Plenary Reviews

THE s-CHANNEL THEORY OF SUPERCONDUCTIVITY

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ABSTRACT

The essential features and new developments of the s-channel theory of superconductivity are presented. Applications to the μsR and Hall number experiments are analyzed. The relations between small coherence length, Bose-Einstein condensation and high T_c are emphasized.

1. INTRODUCTION

In this talk I wish to discuss a new phenomenological theory of high temperature superconductivity, which has been developed in collaboration with R. Friedberg and H. C. Ren. Our starting point is the experimental observation of a small coherence length $\xi \approx 10\AA$ in high T_c superconductors[1,2]. This is in contrast with a much larger ξ in the usual cold superconductor, typically $\approx 10^4\AA$ for type I and $\approx 10^2\AA$ for type II. In addition, it is known that in all these superconductors the magnetic flux carried by each vortex filament is $2\pi\hbar c/2e$, showing the existence of a pairing state.

A small coherence length ξ in the high T_c superconductors indicates that the pairing between electrons, or holes, is reasonably localized in the coordinate space. Hence, the pair-state can be approximated by a phenomenological local boson field $\phi(\vec{r})$, whose mass m_b is $\approx 2m_e$ and whose elementary charge unit is $2e$, where m_e and e are the mass and charge of an electron. It follows then that the transition

$$2e \rightarrow \phi \rightarrow 2e \tag{1.1}$$

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must occur, in which e denotes either an electron or a hole; furthermore, the localization of ϕ implies that phenomena at distances larger than the physical extension of ϕ (which is $< \xi$) are insensitive to the interior of ϕ. Since ξ is of the same order as the scale of a lattice unit cell, it becomes possible to develop a phenomenological theory of superconductivity based *only* on the local character of ϕ.

Of course, physics at large does depend on several overall properties: the spin of ϕ, the stability of an individual ϕ-quantum, the isotropicity and homogeneity (or their absence) of the space containing ϕ and so on. The situation is analogous to that in particle physics: the smallness of the radii of pions, ρ-mesons, kaons, $\cdots$ makes it possible for us to handle much of the dynamics without any reference to their internal structure, such as quark-antiquark pairs or bag models. Hence, the origin of their formation becomes a problem separate from the description of their mechanics. An important ingredient in this type of phenomenological approach is the selection of the basic interaction Hamiltonian that describes the underlying dominant process. In the usual low-temperature superconductors, the large ξ-value makes the corresponding pairing state ϕ too extended and ill-defined in the coordinate space; therefore, (1.1) does not play an important role. Instead, the BCS theory of superconductivity[5] is based on the emission and absorption of phonons,

$$2e \rightarrow 2e + \text{phonon} \rightarrow 2e. \tag{1.2}$$

In the language of particle physics, (1.1) is an s-channel process, while (1.2) is t-channel. The BCS theory may be called the t-channel theory, and the model that is based on (1.1) the *s-channel* theory.[6,7]

The use of a boson field for the superfluidity of Liquid $HeII$ has had a long history. However, there are some major differences in the following application to (high temperature) superconductors:

1. The ϕ-quantum is charged, carrying $2e$, while the helium atom is neutral.

2. We assume each individual ϕ-quantum to be *unstable*, with 2ν as its excitation energy.

In any microscopic attempt to construct ϕ out of $2e$, because of the short-range Coulomb repulsion it is very difficult to have ϕ stable. The explicit assumption of instability bypasses this difficulty; it also makes the present boson-fermion model different from the theory of Schafroth[8,9] and others.

In the rest frame of a single ϕ-quantum (in isolation), the decay

$$\phi \to 2e \tag{1.3}$$

occurs, in which each e carries an energy

$$\frac{k^2}{2m_e} = \nu .$$

Consequently, in a large system, there are macroscopic numbers of both bosons (the ϕ-quanta) and fermions (electrons or holes), distributed according to the principles of statistical mechanics.

At temperature $T < T_c$, there is always a macroscopic distribution of zero momentum bosons co-existing with a Fermi distribution of electrons (or holes). Take the simple example of *zero* temperature: Let ϵ_F be the Fermi energy. When $\epsilon_F = \nu$, the decay $\phi \to 2e$ cannot take place because of the exclusion principle; therefore, the bosons are present. Even when $\epsilon_F < \nu$, there is still a macroscopic number of (virtual) zero momentum bosons in the form of a static coherent field amplitude whose source is the fermion pairs. This then leads to the following essential features of the s-channel model.

Below the critical temperature T_c the long range order in the boson field can always be described by its zero-momentum bosonic amplitude B, as in the Bose-Einstein condensation (and therefore similar to liquid $HeII$). Because of the transition (1.1), the zero-momentum of the boson in the condensate forces the two e to have equal and opposite momenta, forming a Cooper pair. Therefore, the same long range order B also applies to the Cooper pairs of the fermions. Furthermore, as we shall see, the gap energy Δ of the fermion system is related to B by

$$\Delta^2 = |gB|^2 \tag{1.4}$$

where g is the coupling for $\phi \to 2e$.

Since in reality ϕ is a composite of $2e$, when the average distance between ϕ-quanta becomes less than the diameter of the composite the approximation of treating each ϕ as a single boson breaks down. However, for densities not that high, by representing the 2e-resonance as an independent ϕ-field, we may convert an otherwise strong interaction problem (which forms the resonance and exists at small distances) to one that can be handled by perturbative series in weak coupling

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(i.e., the residual interaction at relatively larger distances). This enables us to give a systematic analysis of such a theory; it also makes transparent the questions of gauge invariance and symmetry breaking.

2. μsR EXPERIMENTS

In the s-channel theory, the long-range order parameter B is due to Bose-Einstein condensation. Consequently, the phase transition can be of statistical origin, in contrast to the usual BCS theory. As we shall see, the critical temperature T_c may then be much higher. Let us first examine the evidence supporting such a picture.

Recently, Uemura *et al.*[10] discovered that in all (high temperature) cupric superconductors there is a universality law:

$$T_c \propto \rho^*/m^* \tag{2.1}$$

where ρ^* is the number density of superconducting charge carriers and m^* their effective mass; the proportionality constant is the same for all materials, about

$$40°K \qquad \text{to} \qquad 4 \times 10^{20} cm^{-3}/m_e\,, \tag{2.2}$$

assuming each carrier bears a charge e. In the s-channel theory, the ρ^* in the μSR experiment[10] should be interpreted as due to bosons of charge $2e$; the proportionality constant would then be reduced by a factor 4, and the experimentally determined proportionality constant (2.2) becomes

$$40°K \qquad \text{to} \qquad 10^{20}\text{cm}^{-3}/m_e\,. \tag{2.3}$$

In these cupric superconductors, the charge carriers concentrate on the two-dimensional CuO_2 plane; their tunneling between these planes gives rise to the three-dimensional character. The average separation c between CuO_2 planes is approximately constant for different materials:

$$c \cong 6\mathring{A}\,.$$

Let d be the average distance between neighboring bosons in the same CuO_2 plane; the boson density n_b is

$$n_b = (d^2c)^{-1}. \tag{2.4}$$

At temperature T_c, each boson of mass m_b has a thermal deBroglie wavelength ($\hbar = 1$).

$$\lambda_T \equiv \sqrt{\frac{2\pi}{m_b\kappa T}}. \tag{2.5}$$

Setting

$$m^* = m_b \quad \text{and} \quad \rho^* = n_b \tag{2.6}$$

in (2.1), one sees that the product m_bT_c is $\propto d^{-2}$; from the definition of λ_T, the same product is also $\propto \lambda_T^{-2}$. Eliminating m_bT_c and using the μsR experimental data we arrive at

$$(\lambda_T/d)_{\text{exp}} \cong 2.8 \tag{2.7}$$

for all cupric superconductors. Furthermore, this value is independent of m_b.

In the s-channel theory, the pair state is represented phenomenologically by a local field ϕ which can propagate relatively freely within each CuO_2 plane; let m_b denote its mass in the CuO_2 plane. Because the ϕ-quantum, in reality, is shaped like a flat disc with its face parallel to the plane, this makes it difficult to tunnel across different CuO_2. The effective boson mass M_b in the direction $\perp$ to the CuO_2 plane should therefore be much larger than m_b:

$$M_b >> m_b. \tag{2.8}$$

The criterion of Bose-Einstein condensation for such a configuration is[11]

$$n_b \cong (\lambda_T^2 c)^{-1}\, ln\,(2M_bc^2\kappa T_c) \tag{2.9}$$

and, because of (2.4),

$$\lambda_T/d \cong \left[ln\,(2M_bc^2\kappa T_c)\right]^{\frac{1}{2}}. \tag{2.10}$$

When $M_b \to \infty$, or $c \to \infty$, λ_T/d becomes infinite and $T_c = 0$; this gives the well-known result that there is no Bose-Einstein condensation in two dimensions. Because of the log-factor in (2.9)-(2.10), we expect that while λ_T is $O(d)$, the ratio λ_T/d can be ~ 2.8, larger than 1.

The theoretical formula (2.10) gives a slight variation of λ_T/d, consistent with experimental data, as shown in Figure 1.

3. BOSE-EINSTEIN CONDENSATION AND HIGH T_c

An essential feature of the Bose-Einstein condensation is that the thermal de Broglie length λ_T should be comparable to the interparticle distance d, so that the effect of symmetric statistics becomes manifest. It is instructive to put side by side the ratios λ_T/d for the ideal bosons, liquid Helium II together with cupric superconductors:

$$\lambda_T/d = \begin{cases} 1.377 & \text{ideal bosons} \\ 1.65 & \text{He II} \\ 2.8 & \text{cupric superconductors}. \end{cases} \tag{3.1}$$

In the BCS theory, T_c depends sensitively on the interaction between electrons and phonons (or other excitations); $T_c \to 0$ when there is no interaction. In the Bose-Einstein condensation, T_c is determined by $\lambda_T \sim d$, which is of statistical origin and therefore can be much higher (T_c exists even without interaction). In the boson picture, on account of (2.5) and (3.1), we have

$$(m_b T_c)^{\frac{1}{2}} d \approx \text{constant}. \tag{3.2}$$

This product varies by only a factor less than, or ~ 2, from ideal boson to He, and from He II to cupric superconductors. For He, $d \cong 3.58\mathring{A}$, $T_c \cong 2.2°K$ and $m_b \cong 8000 m_e$, whereas for cupric superconductors the relevant m_b is only a few times m_e, the electron mass. (See (8.7) below.) Thus, between He and cupric superconductors there is a change in m_b of *three* orders of magnitude. The relative constancy of the product $(m_b T_c)^{\frac{1}{2}} d$ naturally leads to a much higher T_c for cupric superconductors. In addition, if one could have smaller d, then T_c would increase accordingly. Of course, d must not be too small; otherwise, the pair-states overlap, and the boson approximation breaks down (as in the case of cold superconductors, because of their large coherence lengths).

4. A PROTOTYPE s-CHANNEL MODEL

As a prototype of the s-channel theory of superconductivity, we assume ϕ to be of spin 0 and that the space containing ϕ is a three-dimensional homogeneous and isotropic continuum. For realistic applications[11,12], as emphasized before, a more appropriate approximation of the latter would be the product of a two-dimensional x, y-continuum (simulating the CuO_2 plane) and a discrete lattice of spacing c along the z-direction. The two-dimensional layer character of CuO_2 planes helps in the localization of the pair state in the z-direction, making the ϕ-quantum disc-shaped. The space that ϕ moves in becomes a three-dimensional continuum when $c \to 0$, but two-dimensional when $c \to \infty$.

Here we consider an idealized isotropic and homogeneous space; the system consisting of the local scalar field ϕ of mass M and the electron (or hole) field ψ_σ of mass m, with $\sigma = \uparrow$ or $\downarrow$ denoting the spin. The Hamiltonian is $(\hbar = 1)$

$$H = H_0 + H_1 \tag{4.1}$$

in which the free Hamiltonian is

$$H_0 = \int \left[\phi^\dagger (2\nu_0 - \frac{1}{2M} \nabla^2)\, \phi + \psi_\sigma^\dagger (-\frac{1}{2m} \nabla^2)\, \psi_\sigma \right] d^3 r \tag{4.2}$$

with the repeated spin index σ summed over and $\dagger$ denoting the hermitian conjugate. The interaction H_1 can be simply

$$H_1 = g \int (\phi^\dagger \psi_\uparrow \psi_\downarrow + \text{h.c.})\, d^3 r\,. \tag{4.3}$$

Both ϕ and ψ_σ are the usual quantized field operators whose equal-time commutator and anticommutator are

$$[\phi(\vec{r}),\, \phi^\dagger(\vec{r}\,')] = \delta^3(\vec{r} - \vec{r}\,')$$

and

$$\{\psi_\sigma(\vec{r}),\, \psi_{\sigma'}^\dagger(\vec{r}\,')\} = \delta_{\sigma\sigma'}\, \delta^3(\vec{r} - \vec{r}\,')\,.$$

The total particle number operator is defined to be

$$N = \int (2\phi^\dagger \phi + \psi_\sigma^\dagger \psi_\sigma)\, d^3 r \tag{4.4}$$

which commutes with H and is therefore conserved.

Expand the field operators in Fourier components inside a volume Ω with periodic boundary conditions:

$$\psi_\sigma(\vec{r}) = \sum_k \Omega^{-\frac{1}{2}} a_{\vec{k},\sigma} e^{i\vec{k}\cdot\vec{r}}$$

and (4.5)

$$\phi(\vec{r}) = \sum_k \Omega^{-\frac{1}{2}} b_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$$

where the anticommutator $\{a_{\vec{k},\sigma}, a^\dagger_{\vec{k}',\sigma'}\} = \delta_{\vec{k}\vec{k}'}\delta_{\sigma\sigma'}$ and the commutator $[b_{\vec{k}}, b^\dagger_{\vec{k}'}] = \delta_{\vec{k}\vec{k}'}$. Equation (4.3) can then be written as

$$H_1 = \frac{g}{\sqrt{\Omega}} \sum_{p,k} \left[b^\dagger_{\vec{p}}\, a_{\frac{\vec{p}}{2}+\vec{k},\uparrow}\, a_{\frac{\vec{p}}{2}-\vec{k},\downarrow} + \text{h.c.} \right] . \tag{4.6}$$

In (4.2), $2\nu_0$ is the "bare" excitation energy of ϕ. Because of the interaction, the "physical" (i.e., renormalized) excitation energy 2ν in reaction (1.3) is given by

$$2\nu = 2\nu_0 + \frac{g^2}{2\Omega} \sum_k P \frac{1}{\nu - \omega_k} \tag{4.7}$$

where P denotes the principal value and

$$\omega_k = \frac{k^2}{2m} . \tag{4.8}$$

The decay width Γ of a ϕ-quantum (in vacuum) is given by

$$\Gamma = (g^2/\pi) m^{\frac{3}{2}} \sqrt{\frac{\nu}{2}} . \tag{4.9}$$

5. GAP ENERGY

For $T < T_c$, the zero-momentum occupation number $b_0^\dagger b_0$ of the boson field ϕ becomes macroscopic; hence, we may replace the operator b_0 by a macroscopic constant. Set in (4.5)

$$\Omega^{-\frac{1}{2}} b_0 = B = \text{c number}$$

and write

$$\phi = B + \phi_1 \tag{5.1}$$

where

$$\phi_1 = \sum_{k\neq 0} \Omega^{-\frac{1}{2}} b_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} . \tag{5.2}$$

In the following, we shall treat the effects of ϕ_1 perturbatively. Let μ be the chemical potential. Introduce

$$\mathcal{H} \equiv H - \mu N = \mathcal{H}_0 + \mathcal{H}_1 \tag{5.3}$$

where

$$\mathcal{H}_0 = \sum_k \{ [\frac{k^2}{2M} + 2(\nu_0 - \mu)] b^\dagger_{\vec{k}} b_{\vec{k}} + (\omega_k - \mu) a^\dagger_{\vec{k},\sigma} a_{\vec{k},\sigma}$$

$$+ g[B^* a_{\vec{k},\uparrow} a_{-\vec{k},\downarrow} + B a^\dagger_{-\vec{k},\downarrow} a^\dagger_{\vec{k},\uparrow}] \} \tag{5.4}$$

and

$$\mathcal{H}_1 = g \int (\phi_1^\dagger \psi_\uparrow \psi_\downarrow + \text{h.c.}) \, d^3 r \tag{5.5}$$

is the perturbation.

The zeroth-order Hamiltonian $\mathcal{H}_0$ is quadratic in field operators, and can therefore be readily diagonalized. Its fermion-dependent part can be written as a sum of matrix products, each of the form:

$$(a_{\vec{k},\uparrow}, a^\dagger_{-\vec{k},\downarrow}) \cdot A \cdot \begin{pmatrix} a^\dagger_{\vec{k},\uparrow} \\ a_{-\vec{k},\downarrow} \end{pmatrix}$$

where A is a 2×2 matrix

$$A = \begin{pmatrix} -(\omega_k - \mu) & gB^* \\ gB & \omega_k - \mu \end{pmatrix} .$$

Because of Fermi statistics, the two diagonal elements of A have opposite signs. The eigenvalue E_k of A is determined by

$$E_k^2 - (\omega_k - \mu)^2 = g^2 |B|^2 ;$$

i.e.,

$$E_k = [(\omega_k - \mu)^2 + g^2 |B|^2]^{\frac{1}{2}} . \tag{5.6}$$

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Thus, we establish the formula for the gap energy (1.4):

$$\Delta^2 = g^2 \,|\, B \,|^2 \,.$$

6. THERMODYNAMICS

The zeroth-order grand canonical partition function is

$$\mathcal{Q} = \text{trace } e^{-\beta \mathcal{H}_0} \,,$$

whose logarithm is $p\Omega/\kappa T$, where $\beta = (\kappa T)^{-1}$ and p is the pressure. By using the diagonalized form of $\mathcal{H}_0$, we find

$$p = -2(\nu_0 - \mu)\,|\, B \,|^2 + \Omega^{-1} \sum_k (E_k + \mu - \omega_k)$$
$$+2(\beta\Omega)^{-1} \sum_k ln(1 + e^{-\beta E_k})$$
$$-(\beta\Omega)^{-1} \sum_k ln\{\, 1 - \exp \beta\,[2\mu - 2\nu - (k^2/2M)]\,\} \,. \tag{6.1}$$

In accordance with the general thermodynamical principle, at constant T and μ, the function p should be a maximum with respect to any internal parameter, such as $|\, B \,|$. Setting $(\partial p/\partial |\, B \,|)_{\mu,T} = 0$, we have

$$\nu_0 - \mu - \Omega^{-1} \frac{g^2}{4} \sum_k \frac{1}{E_k} \tanh \frac{1}{2} \beta E_k = 0 \,.$$

By using (4.7), we may express the above formula in terms of the physical excitation energy 2ν of the ϕ-quantum:

$$\nu - \mu = \Omega^{-1} \frac{g^2}{4} \sum_k \left[\frac{1}{E_k} \tanh \frac{1}{2}\, \beta\, E_k + P \frac{1}{\nu - \omega_k} \right] , \tag{6.2}$$

where P denotes the principal value, as before. The right-hand side is convergent in the ultra-violet region since the theory is renormalizable. The particle density ρ is given by $(\partial p/\partial \mu)_{T,B}$, which yields

$$\rho = 2|B|^2 + 2\Omega^{-1}\sum_k [e^{\beta(2\nu+(k^2/2M)-2\mu)} - 1]^{-1}$$

$$+\Omega^{-1}\sum_k [E_k(1+e^{-\beta E_k})]^{-1}[E_k+\mu-\omega_k+(E_k-\mu+\omega_k)e^{-\beta E_k}]. \quad (6.3)$$

From (6.2) and (6.3), μ and $|B|^2$ can be determined as functions of ρ and T. (Equation (6.2) is similar to the gap equation in the BCS theory, and Eq. (6.3) is the generalization of the density equation in the Bose-Einstein condensation.)

Regarding (6.1)-(6.3) as the zeroth approximation, one can develop a systematic expansion using $\mathcal{H}_1$ of (5.5) as the perturbation.

It is useful to introduce

$$\rho_\nu \equiv (3\pi^2)^{-1}(2m\nu)^{\frac{3}{2}}, \quad (6.4)$$

the fermionic density when the Fermi-energy equals ν, with the excitation energy of the ϕ-quantum $= 2\nu$. For $\rho << \rho_\nu$, one finds that the gap energy Δ_0 at zero temperature is related to the critical temperature T_c by, as in the BCS theory,

$$\frac{\Delta_0}{\kappa T_c} = \pi e^{-\gamma} = 1.7639 \quad (6.5)$$

where $\gamma =$ Euler's constant $= 0.5772$. For $\rho >> \rho_\nu$

$$\Delta_0^2 = (2.612)\, g^2 \left(\frac{M\kappa T_c}{2\pi}\right)^{\frac{3}{2}} \quad (6.6)$$

and (at any temperature $T < T_c$)

$$\Delta(T)^2 = g^2\,|B(T)|^2 = \Delta_0^2\left[1-\left(\frac{T}{T_c}\right)^{\frac{3}{2}}\right], \quad (6.7)$$

as in the Bose-Einstein condensation.

A detailed study of (6.2) and (6.3) shows that typical BCS and Bose-Einstein formulas can be analytically connected within one single expression. In this way, these two approaches become closely unified. The s-channel theory has an intrinsically simpler structure than the t-channel theory; this makes it possible to take a deductive approach, thereby rendering the analysis attractive on the pedagogical level.

7. COHERENCE LENGTH

Consider the case of a scalar ϕ interacting with an electron (or hole) field ψ through (1.1). Let $\vec{A}$ be the *transverse* electromagnetic field. Assume the space to be isotropic and homogeneous. Define the phase-angle variable $\theta(x)$ by

$$\phi(x) = R(x)\, e^{i\theta(x)} \tag{7.1}$$

with R and θ both hermitian. Write

$$\psi(x) = \psi'(x)\, e^{\frac{1}{2}i\theta(x)}$$

and

$$\vec{V}(x) \equiv \vec{A}(x) - (2e)^{-1}\vec{\nabla}\theta(x). \tag{7.2}$$

At very low temperature we have $R \cong B$, the long-range order parameter (chosen to be real). As shown in Ref. 7, the energy spectra for the transverse and longitudinal modes of $\vec{V}$ are (in units of $\hbar = c = 1$)

$$\omega_t(k) = (\lambda_L^{-2} + k^2)^{\frac{1}{2}} \tag{7.3}$$

and

$$\omega_\ell(k) = \left[\lambda_L^{-2} + k^2 v^2 + (k^2/2M)^2\right]^{\frac{1}{2}} \tag{7.4}$$

where k is the momentum (or wave nunber),

$$\lambda_L^{-2} = (2e\,B)^2/M \tag{7.5}$$

is the inverse square of the London length, $e^2 = 4\pi/137$, v is the "sound" velocity of the boson-fermion system and M the mass of ϕ.

Equations (7.3)-(7.4) also follow from general arguments: (i) At zero momentum $k = 0$, as in the Higgs mechanism,[13] the energies of the three spin-components of the massive vector field $\vec{V}$ become the same; i.e., they are all equal to the rest mass m_V, given by

$$m_V = \lambda_L^{-1}. \tag{7.6}$$

(ii) When $e = 0$, we have $m_V = 0$ and the transverse mode is the usual photon with $\omega_t = k$ (since the velocity of light c is 1). On the other hand, the longitudinal mode describes the Goldstone-Nambu boson[13] which, for $e = 0$, corresponds to the vibration of ϕ, propagating with the sound velocity $v << 1$ (i.e., $\omega_\ell \to kv$ as $k \to 0$). (iii) For very large k, the excitation of ϕ approaches the free boson spectrum $k^2/2M$,

$$\omega_\ell \to \frac{k^2}{2M} \qquad \text{for} \qquad k >> 2Mv \quad \text{and} \quad (2M/\lambda_L)^{\frac{1}{2}} . \tag{7.7}$$

For $e \neq 0$, the Goldstone-Nambu boson joins with the transverse photon to form a massive vector field $\vec{V}$, which leads to the above formulas for ω_ℓ and ω_t, consistent with (i)-(iii).

For the coherence length ξ, we may set $\omega_\ell(k) = 0$ and k becomes complex, which gives a boson-amplitude, say exp (ikx), that varies exponentially with distance (e.g., along the radius of a vortex filament). The decay rate in x determines ξ. From (7.4), the root

$$k \equiv i\sqrt{2}\,\mu_\pm \qquad \text{for} \qquad \omega_\ell(k) = 0 \tag{7.8}$$

satisfies

$$\mu_\pm^2 = (Mv)^2 \pm \left[(Mv)^4 - (M/\lambda_L)^2\right]^{\frac{1}{2}} . \tag{7.9}$$

The amplitude exp (ikx) becomes, then, exp $(-\sqrt{2}\,\mu_\pm x)$. To conform to the usual definition, the coherence length ξ is given by $[Re(\mu_-)]^{-1}$, which is always $\geq [Re(\mu_+)]^{-1}$.

(1) For $v^2 > (M\lambda_L)^{-1}$, μ_+ and μ_- are real and

$$\xi = 1/\mu_- . \tag{7.10}$$

(2) For $v^2 < (M\lambda_L)^{-1}$, $\mu_\pm$ are complex with

$$\mu_+ = \mu_-^* = (M/\lambda_L)^{\frac{1}{2}} e^{i\alpha} \tag{7.11}$$

where

$$\cos 2\alpha = M\,\lambda_L\,v^2 \tag{7.12}$$

and

$$\sin 2\alpha = [1 - (M\,\lambda_L\,v^2)^2]^{\frac{1}{2}} ; \tag{7.13}$$

correspondingly,

$$\xi = \sqrt{\frac{\lambda_L}{M}} \sec\alpha . \tag{7.14}$$

A complex $\mu_\pm$ implies the condensate amplitude inside a vortex filament also contains an oscillatory component, which may lead to new observational possiblities.

In the case $v^2 < (M\lambda_L)^{-1}$, according to (7.12) cos 2α varies from 0 to 1; therefore, cos α is between $\frac{1}{\sqrt{2}}$ and 1. Hence

$$\sqrt{\frac{2\lambda_L}{M}} \geq \xi \geq \sqrt{\frac{\lambda_L}{M}} . \tag{7.15}$$

(Recall that $\lambda_L^{-2} = (2eB)^2/Mc^2$. The product λ_L times the Compton wavelength $\hbar/Mc$ is independent of c, the velocity of light.) Assume a boson condensate density B^2 (at $T << T_c$) between $10^{20} - 10^{21}\text{cm}^{-3}$. On account of (7.5), $M \cong 2m_e$ and $e^2/4\pi = 1/137$, the London length is

$$\lambda_L \sim 1200\text{Å} \qquad \text{for} \qquad B^2 \sim 10^{21}\text{cm}^{-3} \tag{7.16}$$

and $\lambda_L \sim 3800\text{Å}$ for $B^2 \sim 10^{20}\text{cm}^{-3}$. Since the Compton wavelength M^{-1} is $\sim 2 \times 10^{-3}\text{Å}$, we see that in case (2), (7.14)-(7.15) give

$$\xi \sim \text{few Å} . \tag{7.17}$$

Case (1) holds only if v is larger than $(M\lambda_L)^{-\frac{1}{2}} \sim 10^{-3}$ times the velocity of light; hence, depending on v, $\xi \sim (Mv)^{-1} <$ few Å, or $\xi \sim \sqrt{2}v\lambda_L << \lambda_L$. In either case, the theory predicts a very small ξ, consistent with experimental observations. Because $\lambda_L >> \xi$, the s-channel theory gives, in general, a type II superconductor.

The s-channel theory is based on the observation that the coherence length ξ is small for all recently discovered high T_c superconductors. It is indeed satisfying that within the s-channel theory, the smallness of ξ can in turn be calculated.

8. HALL ANOMALY

The Hall number $n_H(T_c, T)$ has been extensively measured[14–20] for a variety of cupric superconductors and for various T_c and $T > T_c$. In the s-channel theory, the fermion carries charge e and the boson $2e$. Let n_f, μ_f be the density and mobility of the fermion, and n_b, μ_b be those of the boson. In a simple two-carrier model, n_H is given by

$$n_H(T_c, T) = \frac{(n_f\mu_f + 2n_b\mu_b)^2}{n_f\mu_f^2 + 2n_b\mu_b^2}. \tag{8.1}$$

From either the μsR experiment (equations (2.1) and (2.6)) or the theoretical formula (2.9), we know that as $T_c \to 0$, $n_b \to 0$ and therefore

$$n_H \rightarrow n_f. \tag{8.2}$$

In addition, since the fermions form a degenerate Fermi sea with top energy $= \nu$, let its density at $T = 0$ be n_ν:

$$n_\nu \cong \frac{m_f \nu}{\pi c} \tag{8.3}$$

where m_f is the fermion mass and c is, as before, the average spacing between neighboring CuO_2 planes. In the temperature range of interest, we may neglect the small temperature variation of the fermion density n_f and set

$$n_f \cong n_\nu = \text{constant}, \tag{8.4}$$

which is the same parameter for all cupric superconductors.

For a given sample ($La - 214$, or $Y - 123$, or $\cdots$), the total charge density (in units of e)

$$n = n_f + 2n_b \cong n_\nu + 2n_b$$

is independent of T but, because of doping, varies with T_c. Therefore,

$$n_b \cong \frac{1}{2}(n - n_\nu) = n_b(T_c) \tag{8.5}$$

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