Nonlocal Quantum Field Theory and Stochastic Quantum Mechanics

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Nonlocal Quantum Field Theory and
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Fundamental Theories of Physics

A New International Book Series on the Fundamental Theories of Physics: Their Clarification, Development and Application

Preface

In this book we attempt to give a physical foundation for the concept of nonlocality and to introduce the fundamental length in physics by the hypothesis of stochasticity of space-time in the microworld. The structure of space-time has always played an important role in physical theory. It is used to simplify concepts connected with symmetry and to give understanding about a great many complicated phenomena in a unified way, in terms of a few simple principles. For example, Einstein's courageous idea of unification of space and time was in fact a starting point for the general theory of gravitation and became the cornerstone of modern relativistic quantum theory. At present, further development of high energy physical experiments and theories dictates a deeper level in the understanding of the structure of space-time and its properties at small distances. However, our concepts of space-time are confirmed experimentally to be valid to distances of the order 10^{-15}–10^{-16} cm.

Among the proposed properties of space-time at small distances [for example, the concept of superspace and pregeometry, lattice (discrete) and cellular structures of space-time, higher-dimensional geometry, etc.] an important role is played by the idea of the stochastic character of space-time. This idea is based on the fact that the quantum fluctuations in geometry are inescapable if one believes in the quantum principle and Einstein's theory. A possible deeper connection between the concept of nonlocality and the hypothesis of space-time stochasticity, originally due to D. I. Blokhintsev, stimulates the *physical foundation* for the *nonlocal quantum field theories* constructed by G. Wataghin, A. Pais and G. E. Uhlenbeck, H. Yukawa, G. V. Efimov, and others.

This book is devoted to the construction of a self-consistent model of nonlocal quantum field theory and stochastic quantum mechanics based on the hypothesis of space-time stochasticity and nonlocality in the microworld. In that case the occurrence of form-factors in the theory (i.e., *violation of the concept of locality* at small distances) and the *random behavior of a physical system* are caused by the stochastic nature of space-time on a small scale. The averaging of any fields independent of their nature (i.e., mass, spin, charge, etc.)

over this stochastic space-time leads to the nonlocal fields considered by G. V. Efimov. In other words, stochasticity of space-time (after being averaged on a large scale) as a self-memory makes the theory nonlocal. This allows one to consider in a unified way the effect of stochasticity (or nonlocality) in all physical processes. Moreover, the universal character of this hypothesis of space-time at small distances enables us to re-interpret the dynamics of stochastic particles and to study some important problems of the theory of stochastic processes [such as the relativistic description of diffusion, Feynman-type processes, and the problem of the origin of self-turbulence in the motion of free particles within nonlinear (stochastic) mechanics]. In this direction our approach (Part II) may be useful in recent developments of the stochastic interpretation of quantum mechanics and fields due to E. Nelson, D. Kershaw, I. Fényes, F. Guerra, de la Peña-Auerbach, J.-P. Vigier, M. Davidson, and others. In particular, as shown by N. Cufaro Petroni and J.-P. Vigier, within the discussed approach, a *causal action-at-distance interpretation* of a series of experiments by A. Aspect and his co-workers indicating a possible *nonlocality property of quantum mechanics*, may also be obtained. Aspect's results have recently inspired a great interest in different nonlocal theories and models devoted to an understanding of the implications of this nonlocality.

This book consists of two parts. The first part is an attempt to give the basic principles of nonlocal theory of quantized fields and to construct the theory of electromagnetic and four-fermion weak interactions of leptons from the stochastic space-time point of view. In the second part, we present to the reader the main ideas of stochastic quantum mechanics and stochastic quantization methods for field theory. The core of the book is based on the original results obtained during the years (1877–83) in collaboration with Professor G. V. Efimov and his students; it covers nonlocal theory of quantized fields, the relativistic and nonrelativistic dynamics of stochastic particles, stochastic quantum mechanics and quantum fields. Since the book is addressed primarily to theoretical and mathematical physicists, it emphasizes a conceptual structure – the definition and formulation of the problem and techniques of solution. Since the emphasis differs from that of conventional physics texts, students may find this book useful as a supplement to their texts.

In both parts we include the mathematical fundamentals of quantum field theory and stochastic processes: generalized function and functional methods, and their applications in quantum field theory, the basic concepts of probability theory and random processes, and some new developments of field theory and stochastic quantization methods. We have organized the book so that it will be useful to new students as well as to experienced researchers. It is neither self-contained nor complete, but it intends to develop the central ideas, to explain the main results of a physical and mathematical nature, and to provide an introduction to the related literature.

Physicists concerned with condensed matter may be interested in the discussion of the stochastic solution to the classical Yang–Mills equation and of vacuum structure; the behavior of vacuum energy density in the strong coupling limit; the mechanism of vacuum tunneling phenomena in the non-Abelian gauge theories; and some problems of cosmic rays (their acceleration mechanism and the energy spectrum, and the ratio of the intensities of the electron component to the proton component at the same energy level). The central matters are functional integral techniques, stochastic quantization and space-time metric fluctuational methods.

I would like to thank Professor G. V. Efimov (JINR) for helpful and stimulating discussions, valuable comments, and for teaching me the principles of scientific work. The author is truly indebted to Professors N. N. Bogolubov and V. A. Meshcheryakov for their hospitality at the Joint Institute for Nuclear Research, Dubna. I wish to thank Doctors M. Dineykhan (Mongolian Academy of Sciences, Ulan-Bator, Mongolia) and P. Exner (JINR and Nuclear Centre, Charles University, Prague, Czechoslovakia) for useful comments and help in preparing the manuscript. I would also like to thank Professors Ch. Tseren and Sh. Tsegmid (Academy of Sciences, Ulan-Bator, Mongolia); N. Sodnom, E. Damdinsuren, and O. Lkhavga (Mongolian State Univ. Ulan-Bator, Mongolia); B. M. Barbashov, V. G. Kadyshevsky and A. N. Sisakhyan (JINR); V. Ya. Fainberg and D. A. Kirzhnits (Lebedev Physical Institute, Moscow, USSR), Ya. A. Smorodinskii (Kurchatov Institute, Moscow, USSR); N. E. Turin (High Energy Institute, Serpukhov, USSR); G. M. Zinovjev (Institute of Theoretical Physics, Kiev, USSR); P. N. Bogolubov (Institute for Nuclear Research, Academy of Sciences, Moscow, USSR); Hans-J. Treder (German Academy of Sciences, Babelsberg, DDR); Alwyn van der Merwe (Univ. of Denver, Colorado, USA); and J.-P. Vigier (Henri Poincaré Institute, Paris, France) for constant interest in my work and valuable comments and encouragement in my investigations.

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KHAVTGAIN NAMSRAI

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Notations used in the book

To introduce the general notation, all four-component vectors are chosen real. The metric is given by the Minkowski one, $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$), for which

$$g_{\mu\nu} = 0 \qquad \text{for} \quad \mu \neq \nu,$$

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1.$$

The product of two four-vectors p and q with components

$$p = (p_0, \mathbf{p}) = (p_0, p_1, p_2, p_3), \qquad q = (q_0, \mathbf{q}) = (q_0, q_1, q_2, q_3)$$

is defined as

$$(pq) = p_\mu q_\nu g_{\mu\nu} = p_\mu q_\mu = p_0 q_0 - \mathbf{pq} = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3.$$

Summation is carried out by repeating indices, omitting the symbol of summation.

Derivatives will be noted as follows:

$$i\frac{\partial}{\partial x_\mu} = i\,\partial_\mu = p_\mu \qquad (\mu = 0, 1, 2, 3),$$

$$\Box = p^2 = -\partial_0^2 + \partial^2 = -\partial^2/\partial x_0^2 + \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2,$$

$$\partial^s = \partial_{\mu_1} \cdots \partial_{\mu_s} = \partial^s/\partial x_{\mu_1} \cdots \partial x_{\mu_s}.$$

The equivalent notation

$$f(x) = f(x_0, \mathbf{x}) = f(x_0, x_1, x_2, x_3)$$

will be used for a function $f(x)$ defined in four-dimensional space-time. Sometimes $x_0 = t$ will be used.

The following equivalent notations

$$\int d^4 x f(x) = \int dx_0 \int d\mathbf{x} f(x_0, \mathbf{x}) = \int dx_0 \int d^3 x f(x_0, \mathbf{x})$$

$$= \int dx_0 \int dx_1 \int dx_2 \int dx_3 f(x_0, x_1, x_2, x_3)$$

are used for integrals of some function $f(x)$ over all four-dimensional space-time.

Four- and three-dimensional δ-function will be read

$$\delta^{(4)}(x) = \delta(x) = \delta(x_0)\,\delta^{(3)}(\mathbf{x}) = \delta(x_0)\,\delta(\mathbf{x}) = \delta(x_0)\,\delta(x_1)\,\delta(x_2)\,\delta(x_3),$$

omitting upper indices (3) and (4).

The Dirac γ_μ-matrices are chosen in the form:

$$\gamma_\mu = (\gamma_0, \boldsymbol{\gamma}),\ (\mu = 0, 1, 2, 3), \qquad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu},$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3, \qquad \gamma_5^2 = 1,$$

$$0_\mu = \gamma_\mu(1 + \gamma_5), \qquad \hat{p} = p_\mu\gamma_\mu = p_0\gamma_0 - \mathbf{p}\boldsymbol{\gamma}.$$

In this book the unit system (with rare exceptions) in which the light velocity c and the Planck constant h divided by 2π ($\hbar = h/2\pi$) are both equal to unity, i.e., $\hbar = c = 1$, is used. In this unit system, energy, momentum and mass have the dimension of inverse length and time $x_0 = t$ has the dimension of length.

Open sets in the real n-dimensional space $\mathbb{R}^n$ will be denoted by G, Ω, Γ, etc., and $\bar{G}$, $\bar{\Omega}$, $\bar{\Gamma}$, etc., are then their closure in $\mathbb{R}^n$. In this notation $S = \bar{G}\setminus G$ means boundary of the set G.

Sometimes the symbols $\forall$ and $\exists$ will be used, which mean: $\forall$ – 'for any' and $\exists$ – "there exist". For example, $\forall\ a \in \Gamma$, $f(a) = 1$ means 'for any a belonging to Γ, $f(a) = 1$ always', and $\exists\ a \in \Gamma$, $f(a) = 1$ means 'there exists an a belonging to Γ, for which $f(a) = 1$'.

The symbol $[a]$ means the enteger part of the number a, for example, $[5] = 5$, $[5.3] = 5$, $[\pi] = 3$.

All other mathematical symbols and notations used in the book should be self-explanatory.

Table of Contents

Part I

Nonlocal Quantum Field Theory

Part I is devoted to the construction of a nonlocal theory of quantized fields and presents systematically the results of studies in this direction. In Chapter 1, a method is formulated for introducing stochasticity into space-time with indefinite metric (i.e., Minkowski space) and of averaging in it. This method allows us to solve the problems of conserving relativistic invariance and of eliminating ultraviolet divergences in constructing quantum field theory from a stochastic space-time point of view. A class of test functions and form factors (or generalized functions) arising from the averaging prescription in a measured stochastic space-time is also considered.

Chapter 2 deals with the formulation and proof of basic principles (the quantization problem, the causality condition and unitarity) of the relativistic quantum field theory in the case of nonlocal interactions. The physical meaning of the form factors is discussed too. It is shown that the finite and unitary S-matrix describing the nonlocal interactions of quantized fields is a solution of the Cauchy problem for the evolution equation (Schrödinger equation in the interaction picture at imaginary time) with retardation.

The finite S-matrix obeying the basic theoretical principles is constructed for the case of the electromagnetic and four-fermion weak interactions (Chapters 3 and 4). The different characteristics of electromagnetic and weak processes are calculated and restrictions on the magnitude of the fundamental length are obtained.

Functional methods and their applications in quantum field theory are considered in Chapter 5. Here attention is focussed on functional analysis of Grassmann algebra and on a definition of the functional integrals in quantum field theory. Vacuum energy estimates have been obtained in the φ_4^4-nonlocal model and the anharmonic oscillator case.