
# The Role of Mathematics in the Rise of Science

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# PREFACE

THE CHAPTERS of this book were originally separate essays for several purposes, and all but one have been published before, beginning in 1961. The essays have now been revised, and linked up; and an Introduction, and a collection of Biographical Sketches in lieu of an Epilogue, have been added.

This is, throughout, a book about mathematics, even if in various contexts it is seemingly concerned with other matters. It is a book about the uniqueness of mathematics as a force of our intellectuality, and about the mystique of its creativity; about the growing efficacy of mathematics, its widening importance, and its continuing spread. It is intended to be a book not *in* mathematics but *about* mathematics, even if some parts of it, which do not affect the rest of the book, are involved with outright mathematical technicalities; but no attempt is made to forcibly translate snippets or samples of technical mathematics into non-technical vernacular.

Ours is an age in which scientists are Wise Men, and the root of this Wisdom is in Mathematics. But mathematics, if taken by itself, is almost a pastime only, albeit an esoteric one; or so it seems. What makes mathematics so effective when it enters science is a mystery of mysteries, and the present book wants to achieve no more than to explicate how deep this mystery is.

Mathematics is both young and old; and in order to comprehend its role in our Knowledge of today, it is fruitful, and even imperative, to understand the portends of the vicissitudes of the mathematics of yesterday. Thus, we will frequently turn, for comparison and contrast, to Greek mathematics, and also Greek physics, and even to the enveloping Greek Rationality of which they are token and texture.

## PREFACE

It gives me pleasure to record that after the first essays of this collection began to appear in periodicals, Mr. Charles Scribner, Jr., president of Princeton University Press, noticing them, soon suggested that they be made into a book.

*August, 1965* S. B.

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# THE ROLE OF MATHEMATICS IN THE RISE OF SCIENCE

# INTRODUCTION

THE ESSAYS of this collection are all concerned with the role of mathematics in the rise and unfolding of Western intellectuality, with the sources and manifestations of the clarity and the mystique of mathematics, and with its ubiquity, universality, and indispensability.

We will frequently confront the mathematics of today with the mathematics of the Greeks; and in such a confrontation it is pertinent to take the entire mathematical development since A.D. 1600 as one unit. Therefore, "modern mathematics" will mean for us, invariably, mathematics since 1600, and not since some later date, even though, for good reasons, the mathematics of most of the 20th century is clamoring for an identity and definition of its own.

For our retrospections, the sobering fact that Greek mathematics, the mighty one, eventually died out in its own phase is of much greater import than the glamorous fact that Greek mathematics had come into being at all, although historians rather exult in its birth than grieve at its death. T. L. Heath in his standard history of Greek mathematics [1] finds it awe-inspiring to contemplate how much Greek mathematics achieved "in an almost incredibly short time"; and the context makes it clear that Heath means the time from 600 B.C., when the first Thales-like geometry began to stir, to 200 B.C., when Apollonius wrote his treatise on *Conics*. This makes 400 long years, and pronouncements like that of Heath are insultingly condescending to the Greeks, especially painfully so since immediately after the *Conics,* and thus 50 years after the *floruit* of Archimedes, Greek mathematics was at a loss where to turn next. In irreconcilable contrast to this, in the 400 years since A.D. 1565 mathematical intellectuality has turned the world topsy-turvy many times over, and shows no sign of abating.

[1] T. L. Heath, *A History of Greek Mathematics* (1921), I, 1.

Any major intellectual growth has to have, from time to time, bursts of unconscionably fast developments and stages of dizzying precipitousness, unless the developments are in anthropologically early phases of pre-intellectuality only. Much more "incredible" than the development of Greek mathematics in the course of 400 years was the reorientation of 20th-century physics, by a handful of youngsters, in the course of the four years from 1925 to 1928. Or, if to stay within Greek antiquity, nothing in the development of Greek mathematics can match, in sheer speed, Aristotle's amassment and *articulation* of logical, metaphysical, physical, cosmological, biological, and sociological knowledge within at most 40 years of his life (he died at the age of 62). As regards the growth of intellectuality, the plea of G. B. Shaw that man's life span ought to be at least 300 years [2] is of dubious merit.

Greek mathematics, whatever its inspiredness and universality, was slow, awkward, clumsy, bungling, and somehow sterile; and the limitations of Greek mathematics have wide implications. By its nature and circumstances Greek mathematics was part of Greek philosophy, that is, of the philosophies of Parmenides, Plato, and Aristotle, and it thus was a faithful image of a large segment of Greek intellectuality as a whole. Therefore, weaknesses of the mathematics of the Greeks were weaknesses of their intellectuality as such; and just as the death of the Great Pan signified the end of Greece's chthonic vitality,[3] so also the death of the mathematics of Archimedes signified, perforce, the end of Olympian [4] intellectuality.

[2] In the Preface to *Back to Methuselah.*

[3] Jane Harrison, *Prolegomena to the Study of Greek Religion,* 3rd ed. (1922), also reprinted as Meridian Book (1957), p. 651; Archer Taylor, "Northern Parallels to the Death of Pan," *Washington University* (*St. Louis*) *Studies,* Vol. 10, Part 2, No. 1 (October 1922), p. 3.

[4] Our distinction between "chthoric" and "Olympian" is that of the work of Jane Harrison, n. 3.

Yet the Greeks had extraordinary anticipations of the role of mathematics. It is an attribute of our times that mathematics is growing differently from other disciplines. All areas of knowledge are growing by expansion from within and accretion from without, by internal subdivision and external overlappings with neighboring areas. Mathematics, in addition to that, is also penetrating into other areas of knowledge one-sidedly, for their benefit, and more by invitations and urgings of the recipient fields than from an expansionist drive. Now, a glimmer of this peculiar and unique universality of mathematics was noticed, however dimly and inarticulately, by the "typically" Greek mathematics, even when still in the Pythagorean swaddling clothes of its infancy.[5] Aristotle repeatedly states that Pythagoreans were saying that all things are numbers, that substance (that is, matter in our sense) is number, etc.; now, this has such a modern ring that it could be lifted out of a book on elementary particles. Aristotle also reports that the Pythagoreans found that acoustical "harmonies" only depend on (commensurate) ratios of lengths and on nothing else, and that they generalized from this experience in acoustics to physics and knowledge in general. This was an uncanny anticipation; the "linear oscillator" is a simple device of acoustical provenance, but it also is an elemental skeletal part of theoretical physics of today. It is true that, against their own background, the Pythagoreans were recklessly imprudent when concluding so much from so little. But Isaac Newton was also "reckless" when proposing that there is a universal gravitational law which subsumes both terrestial falls of bodies and celestial orbitings of planets. In fact, a first confirmation of Newton's law came only in the 20th century when man-made satellites did indeed move according to the law of Newton; until then the law was a "feigned" hypothesis, all Newton's protestations notwith-

[5] A satisfactory recent report on Pythagoreans is in W. K. C. Guthrie, *A History of Greek Philosophy* (1962).

standing. Finally, it is undoubtedly true that the Pythagoreans drew their conclusions about the role of certain commensurate ratios in physics when they were still under the ingenuous misconception that all ratios are automatically commensurate, that is, before they discovered the existence of noncommensurate ratios of ordinary lengths. But this is of no consequence; Planck, Einstein, Niels Bohr, etc., knew that there are irrational numbers, but they laid the foundation of quantum theory on a groundwork of integer numbers and commensurate ratios nonetheless.

There is a common thesis that due to political, technological, sociological, and similar inadequacies the entire fabric of Hellenistic civilization gradually disintegrated and that the decline of mathematics was a part of this decline in general. If one accepts this thesis then one can even argue that, due to its greater strength, mathematics withstood the onslaught of the decline longer than other areas of intellectuality. In fact, Greek tragedy did not maintain itself on the level of Aeschylus, Sophocles, and Euripides beyond 400 B.C., nor did Greek history maintain itself on the level of Herodotus and Thucydides beyond that date. Greek philosophy did maintain itself on the level of Parmenides and Socrates beyond 400 B.C., through Plato and Aristotle, but only till 322 B.C. when Aristotle died. However, just then, Greek mathematics, which had begun in the 6th century B.C., made an extraordinary exertion and maintained itself on a high level for another 150 years or so. Indeed, one might even say that this post-Aristotelian phase was the true Golden Age of Greek mathematics, if a golden age there must be. In fact, sometime between 322 and 300 B.C. Euclid wrote his *Elements,* which was the greatest "primer" in anything, ever; the middle of the 3rd century B.C. was the age of Archimedes, in whose style Newton still composed his *Principia*; and around 200 B.C.

Apollonius wrote his *Conics,* with the aid of which, 1,800 years later, Kepler finally smashed the confining planetary circle which Plato had proclaimed to be divinely beautiful and eternal.

But if, granting all this, one nevertheless persists and asks the very pointed question how it came about that Greek mathematics immediately after the *Conics,* when still at its height, and seemingly at its healthiest, suddenly began to falter, then no explanation from general causes is available.

Explanations from sociological causations are also unavailable for problems of modern mathematics. For instance, it seems impossible to explain from sociology why in the 18th century and only then, when there was little industrial inducement and very little attention to experimental verification, there was a near-perfect, and richly yielding, fusion between mathematics and mechanics, to the immeasurable advantage of both, and of all physics and technology to come; whereas in the 19th century, under the Argus-like stare of advantage-seeking industrial capitalism, a so-called Applied Mathematics was beginning to splinter off, about which it cannot be said even today whether it was necessary or fortuitous, a good thing or a bad thing. And it is not easy to gauge what the eventual impact of the presently proliferating mathematics-to-various-purposes will be. The proliferation might even recede; there is a precedent for this. At the turn of the century there was a widespread movement for the propagation of Hamiltonian quaternions, on the grounds, apparently, that it is a most applicable technique. The movement has long since been extinct.

Most of the growing mathematics-to-a-purpose will probably endure. But we wish to say that the "purer" mathematics is, the more it embodies the significant designs of the texture of knowledge; for this reason, in the past, more significant applications to basic science came from mathematics that had been pursued for its own sake than from mathematics that had been pursued to a purpose.

A heavy sociological accent rests on the contemporary movement, especially in the United States, for reforming the mathematical curriculum in pre-college schools. The movement will undoubtedly prevail; but there may be an "disadvantaged generation" of youngsters who will have to pay a transition price, before a certain retardation in the learning of basic "operational" skills, which seems to be incident to the reform, is minimized. The last such reform was, at the turn of the century, Felix Klein's introduction of the infinitesimal calculus into the final stages of school training, and this reform began to erase the distinction between lower and higher mathematics which had been institutionalized in the course of the 19th century. The present-day movement aims, in substance, at introducing into school training Georg Cantor's set theory, and at an early stage, too. The tempo of developments quickens; the calculus entered schools after 200 years, set theory after 75 years. Cantor's set theory is a fountainhead of modernism-by-universality, or of universality-by-modernism, in 20th-century mathematics; the theory sets in at precisely the point at which mathematics proper sets off from rationality in general. Cantor well knew what his mission was; his memoirs have, in their footnotes, pertinent references to Plato, Aristotle, and other ancients, and to earlier figures in modern mathematics.[6] He asserts that he is succeeding where others have failed; but there is an undertone of awareness that he is succeeding because there were others who tried before him, even if they failed. And Cantor's attention to "ancient history," when himself on the height of achievements for the future, might be viewed as a lesson too.

[6] Georg Cantor, *Gesammelte Abhandlungen* [Collected Works], edited by E. Zermelo (1932).