kentucky University; Judy Clark, California State University at Chico; Hope Florence, College of Charleston; D.W. Hall, Michigan State University; Ray Hamlett, East Central University; William Heaslip, Bucks County Community College; Lotus Hershberger, Illinois State University; Herbert Kasube, Bradley University; Gail Koplin, Ocean County College; Jimmie McKim, University of Central Arkansas; Curtis McKnight, University of

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GARY L. PETERSON

Contents

1 Fundamentals of Algebra

□ This chapter deals primarily with basic concepts and skills of algebra. We begin in the first section with some preliminary notions and terminology. The next six sections are concerned with polynomials, rational expressions, exponents, and radicals. In Section 1-8 we shall discuss the complex numbers, and the chapter concludes with some comments about calculators. Much of this chapter reviews concepts and skills that you have encountered in previous algebra courses. Make sure that you master them, however, because they will form the foundation for the remainder of this text as well as future courses involving the use of mathematics.

1-1 Preliminaries

We begin this section by mentioning what we mean by a **real number**. These are the decimal numbers. For instance,

$$1 \qquad 2.4 \qquad -7 \qquad -1\tfrac{1}{4} = -1.25 \qquad \frac{2}{3} = 0.666\ldots \qquad \sqrt{2} = 1.414\ldots$$

are examples of real numbers. In practice, we often refer to real numbers simply as "numbers." The nonzero numbers prefixed by a negative sign such as -7 or $-1\frac{1}{4}$ are called **negative numbers**. Those that are not, such as 1, 2.4, $\frac{2}{3}$, or $\sqrt{2}$, are called **positive numbers**. The number zero is considered to be neither positive nor negative.

Besides distinguishing numbers by their sign, we also separate them into categories. The **positive integers** or **natural numbers** are the numbers

$$1, 2, 3, 4, \ldots$$

The **integers** are the numbers

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

The **rational numbers** are the numbers that can be expressed as a fraction m/n, where m and n are integers and n is not zero. For example,

$$\frac{3}{4} \qquad -1\tfrac{1}{4} = -\frac{5}{4} \qquad 3 = \frac{3}{1} \qquad 0.333\ldots = \frac{1}{3}$$

are rational numbers. The **irrational numbers** are numbers that cannot be expressed as a fraction of an integer divided by a nonzero integer. Some examples of irrational numbers are

$$\sqrt{2} \qquad \sqrt[3]{3} \qquad \pi$$

Note that every positive integer is an integer and every integer is a rational number. The rational and irrational numbers form distinct categories of real numbers. Rational and irrational numbers can also be distinguished by their decimal forms. When rational numbers are expressed in decimal form, their digits either terminate or eventually form a continually repeating pattern. For example:

$$\frac{3}{4} = 0.75 \qquad \frac{1}{3} = 0.3333\ldots \qquad -\frac{137}{110} = -1.2454545\ldots$$

The digits of irrational numbers do not have a continual repeating pattern when expressed in decimal form. For instance, there is no repetition in the first eight digits of $\sqrt{2}$ and π:

$$\sqrt{2} = 1.4142135\ldots \quad \text{and} \quad \pi = 3.1415926\ldots$$

Moreover, including additional digits in either of these numbers will not introduce a continual repeating pattern.

On the real numbers, we have the arithmetic operations of *addition* (denoted by $+$) and *multiplication* (denoted by $\cdot$). That is, for any real numbers a and b there is a unique number $a + b$ called the *sum* of a and b and a unique number $a \cdot b$ (also written ab) called the *product* of a and b. Addition and multiplication of real numbers satisfy the following fundamental laws, where a, b, and c are arbitrary real numbers:

Commutative law of addition: $a + b = b + a$

Commutative law of multiplication: $ab = ba$

Associative law of addition: $(a + b) + c = a + (b + c)$

Associative law of multiplication: $(ab)c = a(bc)$

Distributive law: $a(b + c) = ab + ac$

Identity law of addition: $a + 0 = a$

Identity law of multiplication: $a \cdot 1 = a$

Inverse law of addition: There is a real number $-a$ so that $a + (-a) = 0$.

Inverse law of multiplication: If $a \neq 0$, there is a real number $1/a$ so that $a \cdot (1/a) = 1$.

Because the associative laws of addition and multiplication say that we may group together the numbers in any order without affecting the sum or product, it is unnecessary to do so. That is, a sum of three numbers a, b, and c can be written as $a + b + c$ and their product can be written as abc. For example, in the sum $2 + 3 + 4$, it does not matter whether we first add 2 and 3—that is, perform the addition as $(2 + 3) + 4$—or first add 3 and 4—that is, perform the addition as $2 + (3 + 4)$. Either way, we end up with 9. Similar statements hold for sums or products involving more than three numbers. For instance, it is unnecessary to indicate how we group together the numbers in pairs in the product $abcde$. Moreover, the commutative laws of addition and multiplication allow us to rearrange the numbers in a sum or product in any order we wish. For example:

$$a + b + c + d = a + c + b + d \quad \text{and} \quad abcde = edcba$$

The form of the distributive law that we have stated is sometimes more properly called the *left-hand distributive law*. In the following example, we shall use it along with the commutative law of multiplication to show that it holds when a is multiplied on the right.

Example 1. Show that $(b + c)a = ba + ca$.

Solution To help you follow the steps, the law justifying each step is given in parentheses:

$$\begin{aligned}(b + c)a &= a(b + c) && \text{(commutative law of multiplication)}\\ &= ab + ac && \text{(distributive law)}\\ &= ba + ca && \text{(commutative law of multiplication)}\end{aligned}$$ ■

The property that we verified in Example 1 is called the *right-hand distributive law*. Both the right-hand and left-hand distributive laws extend to situations where the sum involves more than two numbers. For example:

$$a(b + c + d + e) = ab + ac + ad + ae$$

The number $-a$ in the inverse law of addition is called the **negative*** of a and is the number with sign opposite to that of a. For instance, the negative of 5 is -5; the negative of 0 is $-0 = 0$; the negative of -2 is $-(-2) = 2$. There are some important properties that can be established about negatives of numbers. First, there is the double negative property:

$$\boxed{-(-a) = a}$$

for any number a. For multiplication, we have the following properties of signs:

$$\boxed{\begin{aligned}(-a)b = a(-b) &= -ab\\ (-a)(-b) &= ab\end{aligned}}$$

for any numbers a and b. As a consequence of the first property of signs for multiplication, note that the negative of a number can be obtained by multiplying it by -1:

* Also called the **opposite** or **additive inverse** of a.

$$(-1)a = -a$$

for any number a.

Subtraction of numbers can be described in terms of addition as follows:

$$a - b = a + (-b)$$

for any numbers a and b. That is, to subtract a number, we add its negative. By expressing subtraction in terms of addition, we can obtain some useful properties. First, we may show that the distributive law holds when addition is replaced by subtraction as follows:

$$\begin{aligned} a(b - c) &= a[b + (-c)] \\ &= ab + a(-c) \\ &= ab + (-ac) \\ &= ab - ac \end{aligned}$$

so that

$$a(b - c) = ab - ac$$

In general, any distributive law holds when some or all of the additions are replaced by subtractions. For instance:

$$(b - c + d)a = ba - ca + da$$

Another useful property deals with the negative of a difference. Since

$$\begin{aligned} -(a - b) &= (-1)(a - b) \\ &= (-1)[a + (-b)] \\ &= (-1)a + (-1)(-b) \\ &= -a + b \\ &= b - a \end{aligned}$$

we have

$$-(a - b) = b - a$$

In words, this property says that *the negative of a difference reverses the order of the difference.*

For a nonzero number a, the number $1/a$ in the inverse law of multiplication is called the **reciprocal** of a.* The reciprocal of a nonzero number plays a role in

* Also called the **multiplicative inverse** of a.

division which is similar to that of the negative of a number in subtraction. To be explicit, if a and b are numbers with $b \neq 0$, then a divided by b (indicated $a \div b$, $\frac{a}{b}$, or a/b) is

$$\boxed{\frac{a}{b} = a \cdot \frac{1}{b}} \qquad (b \neq 0)$$

We shall refer to a/b as the *quotient* of a by b. Note that division by zero is not allowed; that is,

$$\boxed{\frac{a}{0} \text{ is undefined}}$$

Whenever we write a quotient a/b, it will be understood that b is not zero.

We come now to some useful properties that can be established about quotients. First, they obey properties of signs similar to those for multiplication:

$$\boxed{\begin{gathered} \frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \\ \frac{-a}{-b} = \frac{a}{b} \end{gathered}}$$

Other important properties are

$$\boxed{\begin{gathered} \frac{ak}{bk} = \frac{a}{b} \\ \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \\ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \\ \frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \\ \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \end{gathered}}$$

Note that the second group of properties is used extensively when working with fractions. The first property is the rule we use for reducing fractions, the second and third for multiplying and dividing fractions, and the fourth and fifth for adding and subtracting fractions with common denominators. Keep in mind, however, that these properties apply not only to fractions but to *all* quotients.

A convenient way of representing the real numbers pictorially is by using a **number line** as in Figure 1-1. We form the line by choosing a point on it for the location of zero. This point is called the **origin**. We then choose a fixed length that we call a *unit*. Next a positive integer n is located n units to the right of the origin, and the negative integer $-n$ is located n units to the left of the origin. The points associated with rational numbers such as $2\frac{1}{2}$, 1.3, $-\frac{1}{3}$, $-\frac{13}{4}$ are located by subdividing the line segment between the appropriate consecutive integers as illustrated in Figure 1-1. We may approximate the location of the point corresponding to an irrational number by approximating the irrational number by a rational number. Of course, the better the rational approximation of the irrational number, the closer we get to the actual point representing the irrational number. The approximate location of π in Figure 1-1 is obtained by using the approximation 3.14 for π.

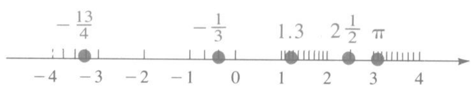

Figure 1-1

For two numbers a and b, we say that a is **greater than** b and write

$$a > b$$

if $a - b$ is a positive number. If $a - b$ is a negative number, we say that a is **less than** b and write

$$a < b$$

For example:

$4 > 3$	since $4 - 3 = 1$ is positive
$-2\frac{1}{3} > -3$	since $-2\frac{1}{3} - (-3) = \frac{2}{3}$ is positive
$-8 < -2$	since $-8 - (-2) = -6$ is negative
$-\sqrt{2} < 1$	since $-\sqrt{2} - 1$ is negative

Graphically, $a > b$ when a lies to the right of b on a number line as illustrated in Figure 1-2. Likewise, $a < b$ when a lies to the left of b on a number line. The symbols $>$ and $<$ are called **inequality symbols**. There are two other inequality symbols: $\geq$ and $\leq$. The symbol $\geq$ denotes **greater than or equal to**; that is, a statement $a \geq b$ is true when either $a > b$ or $a = b$. The symbol $\leq$ denotes **less than or equal to**; that is, $a \leq b$ is true when either $a < b$ or $a = b$. For instance:

$$2 \geq -3 \quad \text{since } 2 > -3$$

$$5 \geq 5 \quad \text{since } 5 = 5$$

$$1 \leq \sqrt{3} \quad \text{since } 1 < \sqrt{3}$$

$$\frac{3}{5} \leq \frac{3}{5} \quad \text{since } \frac{3}{5} = \frac{3}{5}$$

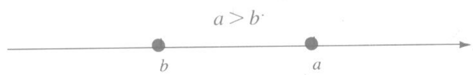

Figure 1-2

A statement involving inequality symbols is called an **inequality**.

The **absolute value** of a number a, denoted $|a|$, is defined as follows:

$$|a| = a \text{ if } a \geq 0$$
$$|a| = -a \text{ if } a < 0$$

In words: The absolute value of a nonnegative number is the number, and the absolute value of a negative number is the negative of the number. For instance:

$$|7| = 7$$

$$|0| = 0$$

$$\left|-\frac{1}{2}\right| = -\left(-\frac{1}{2}\right) = \frac{1}{2}$$

On a number line, the absolute value of a number is its distance from the origin. For instance (see Figure 1-3), 7 is $|7| = 7$ units from the origin and $-\frac{1}{2}$ is $|-\frac{1}{2}| = \frac{1}{2}$ unit from the origin.

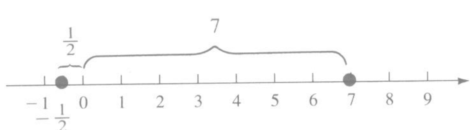

Figure 1-3