FLUID MECHANICS

SECOND EDITION

PIJUSH K. KUNDU • IRA M. COHEN

Fluid Mechanics
Second Edition

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with a chapter on Computational Fluid Dynamics by *Howard H. Hu*

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Cover photo: Karman vortex street behind a circular cylinder at $R = 105$. Photograph by Sadatoshi Taneda

Cover photo: Karman vortex street behind a circular cylinder at $R = 140$. Photograph by Sadatoshi Taneda

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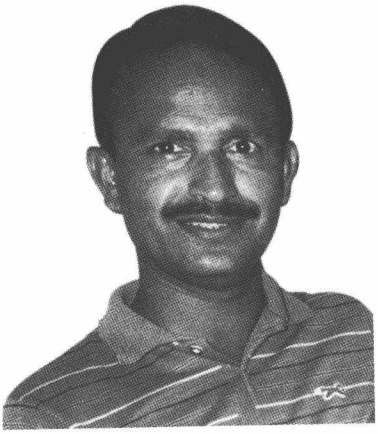
The second edition is dedicated to the memory of Pijush K. Kundu and also to my wife Linda and daughters Susan and Nancy who have greatly enriched my life.

"Everything should be made as simple as possible,
but not simpler."
—Albert Einstein

"If nature were not beautiful, it would not be worth studying it.
And life would not be worth living."
—Henry Poincaré

In memory of Pijush Kundu



Pijush Kanti Kundu was born in Calcutta, India, on October 31, 1941. He received a B.S. degree in Mechanical Engineering in 1963 from Shibpur Engineering College of Calcutta University, earned an M.S. degree in Engineering from Roorkee University in 1965, and was a lecturer in Mechanical Engineering at the Indian Institute of Technology in Delhi from 1965 to 1968. Pijush came to the United States in 1968, as a doctoral student at Penn State University. With Dr. John L. Lumley as his advisor, he studied instabilities of viscoelastic fluids, receiving his doctorate in 1972. He began his lifelong interest in oceanography soon after his graduation, working as Research Associate in Oceanography at Oregon State University from 1968 until 1972. After spending a year at the University de Oriente in Venezuela, he joined the faculty of the Oceanographic Center of Nova Southeastern University, where he remained until his death in 1994.

During his career, Pijush contributed to a number of sub-disciplines in physical oceanography, most notably in the fields of coastal dynamics, mixed-layer physics, internal waves, and Indian-Ocean dynamics. He was a skilled data analyst, and, in this regard, one of his accomplishments was to introduce the "empirical orthogonal eigenfunction" statistical technique to the oceanographic community.

I arrived at Nova Southeastern University shortly after Pijush, and he and I worked closely together thereafter. I was immediately impressed with the clarity of his scientific thinking and his thoroughness. His most impressive and obvious quality, though, was his love of science, which pervaded all his activities. Some time after we met, Pijush opened a drawer in a desk in his home office, showing me drafts of several chapters to a book he had always wanted to write. A decade later, this manuscript became the first edition of "Fluid Mechanics," the culmination of his lifelong dream; which he dedicated to the memory of his mother, and to his wife Shikha, daughter Tonushree, and son Joydip.

Julian P. McCreary, Jr.,
University of Hawaii

Preface

My involvement with Pijush Kundu's *Fluid Mechanics* first began in April 1991 with a letter from him asking me to consider his book for adoption in the first year graduate course I had been teaching for 25 years. That started a correspondence and, in fact, I did adopt the book for the following academic year. The correspondence related to improving the book by enhancing or clarifying various points. I would not have taken the time to do that if I hadn't thought this was the best book at the first-year graduate level. By the end of that year we were already discussing a second edition and whether I would have a role in it. By early 1992, however, it was clear that I had a crushing administrative burden at the University of Pennsylvania and could not undertake any time-consuming projects for the next several years. My wife and I met Pijush and Shikha for the first time in December 1992. They were a charming, erudite, sophisticated couple with two brilliant children. We immediately felt a bond of warmth and friendship with them. Shikha was a teacher like my wife so the four of us had a great deal in common. A couple of years later we were shocked to hear that Pijush had died suddenly and unexpectedly. It saddened me greatly because I had been looking forward to working with Pijush on the second edition after my term as department chairman ended in mid-1997. For the next year and a half, however, serious family health problems detoured any plans. Discussions on this edition resumed in July of 1999 and were concluded in the Spring of 2000 when my work really started. This book remains the principal work product of Pijush K. Kundu, especially the lengthy chapters on Gravity Waves, Instability, and Geophysical Fluid Dynamics, his areas of expertise. I have added new material to all of the other chapters, often providing an alternative point of view. Specifically, vector field derivatives have been generalized, as have been streamfunctions. Additional material has been added to the chapters on laminar flows and boundary layers. The treatment of one-dimensional gasdynamics has been extended. More problems have been added to most chapters. Professor Howard H. Hu, a recognized expert in computational fluid dynamics, graciously provided an entirely new chapter, Chapter 11, thereby providing the student with an entree into this exploding new field. Both finite difference and finite element methods are introduced and a detailed worked-out example of each is provided.

I have been a student of fluid mechanics since 1954 when I entered college to study aeronautical engineering. I have been teaching fluid mechanics since 1963 when I joined the Brown University faculty, and I have been teaching a course corresponding to this book since moving to the University of Pennsylvania in 1966. I am most grateful to two of my own teachers, Professor Wallace D. Hayes (1918–2001), who expressed fluid mechanics in the clearest way I have ever seen, and Professor Martin D. Kruskal, whose use of mathematics to solve difficult physical problems was developed to a high art form and reminds me of a Vivaldi trumpet concerto. His codification of rules of applied limit processes into the principles of "Asymptotology" remains with me today as a way to view problems. I am grateful also to countless students who asked questions, forcing me to rethink many points.

The editors at Academic Press, Gregory Franklin and Marsha Filion (assistant) have been very supportive of my efforts and have tried to light a fire under me. Since this edition was completed, I found that there is even more new and original material I would like to add. But, alas, that will have to wait for the next edition. The new figures and modifications of old figures were done by Maryeileen Banford with occasional assistance from the school's software expert, Paul W. Shaffer. I greatly appreciate their job well done.

Ira M. Cohen

Preface to First Edition

This book is a basic introduction to the subject of fluid mechanics and is intended for undergraduate and beginning graduate students of science and engineering. There is enough material in the book for at least two courses. No previous knowledge of the subject is assumed, and much of the text is suitable in a first course on the subject. On the other hand, a selection of the advanced topics could be used in a second course. I have not tried to indicate which sections should be considered advanced; the choice often depends on the teacher, the university, and the field of study. Particular effort has been made to make the presentation clear and accurate and at the same time easy enough for students. Mathematically rigorous approaches have been avoided in favor of the physically revealing ones.

A survey of the available texts revealed the need for a book with a balanced view, dealing with currently relevant topics, and at the same time easy enough for students. The available texts can perhaps be divided into three broad groups. One type, written primarily for applied mathematicians, deals mostly with classical topics such as irrotational and laminar flows, in which analytical solutions are possible. A second group of books emphasizes engineering applications, concentrating on flows in such systems as ducts, open channels, and airfoils. A third type of text is narrowly focused toward applications to large-scale geophysical systems, omitting small-scale processes which are equally applicable to geophysical systems as well as laboratory-scale phenomena. Several of these geophysical fluid dynamics texts are also written primarily for researchers and are therefore rather difficult for students. I have tried to adopt a balanced view and to deal in a simple way with the basic ideas relevant to both engineering and geophysical fluid dynamics.

However, I have taken a rather cautious attitude toward mixing engineering and geophysical fluid dynamics, generally separating them in different chapters. Although the basic principles are the same, the large-scale geophysical flows are so dominated by the effects of the Coriolis force that their characteristics can be quite different from those of laboratory-scale flows. It is for this reason that most effects of planetary rotation are discussed in a separate chapter, although the concept of the Coriolis force is introduced earlier in the book. The effects of density stratification, on the other hand, are discussed in several chapters, since they can be important in both geophysical and laboratory-scale flows.

The choice of material is always a personal one. In my effort to select topics, however, I have been careful not to be guided strongly by my own research interests. The material selected is what I believe to be of the most interest in a book on general fluid mechanics. It includes topics of special interest to geophysicists (for example, the chapters on *Gravity Waves* and *Geophysical Fluid Dynamics*) and to engineers (for example, the chapters on *Aerodynamics* and *Compressible Flow*). There are also chapters of *common* interest, such as the first five chapters, and those on *Boundary Layers*, *Instability*, and *Turbulence*. Some of the material is now available only in specialized monographs; such material is presented here in simple form, perhaps sacrificing some formal mathematical rigor.

Throughout the book the convenience of tensor algebra has been exploited freely. My experience is that many students feel uncomfortable with tensor notation in the beginning, especially with the permutation symbol ε_{ijk}. After a while, however, they like it. In any case, following an introductory chapter, the second chapter of the book explains the fundamentals of *Cartesian Tensors*. The next three chapters deal with standard and introductory material on *Kinematics*, *Conservation Laws*, and *Vorticity Dynamics*. Most of the material here is suitable for presentation to geophysicists as well as engineers.

In much of the rest of the book the teacher is expected to select topics that are suitable for his or her particular audience. Chapter 6 discusses *Irrotational Flow*; this material is rather classical but is still useful for two reasons. First, some of the results are used in later chapters, especially the one on *Aerodynamics*. Second, most of the ideas are applicable in the study of other potential fields, such as heat conduction and electrostatics. Chapter 7 discusses *Gravity Waves* in homogeneous and stratified fluids; the emphasis is on linear analysis, although brief discussions of nonlinear effects such as hydraulic jump, Stokes's drift, and soliton are given.

After a discussion of *Dynamic Similarity* in Chapter 8, the study of viscous flow starts with Chapter 9, which discusses *Laminar Flow*. The material is standard, but the concept and analysis of similarity solutions are explained in detail. In Chapter 10 on *Boundary Layers*, the central idea has been introduced intuitively at first. Only after a thorough physical discussion has the boundary layer been explained as a singular perturbation problem. I ask the indulgence of my colleagues for including the peripheral section on the dynamics of sports balls but promise that most students will listen with interest and ask a lot of questions. *Instability* of flows is discussed at some length in Chapter 12. The emphasis is on linear analysis, but some discussion of "chaos" is given in order to point out how deterministic nonlinear systems can lead to irregular solutions. Fully developed three-dimensional *Turbulence* is discussed in Chapter 13. In addition to standard engineering topics such as wall-bounded shear flows, the theory of turbulent dispersion of particles is discussed because of its geophysical importance. Some effects of stratification are also discussed here, but the short section discussing the elementary ideas of two-dimensional geostrophic turbulence is deferred to Chapter 14. I believe that much of the material in Chapters 8–13 will be of general interest, but some selection of topics is necessary here for teaching specialized groups of students.

The remaining three chapters deal with more specialized applications in geophysics and engineering. Chapter 14 on *Geophysical Fluid Dynamics* emphasizes the linear analysis of certain geophysically important wave systems. However, elements of barotropic and baroclinic instabilities and geostrophic turbulence are also included. Chapter 15 on *Aerodynamics* emphasizes the application of potential theory to flow around lift-generating profiles; an elementary discussion of finite-wing theory is also given. The material is standard, and I do not claim much originality or innovation, although I think the reader may be especially interested in the discussions of propulsive mechanisms of fish, birds, and sailboats and the material on the historic controversy between Prandtl and Lanchester. Chapter 16 on *Compressible Flow* also contains standard topics, available in most engineering texts. This chapter is included with the belief that all fluid dynamicists should have some familiarity with such topics as shock waves and expansion fans. Besides, very similar phenomena also occur in other nondispersive systems such as gravity waves in shallow water.

The appendixes contain conversion factors, properties of water and air, equations in curvilinear coordinates, and short bibliographical sketches of *Founders of Modern Fluid Dynamics*. In selecting the names in the list of founders, my aim was to come up with a very short list of historic figures who made truly fundamental contributions. It became clear that the choice of Prandtl and G. I. Taylor was the only one that would avoid all controversy.

Some problems in the basic chapters are worked out in the text, in order to illustrate the application of the basic principles. In a first course, undergraduate engineering students may need more practice and help than offered in the book; in that case the teacher may have to select additional problems from other books. Difficult problems have been deliberately omitted from the end-of-chapter exercises. It is my experience that the more difficult exercises need a lot of clarification and hints (the degree of which depends on the students' background), and they are therefore better designed by the teacher. In many cases answers or hints are provided for the exercises.

Acknowledgements

I would like to record here my gratitude to those who made the writing of this book possible. My teachers Professor Shankar Lal and Professor John Lumley fostered my interest in fluid mechanics and quietly inspired me with their brilliance; Professor Lumley also reviewed Chapter 13. My colleague Julian McCreary provided support, encouragement, and careful comments on Chapters 7, 12, and 14. Richard Thomson's cheerful voice over the telephone was a constant reassurance that professional science can make some people happy, not simply competitive; I am also grateful to him for reviewing Chapters 4 and 15. Joseph Pedlosky gave very valuable comments on Chapter 14, in addition to warning me against too broad a presentation. John Allen allowed me to use his lecture notes on perturbation techniques. Yasushi Fukamachi, Hyong Lee, and Kevin Kohler commented on several chapters and constantly pointed out things that may not have been clear to the students. Stan Middleman and Elizabeth Mickaily were especially diligent in checking my solutions to the examples and end-of-chapter problems. Terry Thompson constantly got me out of trouble with my personal computer. Kathy Maxson drafted the figures. Chuck Arthur and Bill LaDue, my editors at Academic Press, created a delightful atmosphere during the course of writing and production of the book.

Lastly, I am grateful to Amjad Khan, the late Amir Khan, and the late Omkarnath Thakur for their music, which made working after midnight no chore at all. I recommend listening to them if anybody wants to write a book!

Pijush K. Kundu

Author's Notes

Both indicial and boldface notations are used to indicate vectors and tensors. The comma notation to represent spatial derivatives (for example, $A_{,i}$ for $\partial A/\partial x_i$) is used in only two sections of the book (Sections 5.6 and 13.7), when the algebra became cumbersome otherwise. *Equal to by definition* is denoted by $\equiv$; for example, the ratio of specific heats is introduced as $\gamma \equiv C_\mathrm{p}/C_\mathrm{v}$. *Nearly equal to* is written as $\simeq$, *proportional to* is written as $\propto$, and *of the order* is written as $\sim$.

Plane polar coordinates are denoted by (r, θ), cylindrical polar coordinates are denoted by either (R, φ, x) or (r, θ, x), and spherical polar coordinates are denoted by (r, θ, φ) (see Figure 3.1). The velocity components in the three Cartesian directions (x, y, z) are indicated by (u, v, w). In geophysical situations the z-axis points upward.

In some cases equations are referred to by a descriptive name rather than a number (for example, "the x-momentum equation shows that . . . "). Those equations and/or results deemed especially important have been indicated by a box.

A list of literature cited and supplemental reading is provided at the end of most chapters. The list has been deliberately kept short and includes only those sources that serve one of the following three purposes: (1) It is a reference the student is likely to find useful, at a level not too different from that of this book; (2) it is a reference that has influenced the author's writing or from which a figure is reproduced; and (3) it is an important work done after 1950. In currently active fields, reference has been made to more recent review papers where the student can find additional references to the important work in the field.

Fluid mechanics forces us fully to understand the underlying physics. This is because the results we obtain often defy our intuition. The following examples support these contentions:

1. Infinitesmally small causes can have large effects (d'Alembert's paradox).
2. Symmetric problems may have nonsymmetric solutions (von Karman vortex street).
3. Friction can make the flow go faster and cool the flow (subsonic adiabatic flow in a constant area duct).
4. Roughening the surface of a body can decrease its drag (transition from laminar to turbulent boundary layer separation).
5. Adding heat to a flow may lower its temperature. Removing heat from a flow may raise its temperature (1-dimensional diabatic flow in a range of subsonic Mach number).

6. Friction can destabilize a previously stable flow (Orr-Sommerfeld stability analysis for a boundary layer profile without inflection point).
7. Without friction, birds could not fly and fish could not swim (Kutta condition requires viscosity).

Every one of these counterintuitive effects will be treated and discussed in this text.

This second edition also contains additional material on streamfunctions, boundary conditions, viscous flows, boundary layers, jets, and compressible flows. Most important, there is an entirely new chapter on computational fluid dynamics that introduces the student to the various techniques for numerically integrating the equations governing fluid motions. Hopefully the introduction is sufficient that the reader can follow up with specialized texts for a more comprehensive understanding.

An historical survey of fluid mechanics from the time of Archimedes (ca. 250 B.C.E.) to approximately 1900 is provided in the Eleventh Edition of *The Encyclopædia Britannica* (1910) in Vol. XIV (under "Hydromechanics," pp. 115–135). I am grateful to Professor Herman Gluck (Professor of Mathematics at the University of Pennsylvania) for sending me this article. Hydrostatics and classical (constant density) potential flows are reviewed in considerable depth. Great detail is given in the solution of problems that are now considered obscure and arcane with credit to authors long forgotten. The theory of slow viscous motion developed by Stokes and others is not mentioned. The concept of the boundary layer for high-speed motion of a viscous fluid was apparently too recent for its importance to have been realized.

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