

PRINCIPLES OF AERODYNAMICS

BY

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PREFACE

Principles of Aerodynamics is an outgrowth of six years' development of a set of printed lecture notes used by the author in undergraduate classes at the University of Washington. The purpose of the book is to present an introduction to some of the more important theoretical and practical aspects of aerodynamics.

In order to provide flexibility in teaching, many chapters have been designed as independent units. Thus, for instance, any of Chaps. 4, 7, 8, 9, 10, 12, and 15 may be omitted without destroying the continuity of the text.

While the assistance of all the scientists and engineers whose work has made this volume possible can scarcely be acknowledged separately, the author takes pleasure in thanking particularly the National Advisory Committee for Aeronautics, the Aeronautical Research Council, and the University of Washington Aeronautical Laboratories for permission to use some of the great wealth of material available in their reports. He wishes also to express his gratitude to his colleagues at the University of Washington and to Prof. F. S. Eastman, Prof. V. M. Ganzer, Prof. A. V. Hall, Dr. Alexander Klemin, and Prof. V. J. Martin for their helpful criticisms during the preparation of the manuscript.

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ABBREVIATIONS

- AAL Ames Aeronautical Laboratory, Moffett Field, Calif.
ACR Advanced Confidential Report (of the NACA)
ARC Aeronautical Research Council, England
ASME American Society of Mechanical Engineers
CAA Civil Aeronautics Administration, United States
DVL Deutsche Versuchsanstalt für Luftfahrtforschung (German Institute for Aeronautical Research), Berlin and Göttingen, Germany
FRA Fuselage reference axis
GALCIT Guggenheim Aeronautical Laboratory, California Institute of Technology
JAS Journal of the Aeronautical Sciences, United States
JRAS Journal of the Royal Aeronautical Society, England
LDT Two-Dimensional Low-Turbulence Pressure Tunnel (of the NACA)
LMAL Langley Memorial Aeronautical Laboratory, Langley Field, Va.
LTT Two-Dimensional Low-Turbulence Tunnel (of the NACA)
MIT Massachusetts Institute of Technology
NACA National Advisory Committee for Aeronautics, United States
NPL National Physical Laboratory, Teddington, England
RAE Royal Aeronautical Establishment, Farnborough, England
R&M Reports and Memoranda (of the ARC). See the Bibliography for list of *R&M* used in this text.
SAE Society of Automotive Engineers
TM Technical Memorandum (of the NACA). See the Bibliography for list of *TM* used in this text.
TN Technical Note (of the NACA). See the Bibliography for list of *TN* used in this text.
TR Technical Report (of the NACA). See the Bibliography for list of *TR* used in this text.
UWAL University of Washington Aeronautical Laboratories
VDT Variable-Density Tunnel (of the NACA)
WR Wartime Report (of the NACA). See the Bibliography for list of *WR* used in this text.

ENGLISH-LETTER NOMENCLATURE

- a* an area, sq ft or sq in.
 speed of sound, fps or mph
 slope of lift curve, $dC_L/d\alpha$, per deg (see *m*)
 factor denoting fractional increase in velocity through propeller
 disk, $b/2$, dimensionless
 equivalent flat-plate area, $D_p/1.28q$, sq ft
o acceleration, fps per sec
 area downswept by wing, $(\pi b^2/4) + bG$, sq ft
ac aerodynamic center, fraction of chord from leading edge
A cross-sectional area of stream tube, wind tunnel, jet, etc., sq ft
 propeller disk area, $\pi D^2/4$, sq ft
 aspect ratio, b^2/S , dimensionless
A_t tail aspect ratio, dimensionless
AF activity factor, $(100,000/16) \int_{0.2}^{1.0} (\tau/R)^3 (b/D) d(\tau/R)$, dimensionless
b factor denoting fractional increase in velocity behind propeller,
 $2a$, dimensionless
 wing span, ft or in.
 propeller blade width, ft or in.
bhp engine brake horsepower
B number of blades of propeller
c a constant
 chord, ft or in.
 specific fuel consumption, lb per bhp-hr (see *w*)
c_a section drag coefficient, D/qS , positive aft, dimensionless
c_l section lift coefficient, L/qS , positive up, dimensionless
c_m section pitching moment coefficient, M/qSc , positive for climbing
 moment, dimensionless
c_r wing-root chord, ft or in.
c_t wing-tip chord, ft or in.
cp center of pressure, positive aft of leading edge, fraction of chord
cg center of gravity, frequently expressed as fraction of chord from
 leading edge
C a constant
 chordwise force, positive aft, lb
C degrees centigrade

- C_D drag coefficient of wing or airplane, D/qS , positive aft, dimensionless
- C_{D_i} induced drag coefficient, $C_L^2/\pi A$, dimensionless
- C_{D_0} profile drag coefficient = wing drag coefficient for infinite aspect ratio, $C_D - C_{D_i}$, dimensionless
- C_{D_p} parasite drag coefficient, $C_{D_0} + C_{D_r}$ or $C_D - C_{D_i}$, dimensionless
- $C_{D_{pe}}$ "effective" parasite drag coefficient at $C_L = 0$ for representative parabolic polar curve, dimensionless
- C_{D_r} residual drag coefficient, $C_{D_p} - C_{D_0}$, dimensionless
- C_{D_r} proper drag coefficient, dimensionless
- C_F general force coefficient, F/qA , dimensionless
- C_h rate of climb $(P_a - P_r)(33,000/W)$, fpm
- C_H hinge moment coefficient, H/qSc , where Sc is based on surface aft of hinge line, positive when an applied aerodynamic torque on the hinge tends to increase the deflection of a positively deflected flap, dimensionless
- C_l rolling moment coefficient, L/qSb , positive clockwise viewed from rear, dimensionless
- C_L lift coefficient, L/qS , positive up, dimensionless
- C_m pitching moment coefficient, M/qSc , positive for climbing moment, dimensionless
- C_n yawing moment coefficient, N/qSb , positive clockwise viewed from above, dimensionless
- C_N normal coefficient, N/qS , positive up, dimensionless
- C_p pressure coefficient, $\Delta P/q$, dimensionless
- C_{p_0} stagnation pressure coefficient, dimensionless
- C_P power coefficient, $p/\rho n^3 D^5$, or $10^{11}P/2\sigma N^3 D^5$, dimensionless
- C_s speed-power coefficient, $\rho v/\sqrt[5]{pn^2}$, or $0.638V\sigma/\sqrt[5]{PN^2}$, dimensionless
- C_T thrust coefficient, $T/\rho n^2 D^4$, or $10^7 T/6.61\sigma N^2 D^4$, dimensionless
- C_Y side force coefficient, Y/Sq , positive right viewed from rear, dimensionless
- d diameter, ft or in.
- D diameter (propeller, sphere, cylinder, etc.), ft or in.
drag, $C_D S q$, lb (see coefficient definitions above for subscript definitions)
- e Oswald's airplane efficiency factor, $\frac{dC_{D_i}/dC_L^2}{dC_D/dC_L^2}$, dimensionless
- Naperian logarithm base
- e_w wing efficiency factor corresponding to e for an airplane, dimensionless
- E modulus of elasticity (solid or fluid), psi or psf
- endurance, hr

- f equivalent parasite area, D_p/q , sq ft (in performance equations, parasite drag is that at zero lift)
- fhp friction horsepower
- F degrees Fahrenheit
- F force, lb
- f friction force, lb
- T torque force, lb
- F_n tip-speed correction factor for propeller efficiency, dimensionless
- FRA fuselage reference axis
- g acceleration of gravity, 32.2 fps per sec
- G biplane gap, ft
- G Froude Number, v^2/lq , dimensionless
- h height, ft or in.
- h altitude, ft
- H absolute ceiling, ft
- H total pressure, psf
- H hinge moment, $C_H q S c$, lb-in. (see definition of C_H)
- H_s service ceiling, ft
- i_w wing incidence angle usually measured from fuselage reference axis, deg
- i_s stabilizer angle usually measured from either fuselage reference axis or wing chord, deg
- ihp indicated horsepower, $bhp + fhp$
- J propeller advance ratio, v/nD or $88V/ND$, dimensionless
- k constant
- K constant
- K control-surface effectiveness factor, $(\partial\alpha/\partial\delta)C_N$, dimensionless
- l length, ft or in.
- l_t tail length measured from airplane center of gravity to tail aerodynamic center, ft or in.
- l_w wing loading, W/S , psf
- L length, ft or in.
- L lift, $C_L S q$, positive up, lb
- L rolling moment, $C_l q S b$, positive clockwise viewed from rear, lb-in.
- m mass, W/g , slug
- m slope of lift curve, $dC_L/d\alpha$, per radian (see a)
- M mass flow of fluid, slug per sec
- M Mach number, v/a , dimensionless
- M pitching moment, $C_m q S c$, positive for climbing moment, lb-in.
- M_{cr} critical Mach number, dimensionless
- n revolutions per sec
- n distance from leading edge, positive rearward, fraction of chord
- N revolutions per minute

- N yawing moment, $C_n q S b$, positive clockwise when viewed from above, lb-in.
 normal force $C_N q S$, positive up, lb
- p power, ft-lb per sec
 static pressure, psf
- p_s stagnation pressure, psf
- P engine bhp
- P_a thrust horsepower available
- P_{i_w} P_a or P_r , adjusted for altitude and weight, hp
- P_r airplane horsepower required
- q dynamic pressure, $\frac{1}{2} \rho v^2$; or $V^2 \sigma / 391$, psf
- Q torque, lb-ft
- r radius, ft or in.
- R radius, ft or in.
 rolling moment, $C_l q S b$, positive clockwise viewed from rear, lb-in.
 range, miles
 resultant force, lb
 test Reynolds number; also frequently used for effective Reynolds number R_e , $\rho v l / \mu$, dimensionless (l is arbitrary characteristic length that is chosen as chord for airfoil and diameter for cylinder, sphere, etc.)
 resistance force during take-off, $F + D$, lb
- R_e effective Reynolds number, simulating free air conditions, $(R)(TF)$, dimensionless
- s distance, ft or in.
 distance along the surface of a body, commonly measured either from front stagnation point or from leading edge of chord, ft or in.
- sg specific gravity, dimensionless
- S wing area, sq ft
- S_f wing area defined by flap span, sq ft
- S_r proper area (maximum projected frontal area), sq ft
- t thickness, ft, in., or fraction of chord
 time, sec, min, or hr
 temperature, °F (see T)
- thp thrust horsepower, $Tv/550$ or $TV/375$
- T thrust, $C_T \rho n^2 D^4$ or $(6.61 \times 10^{-7}) C_T \sigma N^2 D^4$, lb
 time, sec, min, or hr
 minimum time to climb, min
 temperature, $459.4 + t$, °F abs, (see t)
- T_c thrust coefficient, $T/\rho v^2 D^2$, dimensionless
- TF turbulence factor, R_e/R , dimensionless
- u velocity within boundary layer, fps

- U** velocity just outside boundary layer, fps
 v velocity, fps
 true air speed, fps (see V)
 v_i indicated velocity, $v\sqrt{\sigma}$, fps
 v_l landing velocity, fps
 v_o remote velocity, fps
 v_s slip-stream velocity, $v(1 + b)$, fps
 stalling speed, fps
 v_t take-off velocity, fps
 tangential velocity, fps
 v_w sinking speed, fps
 v_w wind speed, fps
 V_{t_w} air speed adjusted for weight and altitude, mph
 V velocity, mph (see v for subscript definitions)
 true air speed, mph (see v)
 volume, cu ft
 specific volume, cu ft per lb
 w density, lb per cu ft
 down-wash velocity, fps
 velocity introduced by circulation, fps
 width, ft or in.
 fuel consumption, lb fuel per hr per lb thrust (see c)
 W weight, lb
 width, ft or in.
 W_a air flow, lb per sec
 W_f fuel flow, lb per hr
 x distance from aerodynamic center measured parallel to zero-lift
 chord, positive aft, dimensionless
 horizontal distance, ft or in.
 y distance perpendicular to surface for boundary-layer measure-
 ment, ft or in.
 Y side force, $C_Y q S$, positive right when viewed from rear, lb
 z distance perpendicular to zero-lift chord, positive up, dimen-
 sionless

GREEK-LETTER NOMENCLATURE

- alpha α angle of attack measured between chord line and remote velocity, positive for trailing edge down, deg or radians
- α_i induced angle of attack, $C_L/\pi A$, radians, or $18.24C_L/A$, deg
- α_0 angle of attack for infinite aspect ratio, deg or radians
- α_{l_0} section angle of zero lift, deg or radians
- α_{L_0} angle of zero lift, deg or radians
- beta β propeller-blade angle between plane of rotation and chord line, deg (usually measured at three-quarter radius)
- deflection angle of streamlines in passing through a shock wave, deg
- gamma γ ratio of specific heats (1.4 for dry air), dimensionless
- angle between airfoil resultant and lift, deg
- Gamma Γ dihedral angle, positive for wing tips up, deg
- circulation, sq ft per sec
- delta δ control-surface deflection, positive deflection causes positive lift or side force, deg (subscript designates surface considered; *i.e.*, δ_e for elevator, δ_a for aileron, etc.)
- induced drag correction factor for taper and aspect ratio, dimensionless
- boundary-layer thickness defined where 99 per cent of local velocity is attained, ft or in.
- Delta Δ prefix meaning "increment;" *e.g.*, ΔP means "pressure increment"
- epsilon ϵ down-wash angle, deg or radians
- exchange coefficient in turbulent flow, slug per ft-sec
- eta η propeller efficiency, dimensionless
- η_t tail efficiency factor, proportional to the ratio of dynamic pressure at tail to the remote dynamic pressure, dimensionless
- theta θ an angle, deg or radians
- angle of pitch, positive for climbing moment, deg
- glide angle, deg
- climb angle, deg
- shock wave angle, deg
- lambda λ wing-taper ratio, tip chord/root chord, dimensionless
- Lambda Λ sweepback angle, deg

- mu μ coefficient of viscosity, $\frac{\tau_L}{dv/dy}$, slug per ft-sec
 coefficient of friction, dimensionless
 Mach angle, deg
- nu ν kinematic coefficient of viscosity, μ/ρ , sq ft per sec
 frequency, per sec
- xi ξ camber, fraction of chord, dimensionless
- pi π a constant, 3.14
- rho ρ mass density, w/g , slug per cu ft
- sigma σ ratio of any mass density to standard sea-level mass density
 of air, ρ/ρ_0 , dimensionless
- tau τ shearing stress, subscripts L and T refer to laminar and
 turbulent, respectively, psf
 induced angle of attack correction for taper and aspect ratio,
 dimensionless
- phi ϕ propeller helix angle, $\beta - \alpha$, deg
 angle of roll, positive clockwise viewed from rear, deg
- psi ψ angle of yaw, positive clockwise viewed from above, deg

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CHAPTER 1

INTRODUCTION

Aerodynamics is a branch of fluid mechanics that deals with the particular fluid, air. The laws that govern fluid mechanics are fundamentally identical to those used in the usual mechanical studies, but they frequently appear in an unfamiliar or extended form, giving the impression of novelty. In this introductory chapter, some of the general laws of fluid mechanics will be discussed, with particular reference to applications in aerodynamics. It is thought advisable to generalize the discussion wherever possible, to prevent creating the impression that the relations are peculiar to aerodynamics.

1.1 Newton's Laws of Motion. From an engineering standpoint, the most important laws governing both solid and fluid mechanics are those encompassed by the laws¹ of Sir Isaac Newton:

1. Every body continues in a state of rest or uniform motion in a straight line unless acted upon by some external force.
2. An acceleration, which is proportional to the force, will be produced in the direction of the force.
3. Action and reaction are equal and opposite.

The meaning of the first and third laws is evident from physical experience. The implications contained in the second law are not apparent without study. According to the statement of the law,

$$F \propto a \quad (1.1)$$

where F is force

a is acceleration

There is no statement as to the size of the acceleration, but only that one will be produced if there is an unbalanced force on the body. In the form of an equality,

$$F = ma \quad (1.2)$$

where m is a constant called *mass*. Mass is a physical character-

¹ The laws are not stated exactly as originally proposed. They are reworded, without reservation, for simplicity of presentation.

istic of the body that may be regarded as a measure of its material content or "how much matter" it has. By this is meant that the mass of a body is constant if no molecules are added to or subtracted from the body, even though it may undergo changes in state. This definition does not describe all the properties of mass, but has an advantage of giving to it a degree of tangibility.

Mass may be considered from another viewpoint. Let a stationary body be suspended in a vacuum from an infinite string. If a horizontal force is applied for a short time and is then removed, the body attains a constant linear velocity in accordance with Newton's first law. If the force is applied continuously, the velocity increases linearly with time; hence the acceleration is constant. Now let the experiment be repeated on half the body (half the mass). With the same force, the resultant acceleration must be double, in accordance with Newton's second law. Therefore, mass may be regarded as a measure of the *inertia*, or "resistance to acceleration," of a body.

The character of mass has been described; its magnitude will next be discussed. One of the most elementary of algebraic processes is that involved in determining a constant of proportionality in an equation such as

$$x = ky \quad (1.3)$$

where x and y are variables and k is a constant. The procedure is to substitute any known value of x and its corresponding value of y . For instance, if $x = 6$ when $y = 3$, substitution shows that $k = 2$. This constant, once determined, is fixed for all values of x and y within the meaning of the equation. Similarly, Eq. (1.2) may be regarded as an equation involving two variables, F and a , and a constant, m . If *any* force and its corresponding acceleration are known for the body, the constant of proportionality, m , may be found in a manner similar to that described above. One such combination may be chosen if the body is allowed to fall freely in a vacuum, where the force is the weight and the acceleration is that due to gravity, thus

$$m = \frac{W}{g} \quad (1.4)$$

The *magnitude* of mass may be obtained from Eq. (1.4) by use of the standard acceleration of gravity, 32.2 fps per sec.

Mass is a scalar quantity, while weight is a vector quantity. The relation between them is seen to arise only from experimental convenience.

If W is the weight in pounds and g is the acceleration in feet per second per second, the dimensions of mass may be obtained from Eq. (1.4):

$$m \approx \frac{\text{lb-sec}^2}{\text{ft}} \quad (1.5)$$

This unwieldy combination of terms is combined to form the engineering unit of mass called a *slug*; hence,

$$1 \text{ lb force} = (1 \text{ slug mass}) (1 \text{ fps per sec acceleration})$$

Equation (1.2) may be written in several forms dependent upon convenience. For instance,

$$\begin{aligned} F &= ma = m \frac{dv}{dt} \\ F dt &= m dv \end{aligned} \quad (1.6)$$

If F is independent of time and m is independent of velocity, integration of Eq. (1.6) gives

$$Ft = m(v_f - v_i) \quad (1.7)$$

$$= m \Delta v \quad (1.8)$$

$$= mv_f - mv_i \quad (1.9)$$

$$F = \frac{m}{t} (v_f - v_i) \quad (1.10)$$

$$F = M \Delta v \quad (1.11)$$

where F is force, lb

m is mass, slug

a is acceleration, fps per sec

v is velocity, with subscript f indicating final and subscript i indicating initial, fps

t is time, sec

Ft is impulse, lb-sec

mv is momentum, slug-ft per sec

M is mass per unit time, slug per sec

Δv is change in velocity, fps