FUNDAMENTALS OF ELECTRONICS MATHEMATICS

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Preface

FUNDAMENTALS OF ELECTRONICS MATHEMATICS was written to fill a very specific need. Other textbooks on this subject include far more mathematics and more electronics content. In fact, other texts often leave the electronics student bewildered and floundering in too much theory and facts. This textbook is written for the student who needs a foundation of mathematics upon which to build a career as an electronics technician.

The mathematics covered in this book is basic and is common to all areas of electronics technology. The student who has mastered this textbook should have little difficulty in electronics courses.

The book is planned as a self-help workbook. Each chapter begins with learning Objectives for that chapter. When the book is used for self-study, these Objectives help focus the student's attention on the important principles and skills. If the book is used in a classroom setting, the chapter learning Objectives help the instructor plan the lessons. In either case, these Objectives allow for competency-based instruction.

The chapters are divided into small segments of instruction followed by Practice Exercises and Problems and Solutions. By covering each mathematics topic in a segment, the student has time to fully master that topic before moving to another topic. The Practice Exercises include word problems, multiple-choice questions, and matching tests, among other types of questions. The Problems and Solutions, which give the student problems to work out and the solutions, can be found in selected chapters.

Several appendices follow the last chapter of the text. The appendices consist of tables of: the Greek alphabet, unit prefixes, mathematical symbols, letter symbols, trigonometric ratios, powers of *e,* and common logarithms of numbers. A glossary follows the appendices, and can be used to refer to for specific definitions of terms. The answers to all odd-numbered Practice Exercises are found following the glossary, so the students can check their progress before going on to the next segment. Answers to even-numbered Practice Exercises are printed in the Instructor's Guide so they can be used for testing.

Acknowledgments

Several people have contributed to making this text a high-quality aid for future electronics technicians. The author is grateful to each of them. The students and staff at Brown Institute in Fort Lauderdale, Florida, used this material as their textbook, ensuring that each topic is explained clearly and satisfies the needs of electronics students. Advanced students at the National Institute of Technology in Cuyahoga Falls, Ohio, sampled all of the mathematics problems, so that the text is as error-free as possible. The educational approach, technical content, and organization were thoroughly reviewed by the following people.

Mark R. Wright	Dean of Education National Institute of Technology Campus Cuyahoga Falls, OH
Joseph M. Boucher	Electronics Department Head Technical Careers Institute Windsor, CT
John W. Huck	Assistant Director Electronic Technology Brown Institute Minneapolis, MN

The author is also very grateful to Richard C. Brown, founder of Brown Institute, for his educational inspiration.

Contents

chapter 1
Review of Arithmetic

OBJECTIVES

After completing this chapter, you will be able to
- explain the difference between a whole number and a fraction.
- describe a mixed number.
- discuss the similarity between decimal and percents.
- describe a decimal fraction.

The mathematics necessary for the solution of most problems encountered in electronics work is included in this text.

This chapter includes a review of addition, subtraction, multiplication, division, whole numbers, fractions, mixed numbers, and decimals.

TERMS TO KNOW

- *Whole number.* A whole number is a number which contains no fractions or decimals. Examples of whole numbers are: 1, 2, 13, 99, 796, and 843.
- *Fraction.* A fraction is one number over another, and is usually an indication of division. For example, $\frac{1}{2}$ is a fraction and means divide 1 by 2. A few other examples of fractions include: $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{99}{1000}$.
- *Mixed number.* A mixed number is a whole number and a fraction. For example, $1\frac{1}{2}$, $3\frac{7}{8}$, and $99\frac{47}{100}$ are mixed numbers.
- *Decimal.* A decimal is another way of expressing a fraction. For example, the fraction $\frac{1}{10}$ may be expressed as the decimal .1. Other examples are: .7, .99, and .76543.

ADDITION AND SUBTRACTION

Addition.

Example 1. Add the numbers 4732, 21, and 492.

Solution:
```
 4732
   21
  492
 ----
```

Answer: 5245

Example 2. Add the numbers 976, 74, 3986, and 10.

Solution:
```
  976
   74
 3986
   10
 ----
```

Answer: 5046

Subtraction.

Example 1. From 5245, subtract 492.

Solution:
```
 5245
 -492
 ----
```

Answer: 4753

Check:
```
 4753
 +492
 ----
 5245
```

Example 2. From 99,876, subtract 11.

Solution:
```
 99,876
    -11
 ------
```

Answer: 99,865

Check:
```
 99,865
    +11
 ------
 99,876
```

All subtraction problems may be checked by adding the answer obtained to the amount subtracted. If the original number is obtained, the subtraction is correct. In Example 1, 4753 should be added to 492, as shown below the solution. Since the answer to the check, 5245 in this example, is equal to the original number, the subtraction is correct.

MULTIPLICATION AND DIVISION

Multiplication.

Example 1. Multiply 77 by 61.

Solution:

```
  77
 ×61
  77
462
```

Answer: 4697

Check:

```
      77
61)4697
   427
    427
    427
      0
```

Example 2. Multiply 4753 by 492.

Solution:

```
   4753
   ×492
   9506
  42777
 19012
```

Answer: 2338476

Check:

```
          4753
492)2338476
    1968
     3704
     3444
      2607
      2460
       1476
       1476
          0
```

Each multiplication problem may be checked by dividing the answer obtained by one of the numbers originally multiplied together. If the other original number is obtained as the answer to this check, the multiplication is correct. In the check for Example 1, the answer 4697 is divided by 61, and 77 is obtained. Since 77 is the number which was multiplied by 61, the multiplication is correct. If any other number is obtained, it indicates that there is a mistake in the arithmetic.

Division.

Example 1. Divide 99 by 11.

Solution:

```
     9 (Answer)
11)99
   99
    0
```

Check:

```
 11
×9
 99
```

Example 2. Divide 26,677 by 721.

Solution:

```
         37 (Answer)
721)26677
    2163
     5047
     5047
        0
```

Check:

```
  721
  ×37
 5047
2163
26677
```

Example 3. Divide 784 by 53.

Solution:

```
    14
53)784
   53
   254
   212
```

Remainder: 42

There is a remainder of 42. The answer may be written as $14 + \frac{42}{53}$ or $14\frac{42}{53}$. The remainder is expressed as a fraction, $\frac{42}{53}$. The entire answer, $14\frac{42}{53}$, is a *mixed number* because it contains a whole number and a fraction.

All division problems may be checked by multiplying the answer by the *divisor* (the part "divided by" in the problem). If the *dividend* (the part "to be divided" in the problem) is obtained from this multiplication in the check, the arithmetic is correct. In the check for Example 1, the answer 9 is multiplied by the divisor 11, and 99 is obtained. Since 99 is the dividend in the problem, the arithmetic is correct. The check for Example 2 is also shown.

To check a division problem in which the answer is a mixed number, as in Example 3, multiply the whole number of the answer by the divisor, then add the remainder. If the dividend is obtained from this operation, the arithmetic is correct. To check Example 3, the following is done:

```
  53  (divisor)
 ×14  (whole number in answer)
 212
 53
 742
 +42  (remainder)
 784
```

Since 784 is the dividend, the arithmetic is correct.

FRACTIONS

In simple arithmetic it is usually best to express fractions as decimals. However, fractions must be manipulated when working with electronics formulas.

A fraction is made up of two quantities, a top number, or *numerator,* and a bottom number, or *denominator.* In the fraction $\frac{42}{53}$, 42 is the numerator and 53 is the denominator. The answer to the division problem in Example 3 is between 14 and 15. If one is divided into 53 parts and 42 of these 53 parts are taken, the result is the correct amount to add to the 14.

The denominator indicates into how many parts the whole unit has been divided. The numerator indicates how many of these parts have been taken.

Addition and Subtraction of Fractions. It is often necessary to combine fractions. Addition and subtraction are opposite, and the same general rules apply for either operation.

Example 1. Add $\frac{3}{8}$ and $\frac{2}{8}$.

Answer: $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Example 2. Add $\frac{5}{13}$ and $\frac{6}{13}$.

Answer: $\frac{5}{13} + \frac{6}{13} = \frac{11}{13}$

Example 3. Subtract $\frac{2}{8}$ from $\frac{3}{8}$.

Answer: $\frac{3}{8} - \frac{2}{8} = \frac{1}{8}$

Example 4. From $\frac{6}{13}$, subtract $\frac{2}{13}$.

Answer: $\frac{6}{13} - \frac{2}{13} = \frac{4}{13}$

In each of these examples the denominators are the same for both fractions. The same denominator is used in the answer. The numerators in the answers are obtained by adding or subtracting (as instructed in the problem) the numerators of the two fractions.

This is a very simple process if both of the fractions have the same denominator. If the fractions do not have the same denominator, one or more of the fractions in the problem must be changed to obtain a common denominator. *Common denominator* means that all of the fractions have the same denominator.

A common denominator can be found by multiplying the two denominators together. To find a common denominator in the problem $\frac{1}{2} + \frac{3}{5}$, multiply 2×5 and obtain 10. The common denominator is 10. Each of the fractions then must have 10 as a denominator. To change $\frac{1}{2}$ to tenths, multiply both the numerator and the denominator by 5. The result is $\frac{5}{10}$. To convert $\frac{3}{5}$ to tenths, multiply both the numerator and the denominator by 2. The result is $\frac{6}{10}$. To solve the problem, apply the same rule as in Examples 1 and 2 since both of the fractions now have the same denominator. This is shown in Example 5.

Example 5. Add $\frac{1}{2}$ and $\frac{3}{5}$.

$$\frac{1}{2} + \frac{3}{5} =$$

$$\frac{5}{10} + \frac{6}{10} = \frac{11}{10}$$

Example 6. Add $\frac{1}{3}$ and $\frac{1}{4}$.

$$\frac{1}{3} + \frac{1}{4} =$$

$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Note: The number 12 is the common denominator.

Example 7. Subtract $\frac{2}{3}$ from $\frac{7}{8}$.

$$\frac{7}{8} - \frac{2}{3} =$$

$$\frac{21}{24} - \frac{16}{24} = \frac{5}{24}$$

Note: The number 24 is the common denominator.

Example 8. Subtract $\frac{1}{8}$ from $\frac{1}{7}$.

$$\frac{1}{7} - \frac{1}{8} =$$

$$\frac{8}{56} - \frac{7}{56} = \frac{1}{56}$$

Note: The number 56 is the common denominator.

In each of these examples, the common denominator is obtained by multiplying the denominator of each of the fractions together. Sometimes it is possible to use a common denominator that is smaller than the product of the denominators in the problem.

Example 9. Add $\frac{1}{6}$ and $\frac{3}{8}$.

$\frac{1}{6} + \frac{3}{8} =$

Using 48 as the common denominator,

$\frac{8}{48} + \frac{18}{48} = \frac{26}{48}$

In this case the number 24 can be used as the common denominator of the problem.

$\frac{1}{6} + \frac{3}{8} =$

Using 24 as the common denominator,

$\frac{4}{24} + \frac{9}{24} = \frac{13}{24}$

The fraction $\frac{13}{24}$ is equal to $\frac{26}{48}$. Therefore, the problem has been solved using smaller numbers than before.

The smallest number which the 8 and the 6 can each be divided into evenly is 24. In a great majority of cases, this number can be found by inspection.

Multiplication and Division of Fractions. Multiplication and division of fractions is much easier than addition and subtraction of fractions. There is no need to be concerned with common denominators, and often the size of the numbers can be reduced by cancellation.

In multiplication of fractions, multiply all the numerators to obtain the numerator in the answer, and multiply all the denominators to obtain the denominator in the answer.

Example 1. Multiply $\frac{2}{7}$ by $\frac{3}{5}$.

Solution: $\frac{2}{7} \times \frac{3}{5} =$

$\frac{2 \times 3}{7 \times 5} = \frac{6}{35}$

The two numerators, 2 and 3, are multiplied to obtain the numerator 6 of the answer. The two denominators, 7 and 5, are multiplied to obtain 35 for the denominator of the answer.

Example 2. Multiply $\frac{3}{7} \times \frac{6}{7}$.

Solution: $\frac{3}{7} \times \frac{6}{7} = \frac{18}{49}$

Example 3. Multiply $\frac{1}{2}, \frac{2}{3}$, and $\frac{3}{4}$.

Solution: $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} =$

$$\frac{1 \times 2 \times 3}{2 \times 3 \times 4} = \frac{6}{24}$$

Example 4. Multiply $\frac{3}{8}, \frac{1}{3}$, and $\frac{5}{9}$.

Solution: $\frac{3}{8} \times \frac{1}{3} \times \frac{5}{9} =$

$$\frac{3 \times 1 \times 5}{8 \times 3 \times 9} = \frac{15}{216}$$

Division of fractions is also a simple process. To do this, the divisor is inverted (turn it upside down) and the division sign is changed to a multiplication sign, and then the fractions are multiplied.

Example 5. Divide $\frac{1}{2}$ by $\frac{1}{4}$.

Solution: $\frac{1}{2} \div \frac{1}{4} =$

$$\frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

Note that the divisor is inverted (the $\frac{1}{4}$ in this example), then the fractions are multiplied.

Example 6. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

Solution: $\frac{2}{3} \div \frac{3}{4} =$

$$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

Example 7. Divide $\frac{7}{12}$ by $\frac{5}{8}$.

Solution: $\frac{7}{12} \div \frac{5}{8} =$

$$\frac{7}{12} \times \frac{8}{5} = \frac{56}{60}$$

Example 8. Divide $\frac{7}{8}$ by 2.

Solution: $\frac{7}{8} \div 2 =$

$$\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$$

Note: Any whole number, such as 2, may be written as that number over 1, or in this case, $\frac{2}{1}$.

It is permissible to multiply both the numerator and the denominator of a fraction by the same number, as this does not change the value of the fraction. It is also permissible to divide *both* the numerator and denominator by the same number.

Example 1. $\frac{6}{10}$ can be written

$$\frac{6 \div 2}{10 \div 2};$$

$6 \div 2 = 3$, and $10 \div 2 = 5$

The fraction is now $\frac{3}{5}$, which is equal to $\frac{6}{10}$. It is simpler to do this by cancellation. To use cancellation, examine the fraction for a number which can be divided into both the numerator and the denominator. In this fraction, $\frac{6}{10}$, 2 can be divided into each. The number 2 goes into 6 three times and into 10 five times. The number 6 is then crossed out, or canceled, and a 3 is written above it. The number 10 is canceled, and a 5 is written below it.

$$\frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}}$$ The new fraction is $\frac{3}{5}$.

This fraction is reduced to its lowest terms. No number will go evenly into both 3 and 5.

Example 2. Reduce $\frac{12}{60}$ to its lowest terms.

$$\frac{\overset{6}{\cancel{12}}}{\underset{30}{\cancel{60}}}$$ The fraction is divided by 2. This fraction is still not in its lowest terms since 2 can still be divided into each of the terms.

$$\frac{\overset{3}{\cancel{6}}}{\underset{15}{\cancel{30}}}$$ Dividing by 2, the result is $\frac{3}{15}$. This fraction is not in its lowest terms, however, since 3 and 15 can both be divided by 3.