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Semigroup Theory & Evolution Equations

semigroup theory and evolution equations

the second international conference

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Preface

The Second International Conference on Trends in Semigroup Theory and Evolution Equations was held September 25 to 29, 1989, at the Department of Technical Mathematics and Informatics of the Delft University of Technology, Delft, The Netherlands. The topics treated in this conference included recent developments in semigroup theory (e.g., positive, dual, integrated), and nonlinear evolution equations (e.g., maximal regularity, Navier-Stokes equations, Thomas-Fermi equations), control theory, and boundary value problems. In comparison with the previous conference in Trieste (1987), more emphasis was given to nonlinear aspects of the subjects.

On behalf of the Organizing Committee (C. J. van Duijn, C. A. Timmermans, and the editors), we express our thanks to the Scientific Committee (H. Amann, M. G. Crandall, G. Da Prato, O. Diekmann, and W. von Wahl) for their advice.

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On a Family of Generators of Analytic Semigroups

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0. INTRODUCTION

Let $\{A(t),\ t\in[0,T]\}$ be a family of generators of analytic semigroups in a complex Hilbert space H, and suppose that both $\{A(t)\}$ and $\{A(t)^*\}$ fulfil the assumptions of (Acquistapace and Terreni, 1987) in a somewhat strengthened form, i.e. assume that:

> for each $t\in[0,T]$, $A(t):D_{A(t)}\subseteq H\longrightarrow H$ is a closed linear operator; in addition there exist $\vartheta\in]\pi/2,\pi[$ and $M>0$ such that $\rho\big(A(t)\big)\supseteq\overline{S(\vartheta)}$, where $S(\vartheta):=\{z\in\mathbb{C}:|\arg z|<\vartheta\}$, and
> $$\|[\lambda-A(t)]^{-1}\|_{\mathcal{L}(H)}\le M[1+|\lambda|]^{-1}\quad\forall\lambda\in\overline{S(\vartheta)},\ \forall t\in[0,T]; \tag{0.1}$$

> there exist $N>0$ and $\alpha,\rho\in]0,1]$ with $\alpha+\rho>1$, such that
> $$\|A(t)[\lambda-A(t)]^{-1}[A(t)^{-1}-A(s)^{-1}]\|_{\mathcal{L}(H)}\le N|t-s|^{\alpha}[1+|\lambda|]^{-\rho}\qquad\forall\lambda\in\overline{S(\vartheta)},\ \forall t,s\in[0,T]; \tag{0.2}$$

> the operators $\{A(t)^*,t\in[0,T]\}$ satisfy (0.1) and (0.2) with the same constants $\vartheta,M,N,\alpha,\rho$. (0.3)

REMARK 0.1 By (0.1), the domains $D_{A(t)}$ are necessarily dense in H, so that $A(t)^*$ is well defined (and densely defined too).□

Denote by $\Sigma(H)$ the set of self-adjoint bounded linear operators on H: $\Sigma(H)$ is a Banach space with the $\mathcal{L}(H)$ norm. Consider for each $t\in[0,T]$ the operator

$$\Lambda(t)P = A(t)^{*}P + PA(t), \quad P\in\Sigma(H), \tag{0.4}$$

whose precise definition will be given in Section 1. It is known (see Sections 6.1, 6.2 in (Da Prato, 1973)) that for each $t\in[0,T]$, $\Lambda(t)$ generates an analytic semigroup in $\Sigma(H)$, and in addition $\Lambda(t)$ preserves positivity, i.e. if $P\in D_{\Lambda(t)}$ and $P\geq0$, then $\Lambda(t)P\geq0$.

Our goal is to show that under the above assumptions the family $\{\Lambda(t),\ t\in[0,T]\}$ fulfils the assumptions of (Acquistapace and Terreni, 1987), or, more precisely, satisfies (0.1) and (0.2), with ρ replaced by any smaller number, in the Banach space $\Sigma(H)$.

As an application of this result, we are able to show existence of classical solutions for an abstract non-autonomous Riccati equation arising in the study of the Linear Quadratic Regulator Problem for parabolic systems with boundary control. Due to lack of space, this application will appear in a forthcoming paper (Acquistapace and Terreni, in preparation).

REMARK 0.2 We may replace (0.2) by the slightly weaker condition

$$\|A(t)[\lambda-A(t)]^{-1}[A(t)^{-1}-A(s)^{-1}]\|_{\mathcal{L}(H)}\leq N\sum_{i=1}^{k}|t-s|^{\alpha_i}[1+|\lambda|]^{-\rho_i}$$
$$\forall\lambda\in\overline{S(\vartheta)},\ \forall t,s\in[0,T],$$

where $\alpha_i,\rho_i\in]0,1]$ and $\alpha_i+\rho_i>1$ for $i=1,\dots,k$; what is crucial here is that $\rho_i>0$, and this requirement makes such assumption stronger than that of (Acquistapace and Terreni, 1987), where on the contrary the ρ_i's are allowed to be possibly 0.

1. THE OPERATOR $\Lambda(t)$ FOR FIXED t.

A precise definition of the operator (0.4), for fixed $t\in[0,T]$, can be given in the following way (compare with (Da Prato, 1973)). Fix $P\in\Sigma(H)$ and consider the sesquilinear form defined on $D_{A(t)}\times D_{A(t)}$ by:

$$\Phi_P(t;x,y):=\left(A(t)x,Py\right)_H+\left(Px,A(t)y\right)_H\ , \quad x,y\in D_{A(t)}. \tag{1.1}$$

We set

$$D_{\Lambda(t)} := \{P\in\Sigma(H):\ \exists\ c(t;P)>0 \text{ such that}$$
$$|\Phi_P(t;x,y)| \le c(t;P)\|x\|_H\|y\|_H \quad \forall x,y\in D_{A(t)}\}. \tag{1.2}$$

If $P\in D_{\Lambda(t)}$, then $\Phi_P(t;\cdot,\cdot)$ has a unique extension $\hat{\Phi}_P(t;\cdot,\cdot)$ to $H\times H$ such that

$$\left.\begin{aligned} &\hat{\Phi}_P(t;x,y) = \Phi_P(t;x,y) \quad \forall x,y\in D_{A(t)}\ , \\ &|\hat{\Phi}_P(t;x,y)| \le c(t;P)\|x\|_H\|y\|_H \quad \forall x,y\in H\ ; \end{aligned}\right\} \tag{1.3}$$

hence by Riesz' Representation Theorem there exists an operator $Q_P(t)\in\mathscr{L}(H)$ such that

$$\hat{\Phi}_P(t;x,y) = \big(Q_P(t)x,y\big)_H \quad \forall x,y\in H. \tag{1.4}$$

Now we define

$$\Lambda(t)P := Q_P(t) \quad \forall P\in D_{\Lambda(t)}\ , \tag{1.5}$$

i.e.

$$\big(\Lambda(t)Px,y\big)_H = \hat{\Phi}_P(t;x,y) \quad \forall x,y\in H. \tag{1.6}$$

We remark that if $P\in D_{\Lambda(t)}$ and $x\in D_{A(t)}$ then in particular

$$\big|\big(Px,A(t)y\big)_H\big| = \big|\hat{\Phi}_P(t;x,y)-\big(A(t)x,Py\big)_H\big| \le$$
$$\le \big[c(t;P)\|x\|_H+\|A(t)x\|_H\big]\|y\|_H;$$

this means $Px\in D_{A(t)^*}$ and

$$\Lambda(t)Px = A(t)^*Px + PA(t)x \quad \forall x\in D_{A(t)},\ \forall P\in D_{\Lambda(t)}, \tag{1.7}$$

i.e. (0.4) holds when evaluated at any $x\in D_{A(t)}$. In particular, by (1.4), (1.3), (1.1) and (1.7) it follows easily that

$$\big(Q_P(t)x,y\big)_H = \big(x,Q_P(t)y\big)_H \quad \forall x,y\in D_{A(t)},$$

and therefore $\Lambda(t)P\equiv Q_P(t)\in\Sigma(H)$ for each $P\in D_{\Lambda(t)}$.

The operator $\Lambda(t)$ generates the semigroup $\{e^{\xi\Lambda(t)},\xi\ge 0\}\subseteq\mathscr{L}(\Sigma(H))$, defined by

$$e^{\xi\Lambda(t)}P:=e^{\xi A(t)^*}Pe^{\xi A(t)}, \quad P\in\Sigma(H); \tag{1.8}$$

indeed, we have:

PROPOSITION 1.1 Denote by 1 the identity operator on $\Sigma(H)$. We have:

(i) $D_{\Lambda(t)} = \{P\in\Sigma(H):\ \exists \lim_{\xi\to 0}\left(\frac{e^{\xi\Lambda(t)}-1}{\xi}Px,y\right)_H = \left(\Lambda(t)Px,y\right)_H\ \forall x,y\in H\}$;

(ii) $\overline{D_{\Lambda(t)}} = \{P\in\Sigma(H):\ \exists \lim_{\xi\to 0}\|\left(e^{\xi\Lambda(t)}-1\right)P\|_{\Sigma(H)} = 0\}$;

(iii) $\{P\in D_{\Lambda(t)}:\Lambda(t)P\in\overline{D_{\Lambda(t)}}\} =$

$$= \{P\in D_{\Lambda(t)}:\ \exists \lim_{\xi\to 0}\left\|\frac{e^{\xi\Lambda(t)}-1}{\xi}P-\Lambda(t)P\right\|_{\Sigma(H)} = 0\}.$$

Proof. (i) By (1.8) and (0.1)-(0.3) it follows that

$$\|e^{\xi\Lambda(t)}\|_{\mathscr{L}(\Sigma(H))} \le c(\vartheta,M) \quad \forall\xi>0,\ \forall t\in[0,T];$$

hence the argument of Chapter 9, Remark 1.5 of (Kato, 1966) shows that if $P,Q\in\Sigma(H)$ and

$$\lim_{\xi\to 0}\left(\frac{e^{\xi\Lambda(t)}-1}{\xi}Px,y\right)_H = (Qx,y)_H \quad \forall x,y\in H,$$

then $P\in D_{\Lambda(t)}$ and $\Lambda(t)P=Q$. Suppose conversely that $P\in D_{\Lambda(t)}$: then by (1.7) it is easy to get for each $x\in D_{A(t)}$ and $y\in H$:

$$\lim_{\xi\to 0}\left(\frac{e^{\xi\Lambda(t)}-1}{\xi}Px,y\right)_H = \lim_{\xi\to 0}\Big(\left(e^{\xi A(t)^*}-1_H\right)P\left[\xi^{-1}\left(e^{\xi A(t)}-1_H\right)\right]x +$$

$$+ \xi^{-1}\left(e^{\xi A(t)^*}-1_H\right)Px + P\left[\xi^{-1}\left(e^{\xi A(t)}-1_H\right)\right]x,y\Big)_H = \left(\Lambda(t)Px,y\right)_H;$$

hence by (1.6) we get the result since $D_{A(t)}$ is dense in H.
(ii)-(iii) See Proposition 1.2(i)-(iii) of (Sinestrari, 1985). □

EXAMPLE 1.2 $D_{\Lambda(t)}$ is not dense in $\Sigma(H)$ in general (unless, of course, the $A(t)$'s are bounded. Indeed, set $H:=L^2(0,\pi)$, and $A(t)\equiv A:=d^2/dx^2$, with $D_A:=W^{2,2}(0,\pi)\cap W_0^{1,2}(0,\pi)$; then we have

$$e^{\xi A}f = e^{\xi A^*}f = \sum_{n=1}^{\infty}\exp(-n^2\xi)f_n e_n \quad \forall\xi>0,\ \forall f\in H,$$

where $e_n(x) := (2/\pi)^{1/2}\sin(nx)$, $f_n := (f, e_n)_H$. Now if D_Λ were dense in $\Sigma(H)$, then we should have, choosing $P := 1_H$:

$$\lim_{\xi \searrow 0} \left\| \left(e^{\xi\Lambda} - 1_{\Sigma(H)}\right) 1_H \right\|_{\Sigma(H)} = 0,$$

i.e. for each $\varepsilon > 0$ there should exist $\delta_\varepsilon > 0$ such that

$$\sup \left\{ \left\| \left(e^{2\xi A} - 1_H\right) f \right\|_H : \ \|f\|_H = 1 \right\} < \varepsilon \quad \forall \xi \in]0, \delta_\varepsilon[;$$

hence by taking $f := e_n$, $n \in \mathbb{N}^+$, we would get

$$\left\| \left(e^{2\xi A} - 1_H\right) e_n \right\|_H \doteq 1 - \exp(-2n^2\xi) < \varepsilon \quad \forall n \in \mathbb{N}^+, \ \forall \xi \in]0, \delta_\varepsilon[,$$

which is impossible. □

REMARK 1.3 Despite of Example 1.2, we obviously have

$$\lim_{\xi \searrow 0} \left\| \left(e^{\xi\Lambda(t)} - 1_{\Sigma(H)}\right) Px \right\|_H = 0 \quad \forall P \in \Sigma(H), \ \forall x \in H, \ \forall t \in [0, T]. \ \square \tag{1.9}$$

2. MAIN RESULT

By (0.1)-(0.3) and the results of (Acquistapace and Terreni, 1986), (Acquistapace, 1988), (Acquistapace, Flandoli and Terreni, 1990, in press), (Acquistapace and Terreni, 1990) we can construct the evolution operator $U(t,s)$ associated to $\{A(t)\}$, and the following properties hold true:

PROPOSITION 2.1 For $0 \le s < t \le T$ we have:

(i) $U(t,s) = U(t,r)U(r,s) \quad \forall r \in [s,t], \quad U(t,t) = 1_H$;

(ii) $U(t,s) \in \mathcal{L}\left(H, D_{A(t)}\right)$ and $\exists\, dU(t,s)/dt = A(t)U(t,s)$;

(iii) $U(t,s)^* \in \mathcal{L}\left(H, D_{A(s)^*}\right)$ and $\exists\, dU(t,s)^*/ds = -A(s)^*U(t,s)^*$;

(iv) $\exists\, dU(t,s)/ds = -\left[A(s)^*U(t,s)^*\right]^*$;

(v) $\|U(t,s)\|_{\mathcal{L}(H)} + \|U(t,s)^*\|_{\mathcal{L}(H)} + (t-s)\|dU(t,s)/dt\|_{\mathcal{L}(H)} +$
$+ (t-s)\|dU(t,s)/ds\|_{\mathcal{L}(H)} \le c(\vartheta, M, N, \alpha, \rho, T)$.

Proof. (i)-(ii) See Theorem 2.3 of (Acquistapace, 1988).
(iii) See (6.11) of (Acquistapace and Terreni, 1990).
(iv) See Theorem 6.4 of (Acquistapace and Terreni, 1990).

(v) See Theorem 2.3 of (Acquistapace, 1988) and Theorem 6.4 of (Acquistapace and Terreni, 1990). □

Consider now the operator $E(\cdot,\cdot):\Sigma(H)\longrightarrow\Sigma(H)$ defined by

$$E(t,s)P:=U(T-s,T-t)^{*}PU(T-s,T-t), \quad 0\le s\le t\le T, \ P\in\Sigma(H). \tag{2.1}$$

A straightforward computation shows that $E(t,s)$ is strongly continuous in $\Sigma(H)$, and in addition if $0\le s<t\le T$

$$\left.\begin{aligned} &E(t,s) = E(t,r)E(r,s) \quad \forall r\in[s,t], \quad E(t,t) = 1_{\Sigma(H)}; \\ &\frac{d}{dt}E(t,s)P = \Lambda(T-t)E(t,s)P \quad \forall P\in\Sigma(H), \\ &\frac{d}{ds}E(t,s)P = -E(t,s)\Lambda(T-s)P \quad \forall P\in D_{\Lambda(T-s)}; \end{aligned}\right\} \tag{2.2}$$

hence $E(t,s)$ is the (necessarily unique) evolution operator associated to $\{\Lambda(T-t),t\in[0,T]\}$. We will show in our main Theorem 2.3 below that the family $\{\Lambda(T-t)\}$ satisfies (0.1) and (0.2) (with ρ replaced by any smaller number) in the space $\Sigma(H)$. As a consequence of Theorem 2.3, the results of (Acquistapace and Terreni, 1987), (Acquistapace and Terreni, 1986) and (Acquistapace, 1988) immediately imply several regularity properties for the evolution operator $E(t,s)$.

REMARK 2.2 Of course, many smoothness properties for $E(t,s)$ and $E(t,s)^{*}$ may also be directly derived by (2.1), using the regularity results for $U(t,s)$ and $U(t,s)^{*}$ proved in (Acquistapace, 1988), (Acquistapace, Flandoli and Terreni, 1990, *in press*), (Acquistapace and Terreni, 1990). However we believe that Theorem 2.3 has some interest in itself, since it provides a new class of generators of analytic semigroups having a good dependence on t (i.e. satisfying (0.1) and (0.2)); this class is not the "usual" abstract version of some elliptic operator with time-dependent coefficients and homogeneous boundary conditions, acting on sone concrete function space, although its construction in fact starts from an operator of that kind. □

THEOREM 2.3 Under assumptions (0.1)-(0.3) the operators $\Lambda(t)$, defined by (1.2), (1.6), enjoy the following properties: