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Pan-Tai Liu
Robert L. Sternberg

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To all those who
encouraged us to
proceed with the
Second Kingston Conference
on
Differential Games and Control Theory

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FOREWORD

In this volume appear twenty-four of the thirty-six papers presented in person or by title at the Second Kingston Conference on Differential Games and Control Theory held at the University of Rhode Island in Kingston, June 7 to 10, 1976 under sponsorship of the University of Rhode Island with the participation of the International Federation of Automatic Control. Included are Invited Lectures, Contributed Papers, and four papers from the Adjunct Program which were read by title. The selection includes papers by widely known experts and also contributions from beginning scholars just starting out in this field.

While we should have liked to publish all of the papers from the Conference, we were unable to do so for a variety of reasons mostly connected with limitations of time and space, but we are grateful nevertheless for the enthusiastic formal participation in the Conference of Rufus Isaacs of Johns Hopkins University; Mary Ellen Bock of Purdue University; Max Mintz of the University of Pennsylvania; Hubert Hai-Ao Chin of York College of the City University of New York; Musa Yildiz of the University of New Hampshire; John Danskin of Universität Bonn and the École National Supérieure des Télécommunications; N. M. Olgac, R. W. Longman, and C. A. Cooper of Columbia University and the Bell Telephone Laboratories; Wolfgang Carmele of the Technische Universität Darmstadt; D. R. K. Rao of Jundi

Shapur University; Donald W. Tufts and J. T. Francis of University of Rhode Island and the Naval Underwater Systems Center; D. G. Lainiotis of the State University of New York at Buffalo; Howard Blum of Rutgers, State University of New Jersey; and A. G. Lindgren of the University of Rhode Island.

A major purpose of the Conference was to bring together mathematicians, scientists, and engineers from a variety of disciplines having a common interest in the Conference Topic and perhaps special interests in the Theme: Stochastic Problems and Applications. To what extent this effort met with success may perhaps be judged by a perusal of the varied topics of the papers in this book which range from almost purely mathematical considerations to applications in systems analysis, electrical engineering, resource economics, public policy, fisheries management, and harvesting strategies.

The Conference was organized by the three editors of this book with the assistance of Helen M. Sternberg of the University of Connecticut who served as Conference Secretary and Geert Jan Olsder of the Twente University of Technology who served as the IFAC Liaison Representative. Henry J. Kelley of Analytical Mechanics Associates, Inc., while not officially a member of the Organizing Committee, gave invaluable assistance and advice during the several months preparation for the Conference.

Marguerite Ellis prepared the final typescript in her customary exquisite fashion and also prepared most of the illustrations.

In closing, the writer wishes to express the appreciation of the Organizing Committee for the financial support for the Conference kindly provided by the office of the Academic Vice-President, the College of Engineering, the College of Arts and Sciences, the Division of University Extension, the Graduate School, the Development Council through a gift from the Eastman Kodak Company, and the Visiting Scholars Fund of the University of Rhode Island, and wishes to express his own indebtedness to W. R. Ferrante, Douglas Rosie, George J. Dillavou, L. D. Conta, A. A. Michael, C. J. Wilson, Virginia O'Brien, Norman J. Finizio, Rosalind Shumate, June Chandronet, Fred Jackson, Harold Fisher, Frank Dietz, Ghasi R. Verma, James T. Lewis, Charles D. Nash, Jr., Nathaniel McL. Sage, Jr., and Herman E. Sheets, also of the University of Rhode Island, and to Derrill J. Bordelon of the Naval Underwater Systems Center in Newport and A. L. Powell and Ruth Berrett of the Office of Naval Research in Boston for their encouragement, assistance, advice and support during the writer's private labors on the Organizing Committee and in the minutiae of editing these Proceedings.

R. L. S.

CONTENTS

MARKOV GAMES - A SURVEY

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ABSTRACT

Markov games or stochastic games were first introduced by Shapley in an historically important paper that appeared in 1953. Since then many authors have extended their results in various directions. These extensions are discussed in some detail. Results relating to limiting average pay-offs (first considered by Gillette in his Ph.D. thesis around 1953) are discussed. Next, the algorithmic aspects of the problem under consideration are also discussed. In conclusion, some problems which are still open will be mentioned.

§0. INTRODUCTION

The relevant literature on sequential compounding of two-person games dates back to the early 1950's and since that time, independently, a number of workers have attacked variations on the theme of compounding. This of course has led to a good deal of redundancy, both conceptual and technical in nature. In one class of games (recursive and Markov or stochastic) a normalized game is played at each stage, and the player's strategies control not only the (monetary) pay-off but also the transition probabilities which govern the game to be played at the next stage. In another class (survival and attrition games) there is but one component game and it is repeated. The players have limited resources, and these fluctuate in time according to the outcomes of repeated plays of the given game. The overall game is concluded when one of the players is bankrupt. In still another class (compound decision problems) a given game is repeated, and each player attempts to control the average pay-off by exploiting the statistical records of his adversary's previous choices. The final class (economic ruin games) is characterized by the problem of corporate dividend policy: the more generous the dividend policy of the corporation, the less secure it is against future exigencies. An excellent introduction to these topics is given in Luce and Raiffa [72] who indicate some of the interrelations, namely, how the theory of Markov or stochastic games suggested that of recursive games which in turn, is related to the theory of survival and attrition games; how Blackwell's approachability theory, which was motivated by attrition games, can be used to analyze compound decision problems; and how approachability theory is technically similar to a generalization of the theory of survival games.

In this article we will discuss in some depth the theory of stochastic games or Markov games. The term Markov game is due to Zachrisson [138]. (Many authors following Shapley [111] use the term stochastic games.) The theory of Markov games was first introduced by Shapley in an historically important paper [111] during 1953. Around the same time Gillette [144] in his

Ph.D. thesis entitled "Representable infinite games" considered Markov games of perfect information in extensive form. A Markov game is an infinite game in which it is assumed that a pay-off to the players of the game is made at each move. Two types of pay-off are considered in the literature: in one the sum of all pay-offs at different moves--the total expected pay-off--is examined; in the other, a limit of the average expected pay-off over the number of moves made as the number of moves approaches infinity is examined. Each player tries to maximize his expected pay-off by playing optimally. Since the appearance of the Shapley-Gillette results, many authors have extended them in various directions. We will discuss these in some detail. We will also discuss the algorithmic aspects of the problem under consideration.

§1. ZERO-SUM (STOCHASTIC) MARKOV GAMES - STATE SPACE FINITE OR COUNTABLE

A Markov game is determined by five objects, S, A, B, q, and r. Here S denotes the state space of the system. The states will be denoted by s or s'. Once a day players I and II (for simplicity we consider two-person games; theory for n-person games is similar) observe the current state s of the system, and then player I chooses an action a from a finite set A of actions, and player II chooses an action b from a finite set B of actions. As a result of this, two things happen: (i) player I receives an immediate income r(s,a,b), depending on the current state s of the system and the actions a and b chosen, and (ii) the system moves to a new state s' with probability q(s'/s,a,b) which also depends on s,a,b. We assume that $|r(s,a,b)| \leq M$ for all s,a,b. Payments accumulate throughout the course of the play.

Player I wants to maximize his accumulated income while player II wants to minimize the same. The problem is to choose a strategy for player I that will maximize his total expected income and to choose another strategy for player II that will minimize the income of player I.

In order that the total accumulated income be a well-defined number, we introduce a discount factor β, $0 \le \beta < 1$, so that the value of the unit income n days in the future is β^n. In other words, the total income to player I is equal to $\sum_{n-1}^{\infty} \beta^{n-1} r_n$ where r_n is the income to I on the nth day. Shapley assumes $\inf_{s,i,j} q_{ij}^s > 0$ where q_{ij}^s is the probability that the game stops if (i,j) are the actions chosen by the two players at state s. This means the game ends with probability one after a finite number of steps and hence the total accumulated income is well-defined. For simplicity we will use the discount factor to make the total income well-defined.

It follows from the Kuhn-Aumann [66,4] theorem that in a game of perfect recall (and consequently in Markov games) players can restrict themselves to playing only behavior strategies. A behavior strategy π for player I is a sequence $(\pi_1, \pi_2, \cdots, \pi_n)$ where π_n is a conditional probability distribution on A given the past history $h_n = (s_0, a_0, b_0, s_1, a_1, b_1, \cdots, s_{n-1}, a_{n-1}, b_{n-1}, s_n)$. A behavior strategy π is called stationary if $\pi_n = f$ for all $n \ge 1$. Similarly strategies are defined for player II.

The total expected pay-off for player I from (π, Γ) is denoted by $I(\pi, \Gamma)$; the sth coordinate of $I(\pi, \Gamma)$ is the income to player I if the initial state is s.