GRAVITATION AND RELATIVITY

H.-Y. CHIU and W. F. HOFFMANN, Editors

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Foreword

While astronomy and geology have traditionally been sciences involving observation and classification of phenomena in the universe, the other physical sciences have been largely restricted to laboratory investigations of the laws of nature and their manifestations in simple forms of matter. In recent years, however, immense progress has been made in understanding how the laws of nature operate in the universe itself—in the cosmic laboratory—where man cannot perform simple experiments but must attempt to analyze nature as he finds it. Progress has been particularly vigorous in such fields as astrophysics, geophysics, geochemistry, and meteoritics. In particular, the space research program has stimulated large numbers of people from various physical disciplines to participate in the physical exploration of the solar system.

This series of books will be concerned with any line of scientific inquiry which attempts to achieve a better understanding of the physical mechanisms that operate in the universe. Pure investigations of the laws of nature, and laboratory investigations of the properties of matter, will not be included. If a laboratory scientist turns his experimental and theoretical talents to the investigation of his physical environment, the results of his investigations are of interest for this series.

The primary aim of the series will be to further communication between scientists investigating nature, and the mode of publication will be varied to minimize the diversion of a scientist's energy from his active participation in teaching and research. The series will include monographs on various specialized topics, proceedings of conferences and symposia, collections of scientific reprints with critical commentary, and publication of lecture-note volumes.

A. G. W. Cameron
Series Editor

New York, New York
February 1963

Preface

This book is a presentation of fundamental concepts, theoretical structure, and experimental foundations of gravitation theory in the light of present-day experimental and theoretical research. It provides an opportunity for physicists, geophysicists, astronomers, and mathematicians not in the field to become acquainted with several aspects of current research in gravitation. In addition, it provides professional relativists with a unique collection of subject matter, some of which has not been previously available in published form.

Topics range from formal questions concerning the structure of general relativity to possible observable implications of gravitational theory for geophysics, atomic physics, and astrophysics.

The material included, with the exception of Chapter 15, is derived from a series of lectures presented at a seminar on gravitation and relativity at the NASA Goddard Institute for Space Studies, New York City, in 1961–1962. This seminar was organized by Professor Dicke as an introduction to the subject, emphasizing the observational implications of the theory and the potential contributions that modern experimental techniques may make.

The approach in those lectures was conceptual rather than axiomatic. For this reason, a complete mathematical development of the subject was not presented. Rather, a review of the fundamental notions, notation, and equations of general relativity was provided as an introduction to subsequent mathematical elaboration of the conceptual arguments.

The text of Chapters 1 through 14 was prepared from notes and tape recordings with further revision by the lecturers. Chapter 15 was first presented at the Conference on Relativistic Theories of Gravitation, Warsaw, Poland, 1962, and subsequently adapted for lectures at the Summer Institute for Theoretical Physics, University of Colorado, Boulder, Colorado, 1962. The present form of this chapter is derived both from a lecture delivered by Professor Wheeler at the Institute for Space Studies and from the lecture notes of the Summer Institute for Theoretical Physics in Boulder.

A general discussion of current experimental and theoretical work in gravitation physics and a survey of the book's contents is presented in the Introduction. Subsequent chapters follow the order in which the lectures were given. Chapters 1 through 4 introduce the basic content of gravitation theory; the remaining chapters discuss various theoretical and experimental questions according to the interests of the lecturers. This method of choice of topics leads to many omissions. It is hoped that the reader will be rewarded by the freshness and novelty of approach rather than by a comprehensive coverage of the subject.

The editors would like to thank Dr. Robert Jastrow for the hospitality of the NASA Goddard Institute for Space Studies where these lectures were presented, and for his personal encouragement of their publication. We express our appreciation to the National Academy of Sciences and the National Research Council for the fellowships that we held during part of the preparation of this material. We wish to thank the Summer Institute for Theoretical Physics at the University of Colorado for permission to use some of the material appearing in Chapter 15. We should also like to acknowledge our pleasant collaboration with the several lecturers and with the publisher during the preparation of this volume.

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Introduction

Special relativity has been successful in interpreting electromagnetic phenomena and in determining the dynamics of particles not moving in strong gravitational fields. Gravitational phenomena are well described by Newtonian mechanics to the extent that terms involving ϕ/c^2 or v^2/c^2 can be neglected. ϕ is the gravitational potential, v is the velocity of the particle with respect to the source of the gravitational field, and c is the velocity of light. General relativity describes the motion of bodies in strong gravitational fields, and interactions among gravitational fields.

The most important and unique characteristic of general relativity is its identification of gravitational fields with the geometrical structure of the space-time continuum, using concepts developed in Riemannian geometry. These concepts are described in Chapter 2. Although the structure of Riemannian geometry is fairly well established, the contents of general relativity are only partly understood.

Part of the reason is due to the nonlinearity of the Einstein field equations, which makes mathematical treatment difficult. An even greater problem is the difficulty of carrying out experiments to verify the predictions of general relativity, and to distinguish this theory from other theories of gravity.

This volume emphasizes the overriding principles which determine the content and form of the theories of gravity and those experiments and observations which guide the form and verify the predictions of these theories. This introduction provides a general discussion of the subject and a brief survey of the contents of the subsequent chapters.

Theoretical Foundations of General Relativity as a Theory of Gravitation

The Principle of Equivalence

After an extensive study of the invariance properties of Maxwell's equations under Lorentz transformations, and of the nature of the null result of the Michelson-Morley experiment, Einstein postulated two basic principles upon which his geometrical analysis of space and time is founded.[1] These two principles are now known as the principle of equivalence and the principle of covariance. The two principles as Einstein stated them are as follows:

The Principle of Equivalence: "*In a homogeneous gravitational field (acceleration of gravity γ) let there be a stationary system of coordinates K, oriented so that the lines of force of the gravitational field run in the negative direction of the axis of z. In a space free of gravitational fields let there be a second system of coordinates K′, moving with uniform acceleration (γ) in the positive direction of the axis of z* . . . [The equations of motion in the two systems K and K′ are the same.] *But we arrive at a very satisfactory interpretation of this law of experience, if we assume that the systems K and K′ are physically exactly equivalent, that is, if we assume that we may just as well regard the system K as being in a space free from gravitational fields, if we regard K as uniformly accelerated* . . ."

The Principle of Covariance: "*The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are co-variant with respect to any substitutions* [of coordinates] *whatever (generally covariant).*"

Before we proceed to discuss further how the field equations are derived, we must examine these two principles in greater detail.

In Chapter 1 Dicke subdivides the principle of equivalence into two principles, one called the strong principle, and the other called the weak principle. The weak principle has its origin in the following observation. If the gradient of the gravitational field times the square of the dimension of a physical laboratory or the square of the space-time distance over which experiments can be performed, is much less than c^2, then any gravitational effects on the laboratory can be transformed away by letting this physical laboratory fall freely. This characteristic of gravity manifests itself in Riemannian geometry through the possibility of always obtaining a coordinate system which is locally Cartesian.

On the other hand, according to the strong principle of equivalence, in a freely falling, nonrotating laboratory, one observes the same set of physical laws identical in both form and numerical content, independent of the space-time locality and the velocity of the laboratory. Here also it is necessary to neglect the effects of gradients of the gravitational field. In the above sense, the strong principle of equivalence *excludes* the possibility of variation of physical laws in both space and time.

The weak principle of equivalence is supported to a high precision by the Eötvös experiment as performed originally by Eötvös[2] and more recently by Dicke.[3] These experiments verify the following: The ratio of the inertial to the gravitational mass of nearby objects composed of various materials is essentially the same to an accuracy of a few parts in 10^9 (Eötvös) and one part in 10^{11} (Dicke).

Two conclusions may be drawn on the basis of Eötvös's experiment:

1. All material bodies describe unique trajectories in space and time when affected by gravitational fields only, provided the restrictions on the strength of the gradient of the gravitational field described earlier remain true.* This is the weak principle of equivalence. It provides a basis for establishing the structure of space-time.

2. The strong principle of equivalence can be inferred to a limited accuracy. That is, one can infer within some limits that the laws of physics (e.g. the ratios of the strengths of fundamental forces) are the same throughout space-time. This is done by the following argument.

If the ratios of various interaction strengths should vary from place to place, then so would the ratios of their contributions to the inertial mass of different objects vary with position. Such position dependent mass energy would lead to an anomalous force which would be different for materials exhibiting different ratios of mass-energy content due to these various interactions. The experimental absence of such anomalous forces implies the constancy of the interaction ratios to a precision depending on their fractional contribution to the total mass of the various materials. In particular, the contribution of strong interactions, electromagnetic interactions, weak interactions, and gravitation to the energy of a body is roughly $1 : 10^{-2} : 10^{-12} : 10^{-40}$ for an atom (the contribution from the strong interaction is normalized to unity.) For a macroscopic body of mass about 1 gram and density unity the total gravitational energy is about 10^{-8} erg and the above ratios become $1 : 10^{-2} : 10^{-12} : 10^{-29}$.

The Eötvös experiment demonstrates that the inertial and gravitational masses of material bodies have the same ratio to an accuracy of 1 part in 10^{11}. Hence to great accuracies the constancy of the electromagnetic and the strong interactions must follow the strong principles of equivalence. Nothing can be said about weak interactions and the gravitational interaction. Hence, the Eötvös experiment rules out the possibility that the coupling constants of strong interactions and electromagnetic interactions vary appreciably with time and position. It does not rule out the variability of the weak coupling constant and the gravitational coupling constant. This possibility may be closely connected with Mach's principle (Chapters 8 and 15).

One way to test the validity of the equivalence principle in gravitation is to observe the trajectories of objects whose self-gravitational energy is large. Only for astronomical bodies is this possible. For example, for the

* This restriction is necessary since the trajectory of a spinning test particle is different from the trajectory of ordinary particles in the presence of a nonuniform gravitational field.[4]

sun the above interaction strength ratios are $1 : 10^{-2} : 10^{-12} : 10^{-6}$. For Jupiter the ratios are $1 : 10^{-2} : 10^{-12} : 10^{-8}$. A study of the trajectory of Jupiter might lead to more precise limits for the applicability of the strong principle of equivalence to gravitational self-energy.

The Principle of Covariance

This principle states: "*The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant).*"[1]

There are two interpretations of this principle:

1. We may interpret this principle as the statement that a coordinate system is just a particular choice of imaginary lines in space and time whose intersections characterize events. Hence, the choice of coordinate systems has nothing to do with the contents of the theory. This is a restricted sense of covariance. And in a sense is an empty statement; for any physical theory may be made to be covariant without increasing its physical content. For example, Maxwell's equations may be written in a form which appears to be generally covariant [Chapter 11, Eq. (34)], without enlarging its physical content as long as the metric is restricted to that of a flat space. Then the metric can always be restored to the familiar diagonal Lorentz form by an ordinary coordinate transformation. In this restricted sense, all laws of physics may be written in a covariant form without introducing any new physics and the principle of covariance thus interpreted is an empty statement.

2. What Einstein had in mind when he stated the principle of covariance is entirely different (Chapter 9). What he meant is that the laws of nature are geometrical statements about geometrical objects and that such laws must be applicable to spaces of arbitrary geometry. In a completely geometrized theory, the geometry is determined from the theory and is not given a priori (as an *absolute* element). For example, in the Einstein field equations, the geometry is determined from the field equations, whereas in special relativity the geometry is a priori restricted to that which is Lorentz-invariant.

All known descriptions of physical laws utilize geometrical objects and concepts. It may be that the concept of geometry is so deeply imprinted in our minds that we cannot think of other ways to describe physical theories. The distinction we wish to emphasize here is that the geometry used in a theory may be either given a priori (like that for a flat space, used in most physical theories) or determined by the theory. If the geometry is given a priori, it is an absolute element of the theory and all laws of physics which are written in geometrical form only relate different geometrical objects and do not determine the geometry. To this date, only in the theory of relativity is the geometry determined by the theory.

Furthermore, no microscopic theory has been profitably written in a form applicable to spaces of arbitrary geometry. The microscopic laws of physics usually apply to a spatial dimension of around 10^{-8} cm (atoms) or to 10^{-13} cm (nuclei and fundamental particles). Unless the spatial variations of curvature are so large that the curvature changes substantially over such small distances, one can always transform to a coordinate system in which the metric is locally flat and free from gravitational effects. Hence it may not be necessary to require that the physical laws describing the microscopic phenomena be applicable to spaces of arbitrary geometry.*

Moreover, the requirement of covariance introduces additional complications (Chapter 9). The solutions of the Einstein field equations must admit a general coordinate transformation in order to be generally covariant. This is reflected by the fact that the ten equations satisfy four identities (Chapter 4). These are the conservation laws of energy and momentum expressed in the form

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{1}$$

Thus, out of the ten Einstein field equations for ten unknown metric components $g_{\mu\nu}$ only six equations are independent. In this way an arbitrary coordinate transformation is automatically permitted by the theory.

However, no definite solutions can be found from such a set of equations, unless an additional four equations are imposed. These four equations are known as *coordinate conditions*. Once a solution is obtained one must be able to transform the solution freely to other coordinate systems compatible with the geometry. Hence, the role of coordinate conditions is to aid obtaining a solution. Once a solution is obtained they may be discarded. However, without having them imposed first and then discarded, no solution may be found.

Fock[6] argued that a certain set of noncovariant coordinate conditions exists. According to him there exists a set of preferred coordinate systems in general relativity, which are not contained in Einstein's theory but must be obtained from other arguments. This question is discussed in Chapter 9.

Identification of Gravitational Fields with the Geometrical Structure of Space-Time

To the accuracy of the Eötvös experiment, all material bodies follow unique trajectories in space-time, if gravitational forces alone act. Would not this imply that the gravitational field is a property of the space? That is, can one replace gravitational fields by a geometrical structure of space-time? This can and has been done for gravitation theory.

* However, this conclusion may not follow if the topology of the space-time is non-Euclidean.[5]

One way to achieve geometrization of gravitation theory is to define the geometry in such a way that particle trajectories coincide with unique curves contained in the geometry. Einstein suggested that the extremal paths of the geometry should be identified with particle trajectories. Then the gravitational field is replaced by the curvature of space and the equations of motion are equations for geodesics. However, this is not the only way by which the theory of gravitation may be geometrized. It is possible to define a geometry in such a way that particles do not move along geodesics of the metric. Geodesics and other curves are discussed in detail in Chapter 1.

Classical Measurements of Space-Time Distances

In the geometrized theory of gravitation, the gravitational field is replaced by the curvature of space, which is characterized by a set of metric coefficients $g_{\mu\nu}$ (metric tensor). They are defined in such a way that the line element ds (the distance between two neighboring space-time points) is given by

$$(ds)^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{2}$$

where $\{x^\mu\}$ are the coordinates used and a summation over repeated indices is understood. In order to use this notion of distance operationally one must have a scheme for measuring distances. This is necessary not only for infinitesimal distances as described by Eq. (2) but also for large distances.

Classically, Einstein discussed space and time measurements in terms of rods and clocks. In a frame in which the rods and the clocks are at rest relative to each other, the two measuring instruments perform orthogonal measurements, that is, a clock measures the distance along the time axis, and a rod measures the distance along the spatial axis. If the strong equivalence principle is to hold exactly and precisely, then all coupling constants will indeed be constant. Then rods and clocks can be relied on to be unchanged from place to place and the comparison of measurements (such as the velocity of light) from place to place will have a clear meaning.

However, according to one version of gravitational theory (Dicke, Chapter 8), the gravitational coupling constant $Gm_p^2/\hbar c$ is not really a constant, but may vary with time. It is even conceivable that other fundamental constants are not constant. The ratio of distance measurements performed by using a material rod (composed of atoms) to those performed by counting the number of wavelengths of a red spectral line of Kr^{86} is proportional to $e^2/\hbar c$ (Chapter 3). Hence, if the strong principle of equivalence is in doubt (Chapters 1 and 8) all measurements performed by using clocks and rods of various constitution are subject to question. Statements like "the velocity of light has a constant numerical value measured locally in all frames of reference" may not be meaningful.

On the other hand, it is not necessary to use clocks and rods at all to measure space-time intervals. It is possible to combine the measurement of space and time intervals into a single operation. This operation, Marzke and Wheeler show in Chapter 3, can be defined entirely in terms of light beams and particle trajectories independent of the constitution of matter. The constancy of the velocity of light measured in any local system then becomes—as Weyl[7] long ago suggested—a postulated principle by which measurements are interpreted.

Marzke and Wheeler also spelled out one way by which the intercomparison of space-time distance intervals may be carried out over a long distance even in the presence of gravitational fields. The heart of the procedure is a particle moving on a geodesic and a nearby particle moving on a parallel world line which in general is not a geodesic. Light is scattered back and forth between the particles. The number of scatterings yields a well-defined measurement of the space-time interval.

The Einstein Field Equations

The Einstein field equations provide a means for constructing the geometry when the stress tensor (the source of the gravitational field) and suitable boundary conditions are given. From the field equations we can also derive equations of motion of test particles which must reduce to the Newtonian equations of motion in the case of weak fields and low velocities. The Eötvös experiment indicates that the source of gravitational interaction is the total mass energy. Any matter distribution will have in addition to the rest energy of matter, also the stress energy. (Rigid bodies have no invariant meaning since a strain wave cannot be transmitted instantaneously.) Hence the source term for the gravitational equations is usually taken to be the stress-energy tensor $T_{\mu\nu}$, which includes the matter distribution as one of its components. However, one may also use the contracted stress tensor $T = T^{\mu}_{\mu}$ as a source of a scalar field, as is described in Chapter 8.

In Chapter 4 Wheeler discusses the derivation of the field equations. It is demanded that the theory be generally covariant, that it involve only [illegible]tric tensor and various geometrical objects obtained from its components and derivatives of these, and that the equations be the lowest order possible. With these requirements and the additional assumption that the stress-energy tensor $T_{\mu\nu}$ be the source of gravitational effects, the Einstein field equations follow uniquely.

Experimental Foundations of General Relativity and Other Theories of Gravity

General relativity theory, and other theories of gravity, are based on one or more of the following principles:

1. The equivalence principle
2. The local Lorentz character of space and time
3. The principle of covariance (discussed above)
4. The constancy of electric charge independent of its velocity
5. The concept that the ratio of two space-time intervals has a well-defined value independent of the route of intercomparison (Riemannian geometry!).

What is the experimental support for these principles?

The Equivalence Principle and the Principle of the Locally Lorentz Character of Space-Time

Associated with the idea of the local Lorentz character of space-time is the idea of the isotropy of space-time. Mach's principle requires that the inertial properties of matter be determined by the distribution of matter in the universe. A naïve interpretation of Mach's principle is that the inertial property of matter depends on the *local* distribution of matter, and therefore, it is expected to depend on the direction of motion. However, every attempt to detect such anisotropy has failed (Chapter 7).

Beltran-Lopez, Robinson, and Hughes[8] performed experiments to determine if the inertia of matter depends on the spatial direction (Chapter 6). The local source of mass anisotropy is the sun and the galaxy. These experiments have been done both with paramagnetic resonance absorption measurements of the Zeeman splitting in chlorine and oxygen and by nuclear magnetic resonance in lithium. In the case of Zeeman splitting, an atomic electron with nonzero orbital angular momentum moves in different directions with respect to an external magnetic field in different magnetic substates. Hence, if the mass depends on the direction of motion, the energy difference between different magnetic substates will depend on the orientation in space. A similar situation occurs with the different nuclear magnetic states in the lithium experiment. Because of the much higher binding energies in the nucleus compared with those of the atomic electrons, the lithium experiment is the more precise one. The results of this experiment place an upper limit of 1 part in 10^{22} to the anisotropy of mass.

The Constancy of Electric Charge Independent of Its Velocity

The constancy of electric charge independent of its velocity is implied by the invariance of Maxwell's equations under a Lorentz transformation. This constancy is the basis of electrodynamics as well as of special relativity from which general relativity evolved. It has been established to a high degree of precision as a by-product of several experiments carried out to measure the electron-proton charge ratio. Some of these experiments are described by Hughes in Chapter 13. In one of these, performed by Zorn, Chamberlain, and Hughes[9], the deflection of a neutral molecular beam passing through an electric field perpendicular to the direction of motion was measured and found essentially to be zero. From the sensitivity of the apparatus for a beam of cesium atoms the equality of electron and proton charge is established to 1 part in 10^{19}.

In other experiments using a gas efflux method King[10] established the charge equality to about 1 part in 10^{20} in hydrogen and helium and Hillas and Cranshaw[11] determined the equality to about 1 part in 10^{21} with argon and nitrogen. These experiments not only demonstrate that the electric charges for electrons and protons are equal and opposite but also as a byproduct, demonstrate that the electric charge is a constant independent of its velocity to an extremely high degree of precision.

The principle behind this interpretation is the following. In a hydrogen atom the proton and the electron move around their common center of mass. Because of the lighter mass, the velocity of the electron is higher than that of the nucleus by three orders of magnitude. In a many-electron system the velocity ratio is even greater. In the case of argon ($z = 18$), the velocities of the k-shell electrons are about 4×10^9 cm/sec and the charge equality limit is 1 part in 10^{21}. If we use $(v/c)^2$ as a parameter for the dependence of electric charge with respect to its velocity, we find the coefficient of the dependence is less than 10^{-9}. This may be regarded as the accuracy to which the electric charge is independent of the velocity.

The Justification for Using Riemannian Geometry to Describe the Space-Time Structure of the World

One of the most important postulates concerning the structure of space-time is that the ratio of two intervals should be independent of the route of intercomparison. The evidence on this point is discussed by Wheeler in Chapter 3.

Experimental and Observational Tests of Gravitation Theory

For a theory of such seemingly pervasive importance underlying the structure of space-time and, as is sometimes suggested, perhaps the structure of elementary particles as well, general relativity has led to remarkably little successful experimental research. The few predictions the theory does make about observable phenomena require an almost impossible precision for any decisive measurement. Such precision has been realized for only three experiments in the past: Analysis of the orbit of the planet Mercury for a small relativistic precession of the perihelion of the orbit, gravitational bending of starlight passing by the sun, and the red shift of spectral lines emitted and observed at two different gravitational potentials. Significant precision is currently being sought for two more tests: Precession of a gyroscope in the field of the rotating earth, and gravitational radiation. In view of this paucity of tests it is no surprise that gravitation has met with limited interest in the past as an object of experimental research.

In the last few years there has been an increase in activity and interest in the field. This has come about variously by the availability of new and precise experimental techniques (e.g., earth satellites, precise electronics systems); by a broadening base of theoretical work in gravitational theory, and by an increasing feeling among some physicists of the ultimate role gravity may play in other fields of physics, such as the structure of elementary particles. The new experimental work is proceeding along several lines, some of which have already been discussed. All of these approaches are listed here for the sake of completeness.

1. A refinement of the measurements of the "three famous checks" of general relativity: the precession of the perihelion of Mercury, the bending of starlight by the sun, and the gravitational red shift.

2. Attempts to measure previously undetected general relativistic effects, in particular the Lense-Thirring precession of a gyroscope in the gravitational field of the earth, and the gravitational radiation from oscillating laboratory or astronomical masses.

3. Fundamental precision measurements which are prerequisite to any theory of gravity: the experiments concerning the equivalence of gravitational and inertial mass, mass isotropy, and charge equality.

4. Attempts to detect effects not predicted by general relativity but suggested by alternative theories: gravitational "ether drift"; time varying gravitational "constant"; the interaction of scalar matter waves with the solar system.

5. Incorporation of general relativity into the astrophysical analysis of very dense stars with the hope that the comparison of prediction with observation will lead to new conclusions.

6. Search for cosmological effects such as the over-all curvature and closure of space.

Refinements of Measurements of the Three Famous Checks

These standard three experiments are not emphasized in the subsequent chapters of this book, so we shall go into some detail here.

The field equations of general relativity are nonlinear. The principle of superposition cannot be applied to gravitational fields without caution. To discuss general relativistic effects, one examines the motion of very small bodies (test bodies) or light rays whose perturbing effect on the gravitational field is negligible. In particular, the three experimental tests to be discussed here involve the behavior of such test bodies in the Schwarzschild geometry to the order of $(v/c)^2$ or (ϕ/c^2).