# Wavelet Applications in Signal & Image Processing VIII

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Part One of Two Parts

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# *Wavelet Applications in Signal and Image Processing VIII*

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*Chairs/Editors*

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Invited Paper

# Curvelets, Multiresolution Representation, and Scaling Laws

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## ABSTRACT

Curvelets provide a new multiresolution representation with several features that set them apart from existing representations such as wavelets, multiwavelets, steerable pyramids, and so on. They are based on an anisotropic notion of scaling. The frame elements exhibit very high direction sensitivity and are highly anisotropic. In this paper we describe these properties and indicate why they may be important for both theory and applications.

**Keywords:** Edges. Partitioning. Subband Filtering. Local Fourier Transform. Ridge functions. Ridgelets. Multiscale ridgelets. Pyramids.

## 1. INTRODUCTION

Images have edges. Viewing an image as a mathematical object, one might think of an image as an otherwise smooth function with discontinuities along curves, a description which would not be suitable, however, for all kinds of images. Indeed, images of natural scenes are more than just smooth luminance surfaces separated by step discontinuities; for instance, we do not make any claims about the representation of textures, which are typically unsmooth. Our intention is rather to describe a class of images where edges are clearly the dominating features, e.g. cartoons or geometric images.

### 1.1. Our Viewpoint: Harmonic Analysis

The curvelet construction[1,2] was originally developed for providing efficient representations of smooth objects with discontinuities along curves; the underlying motivation being to apply this construction to classical image processing problems such as:

- *Data compression.* Compression of digitally acquired image data.
- *Image restoration, image reconstruction or edge-preserving regularization.* Enhancement (noise removal) of digital images possibly obtained via indirect measurements as in tomography.[3] One of the main challenges here is to develop smoothing or reconstruction techniques that would smooth out the 'flat' part of the image without blurring the edges.

Our viewpoint is that of modern harmonic analysis whose aim is to develop new representation systems. That is, one is searching for new collection of templates or "elementary forms" that will serve for both the analysis and the synthesis of an object under study. The most classical example is, of course, that of

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Fourier series where an object, in $L_2[0,1]^2$ is expanded out in a superposition of sinusoids according to the rule

$$f(x) = \sum_{n \in \mathbb{Z}^2} c_n(f) e^{i2\pi n \cdot x}, \quad c_n(f) = \int f(x) e^{-i2\pi n \cdot x}\, dx,$$

where $n$ indexes the doubles $(n_1, n_2) \in \mathbb{Z}^2$. The Fourier transform offers an alternate viewpoint to the spatial description of the signal and opens up the possibility of developing new schemes based on the processing of Fourier coefficients. This approach is powerful and in image processing, the two-dimensional Fourier transform and analogs are at the core of a countless number of algorithms. For instance, blocking a digital image into disjoint arrays of 8 by 8 pixels and coding the coefficients obtained after applying the Discrete Cosine Transform to those arrays used to be the of basis of JPEG, the former image compression standard.

In the last decades, many representation systems have been introduced as alternatives to the 'classical' Fourier representation; among those, the Gabor system (time-frequency viewpoint) and wavelets (time-scale viewpoint) probably occupy a prominent place. Wavelets have had a wide impact, both in theory and in practice, and especially in the our areas of preoccupation, namely, data compression and signal restoration. The shrinking of wavelet coefficients, originating in the work of Donoho and Johnstone,[4] proved to be a very powerful tool for statistical estimation, from both a theoretical and a practical standpoint. Similarly, wavelet based-coders have found wide applications in various data compression applications and have been included in JPEG-2000, the newly developed still picture compression standard.

### 1.2. Our Grail

Within this framework, we aim at finding a representation that is 'optimal' for representing objects with discontinuities along curves. We need, however, to explain what we mean by 'optimal' representation.

First, the concept of representation is classical in harmonic analysis and consists of finding a system $(f_\mu)$ and a rule such that any function can be represented as follows:

$$f = \sum_\mu c_\mu(f) f_\mu; \tag{1}$$

classically, one would require that

1. the coefficients $c_\mu$ are bounded linear functionals of $f$, and
2. the decomposition is stable in the sense that one has a quasi-Parseval relation

$$\sum_\mu |c_\mu(f)|^2 \sim \|f\|_{L_2}^2.$$

Second, by 'optimal' representation for a class of objects, we mean that the coefficients of the objects in question are as sparse as possible. In nontechnical terms, this means that, when reordered by decreasing amplitude, the coefficient sequence decays as rapidly as possible.

## 1.3. Three Anomalies

About twenty years ago, Burt and Adelson introduced the Laplacian pyramid, a 'revolutionary' idea which triggered a series of completely new algorithms in numerical image processing. Canonical pyramid ideas are perhaps known today under the name of wavelets, although a few of these ideas were introduced before the name 'wavelet' was coined. In that sense, we follow Meyer[5] in viewing wavelet theory as a unifying mathematical language for describing a set of connected ideas that arose in different areas.

Although we recognize the importance and the wide impact of these ideas, we feel, nevertheless, that these canonical pyramid ideas are part of a larger picture; we intend to make clear how much larger the question is than just the portion which wavelets represent. In addition, we also believe that a lot of claims regarding the applicability of wavelets to image processing problems such as those mentioned above have been perhaps overstated.

1. *Inefficient Representations.* From a theoretical viewpoint, wavelet series are not optimal for representing objects with discontinuities along curves. Although the next section develops a heuristic argument of the reason, we do not dwell further on this issue and simply refer the reader to existing literature on the subject, see[6] for example.

2. *Limitations of Existing Pyramid Schemes.* Traditional pyramids have only a fixed number of directional elements, independent of scale.

3. *Limitations of Existing Scaling Concepts.* Traditional pyramids do not have highly anisotropic elements.

The point of this paper is to show that it is possible to build a pyramid that is very different that (2)-(3). In fact, the paper will exhibit a pyramid with scaling properties that are very different.

## 1.4. Outline and Claims

In a recent article,[1] the authors introduced a new system, namely, the curvelet frame which turns out to be very different from existing ideas. Curvelets provide a new multiresolution representation that set them apart from existing representations such as wavelets. The scope of this paper is to review the construction of curvelets and to detail some of their properties. More precisely, our intention is to develop the following claims:

- *Near-optimal representation.* Curvelets provide optimally sparse representation of otherwise smooth objects.
- *Different from existing ideas.* The curvelet transform corresponds to a new way of processing data, unlike any in practice.
- Curvelets have an interesting structure vis-a-vis
  - Scale Space,
  - Pyramids,
  - Non-quadratic image smoothing.

A series of recent articles[1,3,2] addressed the first of these three points and, therefore, we shall here be especially concerned with the other two.