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continued on page 449

PREFACE

On September 23 - 29, 1984 the 8th seminar on stability problems for stochastic models took place in Uzhgorod. It was organized by the Steklov Mathematical Institute, Uzhgorod State University and the Institute for Systems Studies. 78 participants from 19 cities of the Soviet Union attending the seminar represented 30 universities and research institutes. Three papers were presented by mathematicians from Bulgaria, Hungary and Japan.

Both in topic and approach to the problems the seminar followed the tradition of the seven forerunners. The range of problems under discussion was rather wide including theory of probability metrics, queueing theory, limit theorems for sums of random variables, mathematical statistics, to mention only some of them.

As at the previous seminars, most of the problems were despite their diversity unified in their common focus on stability problems and in their search for general approaches to these.

One of the possible general approaches, the so-called method of metric distances was discussed in some detail in the editor's preface to the Proceedings of the 1982 seminar published in 1983 as vol. 982 of the Springer Lecture Notes in Mathematics.

The present volume consists of 23 papers presenting enlarged and revised versions of a part of the papers delivered at the seminar. The other part will be published (in Russian) in 1985 under the same title "Stability problems for stochastic models", Proceedings of the seminar, edited by the Institute for Systems Studies (these Proceedings are translated into English by Plenum Press Corporoation in the series of Journals of Soviet Mathematics).

Several persons were very instrumental in preparing this volume: V.V.Kalashnikov as a co-editor, A.M.Kagan and L.B.Klebanov as editors of the final English text, I.S.Shiganov, V.V.Slavova and S.Rachev assisted in translating some papers into English.

All the authors are grateful to Acad. L.D.Faddeev (the editor of the USSR subseries) and to Dr. A.P.Oskolkov for their efforts that allowed us to meet once again, this time under the cover of Lecture Notes in Mathematics.

V.M.Zolotarev

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Contents

ON PERIODIC DISTRIBUTION OF WAITING-TIME PROCESS

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1. Introduction.

The paper deals with a single-server queueing system with a non-stationary Poisson input. Specifically, the arrival rate function $\lambda(t)$ is periodic with a period T so that $\lambda(nT+t) = \lambda(t)$ for all positive integers n and all t in $[0,T]$. The service time of a customer arriving at an epoch t depends on t and has a distribution function (d.f.) $B_t(x)$. As a function of t it is assumed periodic with the same period T for all x. We also assume $\lambda(t)$ to be a measurable bounded function and $B_t(x)$ is measurable as a function of its two variables.

Queues with non-homogeneous Poisson arrivals have been studied earlier by Takács [1], Reich [2, 3], Hasofer [4], Harrison and Lemoine [5, 6]. Hasofer considered the case of periodic input and, imposing some conditions on the arrival-rate function and the service-time distribution, showed that the probability of server idleness is asymptotically a periodic function of time. Harrison and Lemoine proved limit theorems for the waiting-time process. Lemoine [6] showed that the limit periodic distribution of the virtual-waiting time at a fixed time of the period coincides with the distribution of the supremum, over the time horizon $[0, \infty)$, of the sum of a stationary compound Poisson process with negative drift and a continuous periodic function. Some asymptotic results for the case of periodic input were obtained in [7, 8] .

In this paper we focus attention on the asymptotic properties of the limit periodic distribution in the heavy traffic situation and slowly changing arrival rate functions. First of all we shall obtain a basic relation for the Laplace transform of the virtual waiting-time periodic d.f. $H(t,x)$. This relation also gives a representation of an atbitrary periodic solution of the Takács equation [1] . Further, the connection between $H(t,0)$ and the mean $m(t)$ of the distribution $H(t,x)$ is analysed. In Section 3 we study the customer actual waiting-time. It is evident that its distribution coincides with $\left(\int_0^T \lambda(t)\,dt\right)^{-1} \int_0^T \lambda(t)\, H(t,x)\,dt$ and we find a useful relation between the mean of the customer waiting-time and $m(t)$. It makes possible to analyse the heavy traffic situation. We prove here that under some natural assumptions the normalized waiting-time has the exponential limit distribution. The asymptotic behaviour of $m(t)$ is discussed in Section 4. Some results for the so called slowly changing function $\lambda(t)$ are given in Section 5. It appears that under a suitable normalization the virtual waiting-time converges weakly to the degenerate distribution. We also prove a limit theorem establishing the convergence of $H(t_0, x)$ to the stationary distribution corresponding to the system $M|GI|1$ with a constant arrival rate $\lambda(t_0)$. In Section 5 it is assumed that $B_t(x) \equiv B(x)$.

Out results can be applied to the construction of a computational procedure for $H(t,x)$ (see [7]).

2. Basic relation.

Here we study the virtual waiting-time process $w(t)$. We start with introducing the following notations:

$$\Lambda(t)=\int_0^t \lambda(u)\,du, \quad \lambda=T^{-1}\Lambda(T),$$

$$\beta_k(t)=\int_0^\infty x^k B_t(dx),\ (k=1,2), \quad \overset{*}{b}(t,s)=\int_0^\infty e^{-sx} B_t(dx)\ (\mathrm{Re}\, s\geq 0),$$

$$b_k=(\lambda T)^{-1}\int_0^T \lambda(t)\beta_k(t)\,dt, \quad \pi(t)=\int_0^t \lambda(u)\beta_1(u)\,du-t.$$

Arguing as in Harrison and Lemoine [5] one can prove that there exists a proper distribution $H(t,x)$ such that

$$\lim_{n\to\infty} P\{w(nT+t)\leq x\}=H(t,x)$$

iff

$$\rho=\lambda b_1<1. \tag{1}$$

This is more or less obvious from physical considerations. Under some additional assumptions $H(t,x)$ is a periodic solution of the integro-differential equation due to Takács [1] . It turns out that it is impossible to find $H(t,x)$ in the general case and that is why one can only seek computational procedures for $H(t,x)$ (see [9] in this connection).

The following important observation concerns the function $H(t,0)$ and $H^*(t,s)=\int_0^\infty e^{-sx} H(t,dx)\ (\mathrm{Re}\, s\geq 0)$. With the help of arguments similar to those used in [10] we can see that $H^*(t,s)$ satisfies the following relation :

$$H^*(t,s)=\Big\{ H^*(0,s)-s\int_0^t H(u,0)\exp\Big\{-su+\int_0^u \lambda(y)(1-b^*(y,s))\,dy\Big\}\,du\Big\}\exp\Big\{st-\int_0^t \lambda(u)(1-b^*(u,s))\,du\Big\}. \tag{2}$$

Since $H^*(t,s)$ is a continuous function of t it follows from (2) that

$$H^*(0,s)=\frac{s\int_0^T H(u,0)\exp\{-su+\int_0^u \lambda(y)(1-b^*(y,s))\,dy\}\,du}{1-\exp\{-sT+\int_0^T \lambda(u)(1-b^*(u,s))\,du\}}. \tag{3}$$

Now we consider the system with a Poisson input at a rate $\overset{t}{\lambda}(y) = \lambda(y+t)$ and with a d.f. of the service time $B^t_u(x) = B_{u+t}(x)$ for a customer arriving at the epoch u. Denote by $H^*_t(y,s)$ the function $H^*(y,s)$ for this system. Since $H^*_t(y,s) = H^*(y+t,s)$ we have from (3)

$$\overset{*}{H}(t,s) = \frac{s\int_0^T H(u+t,0)\exp\{-su+\int_t^{t+u}\lambda(y)(1-\overset{*}{b}(y,s))dy\}du}{1-\exp\{-sT+\int_0^T\lambda(u)(1-b^*(u,s))du\}}. \tag{4}$$

Let us formulate this result in the form of a theorem.

Theorem 1. If (1) holds then the Laplace transform of $H(x,t)$ has the form (4).

Hence, it is possible to obtain an equality establishing a connection between $H(t,0)$ and the mean of the waiting-time process.

Theorem 2. If $b_2 < \infty$ and (1) holds the d.f. $H(t,s)$ has a finite mean $m(t)$ for any $t \in [0,T]$ and

$$m(t) = \frac{\lambda b_2}{2(1-\rho)} - \frac{T(1-\rho)}{2} + \pi(t) - \frac{1}{T(1-\rho)}\int_t^{T+t}\pi(y)H(y,0)dy. \tag{5}$$

Proof. Denote by $\tilde{B}(x)$ the d.f. of the total service time of the customers that arrive during one period. It is clear that $\tilde{B}(x)$ is given by

$$\tilde{B}(x) = \sum_{K=0}^{\infty}\frac{(\lambda T)^K}{K!}e^{-\lambda T}\left[\frac{1}{\lambda T}\int_0^T\lambda(u)B_u(x)\,du\right]^{*K}$$

where $[F]^{K^*}$ denotes the K-fold convolution of F with itself. From this relation it follows that

$$\int_0^\infty x^2 \tilde{B}(dx) < \infty \qquad (6)$$

if $b_2 < \infty$. Now we consider a system $\mathcal{D}|G|1$ where customers arrive at the epochs nT $(n = 0, 1, 2, \dots)$ and their service times are mutually independent random variables with the common d.f. $\tilde{B}(x)$. Let $\mathcal{D}(x)$ be the stationary distribution of the virtual waiting time at the epochs $nT + 0$ for this system. By the results of [11] it follows from (6) that $\mathcal{D}(x)$ has a finite mean. Since $H(t, x) \geqslant \mathcal{D}(x)$ for any $t \geqslant 0$, $x \geqslant 0$ the d.f. $H(t, x)$ has a finite mean for any $t \geqslant 0$. Now one can obtain (5) by differentiation of (4) with respect to s .

Note that $m(t)$ is differentiable if $\lambda(t)$ and $\beta_1(t)$ are continuous. From (5) we have

$$m'(t) = H(t, 0) - 1 + \lambda(t)\beta_1(t). \qquad (7)$$

Since $f(t) = m(t) - \pi(t)$ is always differentiable it is possible to find $H(t, 0)$ if $m(t)$ is known.

3. The mean of customer actual waiting time.

We denote by $G_n(x)$ the d.f. of the waiting time of the n-th customer. By the results of [5] there exists a proper d.f. $G(x)$ such that