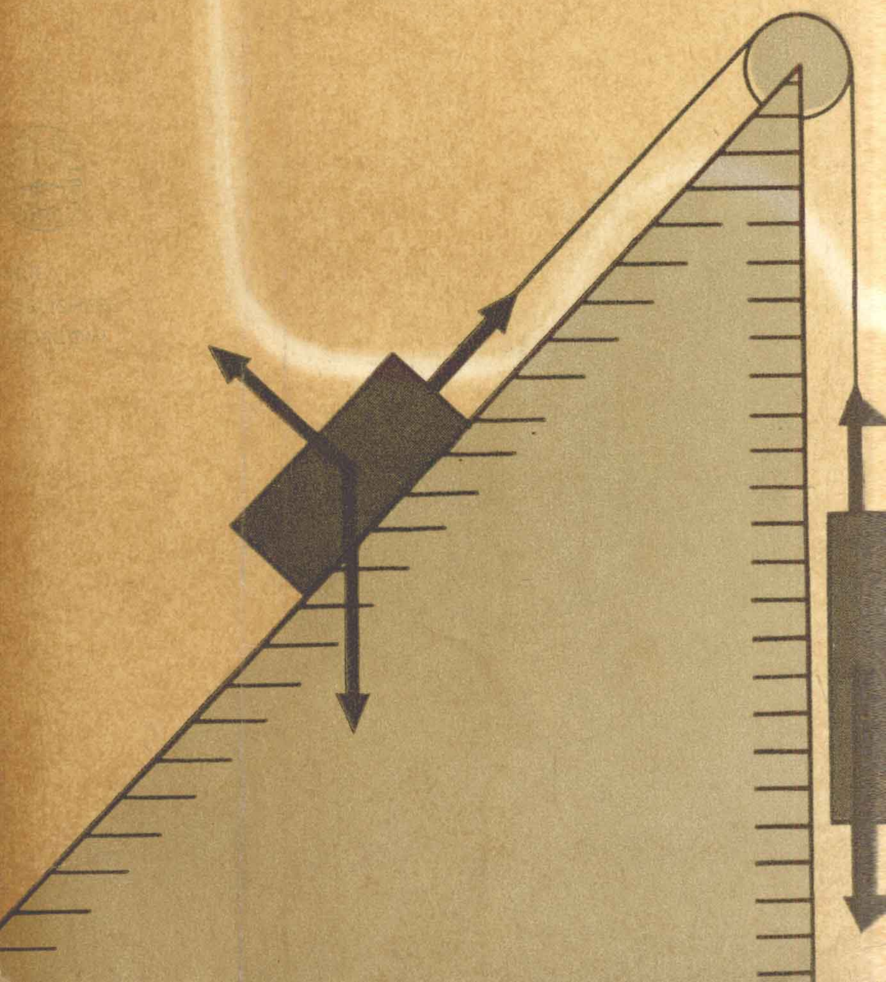
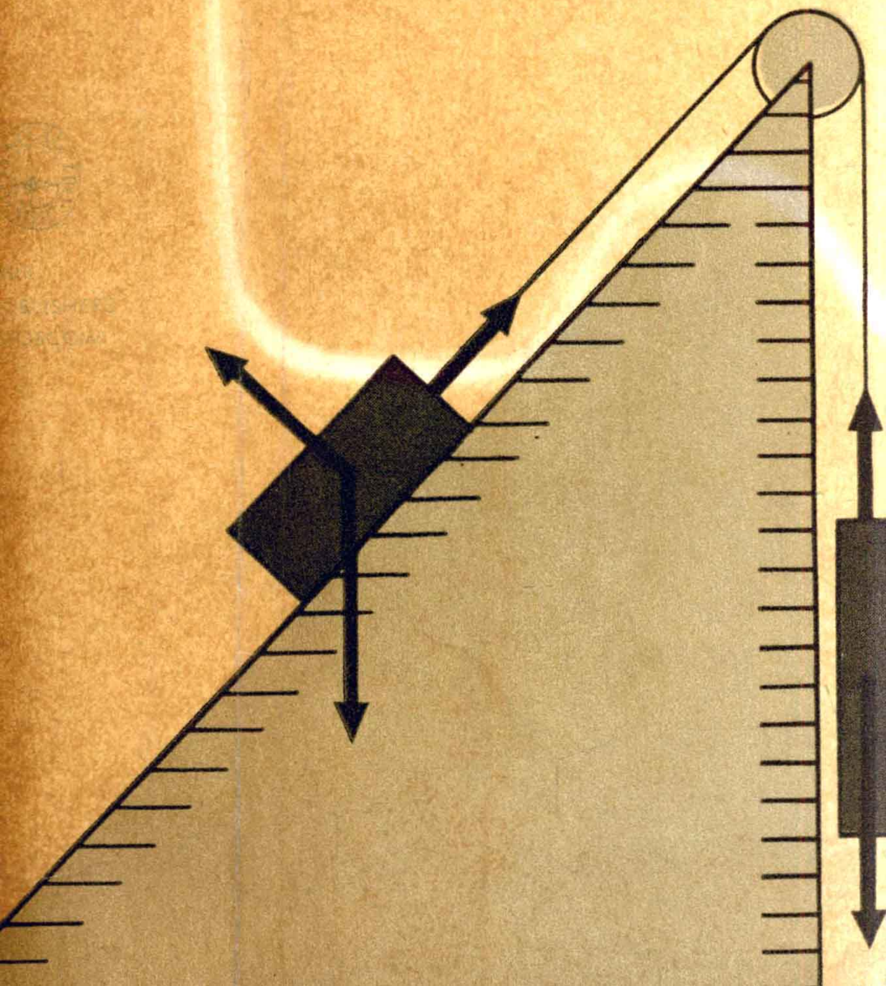
FUNDAMENTAL LAWS OF MECHANICS

I.E. IRODOV



FUNDAMENTAL LAWS OF MECHANICS

I.E. IRODOV



Igor Evgenyevich Irodov, Candidate of Sciences (Physics and Mathematics), Professor of General Physics, has published over 100 scientific papers and books, among which are several manuals: *The Fundamental Laws of Mechanics, Problems of General Physics, A Laboratory Course on Optics.*

His *Problem Book on Atomic and Nuclear Physics* appeared in six Russian editions, and was published in Great Britain, USA, Romania, and twice in Poland.

A Problem Book on General Physics (with I. V. Savelyev and O. I. Zamsha as co-authors) was printed three times in Russian and published in Poland. MIR Publishers have translated in into French; its publication in Arabic and Vietnamese is expected.

I.E. IRODOV

FUNDAMENTAL LAWS OF MECHANICS

Translated from the Russian
by YURI ATANOV

Mir
Publishers
Moscow

First published 1980
Revised from the 1978 Russian edition

На английском языке

CONTENTS

PART TWO
RELATIVISTIC MECHANICS

PREFACE

The objective of the book is to draw the readers' attention to the *basic* laws of mechanics, that is, to the laws of motion and to the laws of conservation of energy, momentum and angular momentum, as well as to show *how* these laws are to be applied in solving various specific problems. At the same time the author has excluded all things of minor importance in order to concentrate on the questions which are the hardest to comprehend.

The book consists of two parts: (1) classical mechanics and (2) relativistic mechanics. In the first part the laws of mechanics are treated in the Newtonian approximation, i.e. when motion velocities are much less than the velocity of light, while in the second part of the book velocities comparable to that of light are considered.

Each chapter opens with a theoretical essay followed by a number of the most instructive and interesting examples and problems, with solutions provided. There are about 80 problems altogether; being closely associated with the introductory text, they develop and supplement it and therefore their examination is of equal importance.

A few corrections and refinements have been made in the present edition to stress the physical essence of the problems studied. This holds true primarily for Newton's second law and the conservation laws. Some new examples and problems have been provided.

The book is intended for first-year students of physics but can also be useful to senior students and lecturers.

I. E. Irodov

И. Е. Иродов

ОСНОВНЫЕ ЗАКОНЫ МЕХАНИКИ

Издательство «Высшая школа»
Москва

NOTATION

Vectors are designated by roman bold-face type (e.g. $\mathbf{r}$, $\mathbf{F}$); the same italicized letters (r, F) designate the norm of a vector.

Mean values are indicated by crotchets $\langle\ \rangle$, e.g. $\langle \mathbf{v} \rangle$, $\langle N \rangle$.

The symbols Δ, d, δ (when put in front of a quantity) signify:

Δ, a finite increment of a quantity, i.e. a difference between its final and initial values, e.g. $\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, $\Delta U = U_2 - U_1$;
d, a differential (an infinitesimal increment), e.g. $d\mathbf{r}$, dU;
δ, an elementary value of a quantity, e.g. δA is an elementary work.

Unit vectors:

$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors of the Cartesian coordinates x, y, z;

$\mathbf{e}_\rho$, $\mathbf{e}_\varphi$, $\mathbf{e}_z$ are unit vectors of the cylindrical coordinates ρ, φ, z;

$\mathbf{n}$, $\boldsymbol{\tau}$ are unit vectors of a normal and a tangent to a path.

Reference frames are denoted by the italic letters K, K' and C.

The C frame is a reference frame fixed to the centre of inertia and translating relative to inertial frames. All quantities in the C frame are marked with a tilde, e.g. $\tilde{p}$, $\tilde{E}$.

A, work,
c, velocity of light *in vacuo*,
E, total mechanical energy, the total energy,
$\mathbf{E}$, electric field strength,
e, elementary electric charge,
$\mathbf{F}$, force,
$\mathbf{G}$, field strength,
$\mathbf{g}$, free fall acceleration,
I, moment of inertia,
$\mathbf{L}$, angular momentum with respect to a point,
L_z, angular momentum with respect to an axis,

l, arc coordinate, the arm of a vector,
M, moment of a force with respect to a point,
M_z, moment of a force with respect to an axis,
m, mass, relativistic mass, m_0 rest mass,
N, power,
p, momentum,
q, electric charge,
r, radius vector,
s, path, interval,
t, time,
T, kinetic energy,
U, potential energy,
v, velocity of a point or a particle,
w, acceleration of a point or a particle,
$\boldsymbol{\beta}$, angular acceleration,
β, velocity expressed in units of the velocity of light,
γ, gravitational constant, the Lorentz factor,
ε, energy of a photon,
$\varkappa$, elastic (quasi-elastic) force constant,
μ, reduced mass,
ρ, curvature radius, radius vector of the shortest distance to an axis, density,
φ, azimuth angle, potential,
$\boldsymbol{\omega}$, angular velocity,
Ω, solid angle.

INTRODUCTION

Mechanics is a branch of physics treating the simplest form of motion of matter, mechanical motion, that is, the motion of bodies in space and time. The occurrence of mechanical phenomena in space and time can be seen in any mechanical law involving, explicitly or implicitly, space-time relations, i.e. distances and time intervals.

The position of a body in space can be determined only with respect to other bodies. The same is true for the motion of a body, i.e. for the change in its position over time. The body (or the system of mutually immobile bodies) serving to define the position of a particular body is identified as the reference body.

For practical purposes, a certain coordinate system, e.g. the Cartesian system, is fixed to the reference body whenever motion is described. The coordinates of a body permit its position in space to be established. Next, motion occurs not only in space but also in time, and therefore the description of the motion presupposes time measurements as well. This is done by means of a clock of one or another type.

A reference body to which coordinates are fixed and mutually synchronized clocks form the so-called *reference frame*. The notion of a reference frame is fundamental in physics. A space-time description of motion based on distances and time intervals is possible only when a definite reference frame is chosen.

Space and time by themselves are also *physical* objects, just as any others, even though immeasurably more important. The properties of space and time can be investigated by observing bodies moving in them. By studying the character of the motion of bodies we determine the properties of space and time.

Experience shows that as long as the velocities of bodies are small in comparison with the velocity of light, linear scales and time intervals remain *invariable* on transition from one reference frame to another, i.e. they do not depend on the choice of a reference frame. This fact finds expression in the Newtonian concepts of absolute space and time. Mechanics treating the motion of bodies in such cases is referred to as classical.

When we pass to velocities comparable to that of light, it becomes obvious that the character of the motion of bodies changes radically. Linear scales and time intervals become *dependent* on the choice of a reference frame and are different in different reference frames. Mechanics based on these concepts is referred to as *relativistic*. Naturally, relativistic mechanics is more general and becomes classical in the case of small velocities.

The actual motion characteristics of bodies are so complex that to investigate them we have to neglect all insignificant factors, otherwise the problem would get so complicated as to render it practically insoluble. For this purpose notions (or abstractions) are employed whose application depends on the specific nature of the problem in question and on the accuracy of the result that we expect to get. A particularly important role is played by the notions of a mass point and of a perfectly rigid body.

A *mass point*, or, briefly, a *particle*, is a body whose dimensions can be neglected under the conditions of a given problem. It is clear that the same body can be treated as a mass point in some cases and as an extended object in others.

A *perfectly rigid body*, or, briefly, a *solid*, is a system of mass points separated by distances which do not vary during its motion. A real body can be treated as a perfectly rigid one provided its deformations are negligible under the conditions of the problem considered.

Mechanics tackles two fundamental problems:

1. The investigation of various motions and the generalization of the results obtained in the form of laws of mo-

tion, i.e. laws that can be employed in predicting the character of motion in each specific case.

2. The search for general properties that are typical of any system regardless of the specific interactions between the bodies of the system.

The solution of the first problem ended up with the so-called dynamic laws established by Newton and Einstein, while the solution of the second problem resulted in the discovery of the laws of conservation for such fundamental quantities as energy, momentum and angular momentum.

The dynamic laws and the laws of conservation of energy, momentum and angular momentum represent the basic laws of mechanics. The investigation of these laws constitutes the subject matter of this book.

PART ONE

CLASSICAL MECHANICS

CHAPTER 1

ESSENTIALS OF KINEMATICS

Kinematics is the subdivision of mechanics treating ways of describing motion regardless of the causes inducing it. Three problems will be considered in this chapter: kinematics of a point, kinematics of a solid, and the transformation of velocity and acceleration on transition from one reference frame to another.

§ 1.1. Kinematics of a Point

There are three ways to describe the motion of a point: the first employs vectors, the second coordinates, and the third is referred to as natural. Let us examine them in order.

The vector method. With this method the location of a given point A is defined by a radius vector $\mathbf{r}$ drawn from a certain stationary point O of a chosen reference frame to that point A. The motion of the point A makes its radius vector vary in the general case both in magnitude and in direction, i.e. the radius vector $\mathbf{r}$ depends on time t. The locus of the end points of the radius vector $\mathbf{r}$ is referred to as the *path* of the point A.

Let us introduce the notion of the *velocity* of a point. Suppose the point A travels from point *1* to point *2* in the time interval Δt (Fig. 1). It is seen from the figure that the *displacement vector* $\Delta\mathbf{r}$ of the point A represents the increment of the radius vector $\mathbf{r}$ in the time Δt: $\Delta\mathbf{r} = \mathbf{r}_2 -$

$-\mathbf{r}_1$. The ratio $\Delta\mathbf{r}/\Delta t$ is called the *mean velocity vector* $\langle\mathbf{v}\rangle$ during the time interval Δt. The direction of the vector $\langle\mathbf{v}\rangle$ coincides with that of $\Delta\mathbf{r}$. Now let us define the velocity vector $\mathbf{v}$ of the point at a given moment of time as the limit of the ratio $\Delta\mathbf{r}/\Delta t$ as $\Delta t \to 0$, i.e.

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}. \quad (1.1)$$

This means that the velocity vector $\mathbf{v}$ of the point at a given moment of time is equal to the derivative of the radius vector $\mathbf{r}$ with respect to time, and its direction, like that of

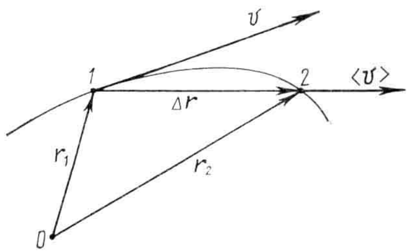


Fig. 1

the vector $d\mathbf{r}$, along the tangent to the path at a given point coincides with the direction of motion of the point A. The modulus of the vector $\mathbf{v}$ is equal to*

$$v = |\mathbf{v}| = |d\mathbf{r}/dt|.$$

The motion of a point is also characterized by *acceleration*. The acceleration vector $\mathbf{w}$ defines the rate at which the velocity vector of a point varies with time:

$$\mathbf{w} = d\mathbf{v}/dt, \quad (1.2)$$

i.e. it is equal to the derivative of the velocity vector with respect to time. The direction of the vector $\mathbf{w}$ coincides with the direction of the vector $d\mathbf{v}$ which is the increment of the vector $\mathbf{v}$ during the time interval dt. The modulus of the

* Note that in the general case $|d\mathbf{r}| \neq dr$, where r is the modulus of the radius vector $\mathbf{r}$, and $v \neq dr/dt$. For example, when $\mathbf{r}$ changes only in direction, that is the point moves in a circle, then $r = \text{const}$, $dr = 0$, but $|d\mathbf{r}| \neq 0$.

vector **w** is defined in much the same way as that of the vector **v**.

Example. The radius vector of a point depends on time t as

$$\mathbf{r} = \mathbf{a}t + \mathbf{b}t^2/2,$$

where **a** and **b** are constant vectors. Let us find the velocity **v** of the point and its acceleration **w**:

$$\mathbf{v} = d\mathbf{r}/dt = \mathbf{a} + \mathbf{b}t, \quad \mathbf{w} = d\mathbf{v}/dt = \mathbf{b} = \text{const}.$$

The modulus of the velocity vector

$$v = \sqrt{\mathbf{v}^2} = \sqrt{a^2 + 2\mathbf{ab}t + b^2t^2}.$$

Thus, knowing the function $\mathbf{r}(t)$, one can find the velocity **v** of a point and its acceleration **w** at any moment of time.

Here the reverse problem arises: can we find $\mathbf{v}(t)$ and $\mathbf{r}(t)$ if the time dependence of the acceleration $\mathbf{w}(t)$ is known?

It turns out that the dependence $\mathbf{w}(t)$ is not sufficient to get a single-valued solution of this problem; one needs also to know the so-called *initial conditions*, namely, the velocity $\mathbf{v}_0$ of the point and its radius vector $\mathbf{r}_0$ at a certain initial moment $t = 0$. To make sure, let us examine the simple case when the acceleration of the point remains constant in the course of time.

First, let us determine the velocity $\mathbf{v}(t)$ of the point. In accordance with Eq. (1.2) the elementary velocity increment during the time interval dt is equal to $d\mathbf{v} = \mathbf{w}\,dt$. Integrating this relation with respect to time between $t = 0$ and t, we obtain the velocity vector increment during this interval:

$$\Delta\mathbf{v} = \int_0^t \mathbf{w}\,dt = \mathbf{w}t.$$

However, the quantity $\Delta\mathbf{v}$ is not the required velocity **v**. To find **v**, we must know the velocity $\mathbf{v}_0$ at the initial moment of time. Then $\mathbf{v} = \mathbf{v}_0 + \Delta\mathbf{v}$, or

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{w}t.$$

The radius vector $\mathbf{r}(t)$ of the point is found in a similar manner. According to Eq. (1.1) the elementary increment of the radius vector during the time interval dt is $d\mathbf{r} = \mathbf{v}\,dt$.