Structural Analysis

A UNIFIED CLASSICAL AND MATRIX APPROACH

A. GHALI

A. M. NEVILLE

SECOND EDITION

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Preface to the Second Edition

Since this book is concerned with analysis, rather than design, the main text does not involve any specific system of units. The same applies to most of the problems set for the reader to solve. However, there are a number of problems where it was thought advantageous to use actual dimensions of members and to specify the magnitude of forces. These problems are set both in the so-called British units (common in North America) and in the SI units (now becoming accepted all over the world). The former version appears at the end of the various chapters and the latter, bearing the same number, is given in a separate section at the end of the book. It is up to the reader to choose which set of problems he wants to solve—and of course the underlying theory is the same. Naturally, answers to all the problems are listed.

We would like to take this opportunity to thank those readers, especially students at the University of Calgary, who assisted in correcting errors in the answers to problems in the first edition. While to expect that no errors are left would be too optimistic, we hope that their number is tending to zero.

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Preface to the First Edition

This book has been written so that it can be followed by anyone familiar with the application of basic mechanics to statically determinate structures—material usually covered in early courses in engineering. The material presented is both elementary and advanced, the emphasis being on a unified presentation of the whole field of structural analysis. Indeed, the book combines the classical and modern methods of analysis of statically indeterminate structures.

The text has been developed in teaching, over a number of years, undergraduate and graduate courses at the University of Calgary, Canada. The first fourteen chapters contain basic material which should be covered in the first course or courses; from the remainder of the book, a suitable choice, depending on requirements, can be made to form a more advanced course. This arrangement was chosen in order to make the book suitable not only for the student but also for the practicing engineer who wants to advance his knowledge by independent study and to obtain guidance on the most convenient methods of analysis of a variety of types of structures.

The methods of analysis introduced in the various chapters are illustrated by fully solved examples in text. At the end of each chapter carefully selected, instructive problems are set, with answers at the end of the book. Data frequently used are presented in appendices, one of which offers a full introduction to matrix algebra for the benefit of those not already familiar with it.

Matrix algebra is particularly useful in structural analysis because it makes possible a formulation of the solution as a series of matrix operations suitable for a digital computer. But even more important is that, by using matrices, structures of all types can be analyzed through a general approach, and because of the organizational properties of matrices, the use of matrix algebra is advantageous even in hand computation. Matrix formulation makes it possible to represent equations in a compact form: this not only saves space but also helps the reader to concentrate on the overall operations without being distracted by algebraic or arithmetical details. For all these reasons the matrix notation is extensively used in the book.

In the first four chapters we introduce two distinct general approaches of analysis: the force method and the displacement method. In the former, the structure is made statically determinate by the removal of restraining forces; a solution satisfying statics is then obtained, and the corresponding inconsistency in geometry is corrected by additional forces. In the displacement method, artificial restraints are added and the corresponding restraining forces are calculated: the equilibrium conditions are then satisfied by the removal of the restraining forces, thus allowing the displacements to occur and returning the structure to its actual condition. Throughout the book emphasis is placed on an overall unifying view, with these two distinct approaches in mind.

Both the force and displacement methods involve the solution of linear simultaneous equations relating forces to displacements. The emphasis in the first chapters is on the basic ideas in the two methods without obscuring the procedure by the details of derivation of the coefficients needed to form the equations. Instead, use is made of Appendices B, C, and D which give displacements due to applied unit forces, forces corresponding to unit displacements, and fixed-end forces due to various loadings. The consideration of the detailed methods of displacement calculation is thus delayed to Chapters 5 to 9, by which time the need for this material in the analysis of statically indeterminate structures is clear. This sequence of presentation of material is particularly suitable when the reader is already acquainted with some of the methods of calculating the deflection in beams. If, however, it is thought preferable first to deal with methods of calculation of displacement, Chapters 5 to 9 should be read before Chapters 2 to 4; this will not disturb the continuity.

The analysis of continuous beams, frames, and trusses is treated in Chapters 10 to 12. These include the methods of column analogy, slope-deflection and moment distribution, and are presented as applications of the force or displacement methods. The numerous solved examples included vary from simple problems to the more complicated types of structures of practical occurrence. Some techniques to speed up the analysis of building frames of several storeys or several bays are discussed in Chapter 12. In essence, these are the classical methods suitable for hand computation but their value to the structural designer is undiminished for preliminary calculations, for checking computer results, and of course when no computer is available.

Influence lines and influence coefficients are a useful means of analysis of structures subjected to moving loads or to loadings which can have varying positions. Chapters 13 and 14 deal with the methods of obtaining influence lines; Müller-Breslau's principle is explained and applied to different types of structures, including gridworks and interconnected bridge systems. Chapter 13 and the major part of Chapter 14 are concerned with influence lines which represent the effect of a unit gravity load. In the remainder of Chapter 14, influence lines for the effect of two other types of loading are discussed; one of these influence lines is applied in the analysis of statically indeterminate prestressed structures.

The effect of axial forces in framed structures is considered in Chapter 15.

Both the effect of the change in length and of the change in stiffness characteristics are discussed and applied to the problem of secondary stresses in trusses and in the determination of the critical buckling loads of continuous frames.

Chapter 16 deals with the analysis of shear walls, commonly used in modern buildings. The chapter summarizes the present knowledge, states the simplifying assumptions usually involved and presents a method of analysis that can be applied in most practical cases.

With the present availability of computers, numerical methods are increasingly used in the practical analysis of structures. The finite-difference method and, to a larger extent, the finite-element method are powerful tools which involve a large amount of computation. Chapter 17 deals with the use of finite differences in the analysis of structures composed of beam elements and extends the procedure to axi-symmetrical shells of revolution. In Chapter 18 the finite-difference method is used in the analysis of plates and in Chapter 19 it is applied to obtain influence, flexibility, and stiffness coefficients useful in a variety of cases.

The finite-element method for the analysis of continuum problems is based on the energy theorems and on the displacement method of analysis, applied to framed structures in earlier chapters. For this reason, it is possible to represent in Chapter 20 the basic general equations with a wide range of applications. Sufficient details of the finite-element method are included for the solution of problems in plane elasticity and in bending of plates. The finite-strip method discussed in the latter part of Chapter 20 is an extension of the finite-element approach which offers some saving in the amount of computation.

Modern design of structures is based on both the elastic and plastic analyses. The plastic analysis cannot replace the elastic analysis but supplements it by giving useful information about the collapse load and the mode of collapse. Chapters 21 and 22 deal with the plastic analysis of framed structures and slabs respectively.

An introduction to structural dynamics is presented in Chapter 23. This is a study of the response of structures to dynamic loading produced by machinery, gusts of wind, blast, or earthquakes. First, free and forced vibrations of a system with one degree of freedom are discussed. This is then extended to multidegree-of-freedom systems, taking advantage of the matrix approach.

In Chapter 24 some computational techniques in the analysis of large structures are discussed. The basic ideas are explained without details of computer programming, for which reference should be made to specialized manuals or texts. But the present book, coupled with practice in computer programming, gives sufficient knowledge to solve a wide variety of problems in structural analysis.

For the convenience of the American and Canadian readers, and of many British, British units are used in numerical examples. However, this should not cause difficulty for other readers as the book is concerned with methods of analysis rather than with design, and all the derivations and expressions are written in symbols.

The authors wish to thank Mr. G. S. Tadros and Mr. B. Langan of the University of Calgary for providing answers to most of the problems; numerous past students of the same university for solution of many problems; Dr. J. Harrop of the University of Leeds for his helpful collaboration in writing Chapter 22; and Dr. Y. K. Cheung of the University of Calgary for criticism and suggestions on some parts of the text, as well as for his work on Chapters 20 and 24, acknowledged on the title page. The typing of the manuscript was carefully done by Miss K. M. F. Dallas, Miss S. M. Fawcett, Mrs. C. I. Hills, Mrs. J. M. Misaelides, and Mrs. G. Donaldson and the manuscript was painstakingly proofread by Mr. P. K. Das of the University of Leeds.

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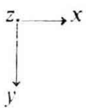
Notation

The following is a list of symbols which are common in the various chapters of the text; other symbols are used in individual chapters. All symbols are defined in the text when they first appear.

A	Any action, which may be a reaction or a stress resultant. A stress resultant at a section of a framed structure is an internal force: bending moment, shearing force or axial force.
a	Cross-sectional area.
D_i or D_{ij}	Displacement (rotational or translational) at coordinate i. When a second subscript j is provided it indicates the coordinate at which the force causing the displacement acts.
E	Modulus of elasticity.
EI	Flexural rigidity.
F	A generalized force: a couple or a concentrated load.
FEM	Fixed-end moment.
f_{ij}	Element of flexibility matrix.
G	Modulus of elasticity in shear.
I	Moment of inertia.
i, j, k, m, n, p, r	Integers.
J	Torsion constant (length^4), equal to the polar moment of inertia for a circular cross section.
l	Length.
M	Bending moment at a section. In beams and grids, a bending moment is positive when it causes tension in bottom fibers.
M_{AB}	Moment at end A of member AB. In plane structures, an end-moment is positive when clockwise. In general, an end-moment is positive when it can be represented by a vector in the positive direction of the axes x, y, or z.
N	Axial force at a section or in a member of a truss.
P, Q	Concentrated loads.
q	Load intensity.
R	Reaction.
S_{ij}	Element of stiffness matrix.
s	Used as a subscript indicates a statically determinate action.
T	Twisting moment at a section.
u	Used as a subscript, indicates the effect of unit forces or unit displacements.
V	Shearing force at a section.

W	Work of the external applied forces.
ε	Strain.
η	Influence ordinate.
ν	Poisson's ratio.
σ	Stress.
τ	Shearing stress.
{ }	Braces indicate a vector, i.e., a matrix of one column. To save space, the elements of a vector are sometimes listed in a row between two braces.
[ ]	Brackets indicate a rectangular or square matrix.
$[\ ]^T_{n \times m}$	Superscript T indicates matrix transpose. $n \times m$ indicates the order of the matrix which is to be transposed resulting in an $m \times n$ matrix.
⟶⟶ (double-headed arrow)	Double-headed arrow indicates a couple or a rotation; its direction is that of the rotation of a right-hand screw progressing in the direction of the arrow.
⟶	Single-headed arrow indicates a load or a translational displacement.
	Axes: the positive direction of the z axis points away from the reader.

The SI System of Units of Measuremen

Length	metre	m
	millimetre = 10^{-3}m	mm
Area	square metre	m^2
	square millimetre = $10^{-6}m^2$	mm^2
Volume	cubic metre	m^3
Frequency	hertz = 1 cycle per second	Hz
Mass	kilogram	kg
Density	kilogram per cubic metre	kg/m^3
Force	newton	N
	= a force which applied to a mass of one kilogram gives it an acceleration of one metre per second per second, i.e. $1N = 1kg\ m/s^2$	
Stress	newton per square metre	N/m^2
	newton per square millimetre	N/mm^2
Temperature interval	degree Celsius	deg C; °C

Nomenclature for multiplication factors

10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

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