Franz Baader
Andrei Voronkov (Eds.)

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Logic for Programming, Artificial Intelligence, and Reasoning

11th International Conference, LPAR 2004
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Preface

This volume contains the papers presented at the 11th International Conference on Logic for Programming, Artificial Intelligence, and Reasoning (LPAR), held from March 14 to 18, 2005, in Montevideo, Uruguay, together with the 5th International Workshop on the Implementation of Logics (organized by Stephan Schulz and Boris Konev) and the Workshop on Analytic Proof Systems (organized by Matthias Baaz).

The call for papers attracted 77 paper submissions, each of which was reviewed by at least three expert reviewers. The final decisions on the papers were taken during an electronic Program Committee meeting held on the Internet. The Internet-based submission, reviewing, and discussion software EasyChair, provided by the second PC co-chair, supported each stage of the reviewing process. But the most important work was, of course, done by the 34 PC members and their external reviewers, who provided high-quality reviews. After intense discussions to resolve conflicts among the reviewers, the Program Committee decided to accept 33 papers.

The conference program also included 4 invited talks, by Jürgen Giesl, Alexander Leitsch, Helmut Seidl, and Igor Walukiewicz, which are documented by short or extended abstracts in these proceedings. In addition, Martín Abadi held a tutorial on Reasoning About Security Protocols, and Ian Horrocks on Description Logic Reasoning.

Apart from the authors, invited speakers, tutorialists, Program Committee members, and external reviewers, we would like to thank the other people and organizations that made this LPAR a success: the Local Arrangements Chair, Alberto Pardo, and all the other people involved in the local organization; the Chair for Automata Theory at TU Dresden, the Kurt Gödel Society, and the European Union (in the Information Society Technologies programme of the European Commission, Future and Emerging Technologies under the IST-2001-33123 CoLogNET project), which provided partial funding for our invited speakers; and the Centro Latinoamericano de Estudios en Informatica (CLEI), which provided scholarships for several Latin American participants of the conference.

January 2005

Franz Baader
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Table of Contents

CERES in Many-Valued Logics*

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Abstract. CERES is a method for cut-elimination in classical logic which is based on resolution. In this paper we extend CERES to CERES-m, a resolution-based method of cut-elimination in Gentzen calculi for arbitrary finitely-valued logics. Like in the classical case the core of the method is the construction of a resolution proof in finitely-valued logics. Compared to Gentzen-type cut-elimination methods the advantage of CERES-m is a twofold one: 1. it is easier to define and 2. it is computationally superior and thus more appropriate for implementations and experiments.

1 Introduction

The core of classical cut-elimination methods in the style of Gentzen [8] consists of the permutation of inferences and of the reduction of cuts to cuts on the immediate subformulas of the cut formula. If we switch from two- valued to many-valued logic, the reduction steps become intrinsically tedious and opaque [3] in contrast to the extension of CERES to the many-valued case, which is straightforward.

We introduce CERES-m for correct (possible partial) calculi for m-valued first order logics based on m-valued connectives, distributive quantifiers [7] and arbitrary atomic initial sequents closed under substitution. We do not touch the completeness issue of these calculi, instead we derive clause terms from the proof representing the formulas which are ancestor formulas of the cut formulas; the evaluation of these clause terms guarantees the existence of a resolution refutation as core of a proof with atomic cuts only. This resolution refutation is extended to a proof of the original end-sequent by adjoining cut-free parts of the original proof. Therefore, it is sufficient to refute the suitably assembled components of the initial sequents using a m-valued theorem prover [2].

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2 Definitions and Notation

Definition 1 (language). *The alphabet Σ consists of an infinite supply of variables, of infinite sets of n-ary function symbols and predicate symbols er σ contains a set W of truth symbols denoting the truth values of the logic, a finite number of connectives $\circ_1, \ldots, \circ_m$ of arity $n_1, \ldots, n_m$, and a finite number of quantifiers $Q_1, \ldots, Q_k$.*

Definition 2 (formula). *An atomic formula is an expression of the form $P(t_1, \ldots, t_n)$ where P is an n-ary predicate symbol in Σ and $t_1, \ldots, t_n$ are terms over Σ. Atomic formulas are formulas.*

If $\circ$ is an n-ary connective and $A_1, \ldots, A_n$ are formulas then $\circ(A_1, \ldots, A_n)$ is a formula.

If Q is quantifier in Σ and x is a variable then $(Qx)A$ is a formula.

Definition 3 (signed formula). *Let $w \in W$ and A be a formula. Then $w{:}\,A$ is called a signed formula.*

Definition 4 (sequent). *A sequent is a finite sequence of signed formulas. The number of signed formulas occurring in a sequent S is called the* length *of S and is denoted by $l(S)$. $\hat{S}$ is called the* unsigned version *of S if every signed formula $w{:}\,A$ in S is replaced by A. The length of unsigned versions is defined in the same way. A sequent S is called* atomic *if $\hat{S}$ is a sequence of atomic formulas.*

Remark 1. Note that the classical sequent $(\forall x)P(x) \vdash Q(a)$ can be written as $\mathbf{f}{:}\,(\forall x)P(x), \mathbf{t}{:}\,Q(a)$.

m-valued sequents are sometimes written as m-sided sequents. We refrain from this notation, because it denotes a preferred order of truth values, which even in the two-valued case might induce unjustified conclusions.

Definition 5 (axiom set). *A set $\mathcal{A}$ of atomic sequents is called an axiom set if $\mathcal{A}$ is closed under substitution.*

The calculus we are defining below is capable of formalizing any finitely valued logic. Concerning the quantifiers we assume them to be of distributive type [7]. Distribution quantifiers are functions from the non-empty sets of truth-values to the set of truth values, where the domain represents the situation in the structure, i.e. the truth values actually taken.

Definition 6. *Let $A(x)$ be a formula with free variable x. The* distribution *$Distr(A(x))$ of $A(x)$ is the set of all truth values in W to which $A(x)$ evaluates (for arbitrary assignments of domain elements to x).*

Definition 7. *Let q be a mapping $2^W \to W$. In interpreting the formula $(Qx)A(x)$ via q we first compute $Distr(A(x))$ and then $q(Distr(A(x)))$, which is the truth value of $(Qx)A(x)$ under the interpretation.*

In the calculus defined below the distinction between quantifier introductions with (strong) and without eigenvariable conditions (weak) are vital.

Definition 8. *A* strong quantifier *is a triple* (V, w, w') *(for* $V \subseteq W$*) s.t.* $(Qx)A(x)$ *evaluates to* w *if* $Distr(A(x)) \subseteq V$ *and to* w' *otherwise. A* weak quantifier *is a triple* (u, w, w') *s.t.* $(Qx)A(x)$ *evaluates to* w *if* $u \in Distr(A(x))$*, and to* w' *otherwise.*

Remark 2. Strong and weak quantifiers are dual w.r.t. to set complementation. In fact to any strong quantifier there corresponds a weak one and vice versa. Like in classical logic we may speak about weak and strong occurrences of quantifiers in sequents and formulas.

Note that strong and weak quantifiers define merely a subclass of distribution quantifiers. Nevertheless the following property holds:

Proposition 1. *Any distributive quantifier can be expressed by strong and weak quantifiers and many valued associative, commutative and idempotent connectives (which are variants of conjunction and disjunction).*

Definition 9 (LM-type calculi). *We define an* LM*-type calculus* **K**. *The initial sequents are (arbitrary) atomic sequents of an axiom set* $\mathcal{A}$. *In the rules of* **K** *we always mark the auxiliary formulas (i.e. the formulas in the premiss(es) used for the inference) and the principal (i.e. the inferred) formula using different marking symbols. Thus, in our definition, classical* $\wedge$*-introduction to the right takes the form*

$$\frac{\Gamma, \mathbf{t}\colon A^{+} \quad \Gamma, \mathbf{t}\colon B^{+}}{\Gamma, \mathbf{t}\colon A \wedge B^{*}}$$

If $\Pi \vdash \Gamma, \Delta$ *is a sequent then* $\Pi \vdash \Gamma, \Delta^{+}$ *indicates that all signed formulas in* Δ *are auxiliary formulas of the defined inference.* $\Gamma \vdash \Delta, w\colon A^{*}$ *indicates that* $A\colon w^{*}$ *is the principal formula (i.e. the inferred formula) of the inference.*

Auxiliary formulas and the principal formula of an inference are always supposed to be rightmost. Therefore we usually avoid markings as the status of the formulas is clear from the notation.

logical rules:

Let $\circ$ *be an* n*-nary connective.For any* $w \in W$ *we have an introduction rule* $\circ\colon w$ *of the form*

$$\frac{\Gamma, \Delta_1^{+} \quad \ldots \quad \Gamma, \Delta_m^{+}}{\Gamma, w\colon \circ(\pi(\hat{\Delta}_1, \ldots, \hat{\Delta}_m, \hat{\Delta}))^{*}} \ \circ\colon w$$

where $l(\Delta_1, \ldots, \Delta_m, \Delta) = n$ *(the* Δ_i *are sequences of signed formulas which are all auxiliary signed formulas of the inference) and* $\pi(S)$ *denotes a permutation of a sequent* S.

Note that, for simplicity, we chose the additive version of all logical introduction rules.

In the introduction rules for quantifiers we distinguish strong *and* weak *introduction rules. Any strong quantifier rule* $Q{:}\,w$ *(for a strong quantifier* (V, w, w')*) is of the form*

$$\frac{\Gamma, u_1{:}\, A(\alpha)^+, \ldots, u_m{:}\, A(\alpha)^+}{\Gamma, w{:}\, (Qx)A(x)^*}\ Q{:}\,w$$

where α *is an eigenvariable not occurring in* Γ*, and* $V = \{u_1, \ldots, u_m\}$.

Any weak quantifier rule (for a weak quantifier (u, w, w')*) is of the form*

$$\frac{\Gamma, u{:}\, A(t)^+}{\Gamma, w{:}\, (Qx)A(x)^*}\ Q{:}\,w$$

where t *is a term containing no variables which are bound in* $A(x)$*. We say that* t *is* eliminated *by* $Q{:}\,w$.

We need define a special n*-ary connective for every strong quantifier in order to carry out skolemization. Indeed if we skip the introduction of a strong quantifier the* m *(possibly* $m > 1$*) auxiliary formulas must be contracted into a single one* after *the removal of the strong quantifier (see definition of skolemization below). Thus for every rule*

$$\frac{\Gamma, u_1{:}\, A(\alpha_1)^+, \ldots, u_m{:}\, A(\alpha_m)^+}{\Gamma, w{:}\, (Qx)A(x)^*}\ Q{:}\,w$$

we define a propositional rule

$$\frac{\Gamma, u_1{:}\, A(t)^+, \ldots, u_m{:}\, A(t)^+}{\Gamma, w{:}\, A(t)^*}\ c_Q{:}\,w$$

This new operator c_Q *can be eliminated by the de-skolemization procedure afterwards.*

structural rules:

The structural rule of weakening is defined like in **LK** *(but we need only one weakening rule and may add more then one formula).*

$$\frac{\Gamma}{\Gamma, \Delta}\ w$$

for sequents Γ *and* Δ.

To put the auxiliary formulas on the right positions we need permutation rules of the form

$$\frac{F_1, \ldots, F_n}{F_{\pi(1)}, \ldots, F_{\pi(n)}}\ \pi$$

where π *is a permutation of* $\{1, \ldots, n\}$ *and the* F_i *are signed formulas .*