HADRON TRANSITIONS IN THE QUARK MODEL

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HADRON TRANSITIONS IN THE QUARK MODEL

Preface

A l'exemple d'icelui vous convient être sages, pour fleurer, sentir et estimer ces beaux livres de haute graisse, légers au pourchas et hardis à la rencontre. Puis, par curieuse leçon et méditation fréquente, rompre l'os et sucer la substantifique moelle, c'est à dire ce que j'entends par ces symboles pythagoriques, avec espoir certain d'être faits escors et preux à la dite lecture, car en icelle bien autre goût trouverez, et doctrine plus absconse, laquelle vous révèlera de très hauts sacrements et mystères horrifiques, tant en ce qui concerne votre religion que aussi l'état politique et vie économique.

RABELAIS, *Gargantua*, prologue

Hadronic physics in the quark model has become an area of investigation so wide and specialized that a study of a particular aspect such as hadron spectroscopy or hadronic transitions at low energy can be justified. As there are already good accounts of the spectroscopy of hadrons in the literature, we have considered it both important and interesting to deal with the topic of hadronic transitions. The aim of this book, then, is to study electromagnetic, weak and strong transitions between hadrons in the context of the quark model.

The level of the book corresponds approximately to that of postgraduate students with a good background in quantum mechanics and also some experience with quantum field theory and the standard $SU(2) \times U(1) \times SU(3)$ model of electroweak and strong interactions. The book is intended to be of use to a wide audience of particle physicists, both experimentalists and theorists. The methods of the quark model developed here are applicable not only to the subjects that we treat in detail, but also to a number of other areas of investigation.

In our choice of the topics we have naturally often been motivated by our own research. Although this choice is thus necessarily somewhat subjective, we have tried to avoid too biased a view and to give a fair account of the main lines of investigation in a limited field, which is precisely defined in the Introduction

(Section 1.1). To complete the references in the text and to guide the reader in the subjects that have been too briefly treated, we provide a survey of the relevant literature at the end of the book. The choice of subjects to cover should not imply a judgement on their value, but is a result of our personal choices.

The book is organized in the following way. Chapter 1 is an introductory review of the basis of the quark model. We have tried first to make precise in Section 1.1 the present status of the naive quark model within the recent developments in particle physics. Then, in Section 1.2, there is an introduction to the quark interactions and their symmetries according to the standard $SU(2) \times U(1) \times SU(3)$ model. Finally, there is a brief but self-contained account of hadron spectroscopy in Section 1.3.

We have made a clear distinction between methods (Chapters 2 and 3) and phenomenological applications (Chapter 4). Chapter 4 is approximately self-contained, and the interested reader who is already familiar with the quark model can read it immediately.

In Chapter 2 we describe methods for calculating transition amplitudes—electromagnetic, weak and strong—within the nonrelativistic quark model. Although electroweak processes (Section 2.1) can be studied by standard methods, this is not the case for strong interactions (Section 2.2), and we discuss in this latter case a number of models that have been proposed.

Chapter 3 is devoted to methods beyond the nonrelativistic approximation. This is perhaps the more specialized contribution of the book. The various methods proposed in the literature for the study of relativistic corrections are described, with a critical account of their advantages and drawbacks: the Dirac equation in a central potential (Section 3.2), Hamiltonian methods (Section 3.3) and the Bethe–Salpeter amplitudes and Mandelstam formalism (Section 3.4). We emphasize which types of corrections to the nonrelativistic quark model of Chapter 2 are obtained. Also, processes like $\pi^0 \to \gamma\gamma$, linked to special concepts in quantum field theory, could not be treated in Chapter 2 and are included here.

Finally, Chapter 4 is an account of phenomenological applications. We have treated different phenomena with a unified set of parameters to make consistent the comparison of the quark model with a great variety of data. We have made a selection of those applications that can give a fair survey of the wide success of the quark model, and also of the open problems in the area. We have also emphasized the distinctive features of the more specific and predictive quark model in comparison with general algebraic schemes. After general remarks on the parameters and the practical application of the model (Section 4.1), we study semileptonic transitions, radiative transitions, strong processes, annihilation processes and finally nonleptonic weak decays, in

Sections 4.2–4.6 respectively. The data and particle symbols are taken from the Particle Data Group (PDG) tables (1984 edition).

The main components of this book were first intended as a contribution to a second volume of the book by Flamm and Schöberl (1982). The reader who wants more detailed descriptions of the fundamentals of the quark model and of spectroscopy than are given in Chapter 1 can profitably consult the latter book. We should like to thank Dieter Flamm for having stimulated us to write this book.

We should also like to thank Mireille Calvet for having typed the manuscript, and Jacqueline Bellone and Patricia Flad for their help in its preparation.

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OLIVIER PÈNE
JEAN-CLAUDE RAYNAL

Contents

CHAPTER 1

Introduction

Hunc igitur terrorem animi tenebrasque necessest non radii solis neque lucida tela diei discutiant, sed naturae species ratioque.†

T. LUCRETI, *Cari De Rerum Natura*, II

Can we actually "know" the universe ? My God, it's hard enough finding your way around in Chinatown.

WOODY ALLEN, *Getting Even*

I tax not you, you elements, with unkindness.

W. SHAKESPEARE, *King Lear*, III.2

1.1 OUTLOOK AND MOTIVATION

1.1.1 Quarks and Hadrons

In the past 20 years, decisive progress in our understanding of particle physics has been achieved. Matter and its interactions are now understood in terms of a few building blocks: the quarks and the leptons, which are fermions, and the *gauge* bosons — the photon, W and Z bosons, and gluons, associated with the gauge invariance group. The fundamental interactions are trilinear interactions between the fermions and the gauge bosons, analogous to that of quantum electrodynamics (QED), plus interactions between the gauge bosons themselves, associated with the nonabelian character of the gauge groups.

This well-defined picture constitutes a whole program that has been widely tested and is already very convincing. Electromagnetic and weak interactions are unified in the Glashow–Salam–Weinberg model based on the $SU(2) \times U(1)$ gauge group: it has been beautifully confirmed by the observation of the W and Z bosons. The point that remains obscure is the status of the additional Higgs particles. We shall frequently call these interactions *electroweak*. However, we

† This dread and darkness of the mind cannot be dispelled by the sunbeams, the shining shafts of day, but only by an understanding of the outward form and inner workings of nature.

are particularly interested in the new picture that has emerged for the strong interactions. Just as electromagnetic interactions are induced by the exchange of virtual photons between charges, so strong interactions are related to the exchange of *gluons* between *color* sources. Color is a set of quantum numbers characterizing the behavior of the source under the gauge group $SU(3)_c$, just as the electric charge characterizes the behavior of sources under the electromagnetic gauge group U(1). A specific feature of strong interactions is that owing to the nonabelian character of $SU(3)_c$ the gluons themselves are colored, whereas photons are not charged. Among the building blocks enumerated above, only quarks and gluons are colored and are affected by the strong interaction. The quantum field theory of strong interactions is known as quantum chromodynamics (QCD). The present theory of the electroweak and strong interactions based on the gauge group $SU(3)_c \times SU(2) \times U(1)$ is called the *Standard Model*.

Hadrons, which are the subject of this book, have a conceptually simple definition. They are composite particles made out of the color constituents, quarks and gluons, but which are themselves *color-neutral*. Just as electrically neutral atoms undergo electromagnetic interactions because of the charge of their constituents, so hadrons, in spite of being colorless, still undergo strong interactions. Of course, these interactions are weaker than those between the colored elementary constituents. In fact, there is strong evidence that the interaction energy between colored sources tends to infinity with separation; this is attributed to the self-interaction of gluons — a distinctive feature of QCD in contrast with QED. The net effect of the very strong repulsions and attractions that are generated by color is the combination of quarks and gluons into color-neutral hadrons. Within hadrons, the forces between the constituents are attractive — so strongly attractive in fact that the constituents cannot be separated: this property is termed *confinement* of quarks and gluons within color-neutral states.

Strictly speaking, the number of quarks and gluons within a given hadron is not defined in quantum field theory. However, within the approximation of the nonrelativistic quark model, one can associate a particular hadron with a definite number of quarks. The usual hadrons are then composed of quarks only: mesons of a quark and an antiquark, and baryons of three quarks. Other hadrons can be constructed in this way, but have not yet been observed. These include *glueballs*, containing only gluons, *hybrids*, containing both quarks and gluons, as well as hadrons containing more than three quarks — *diquonia*, with two quark–antiquark pairs and *dibaryons*, with six quarks. Our discussion here, however, will concentrate on the usual hadrons: mesons and baryons.

1.1.2 Transitions of Hadrons

Consider the various types of particles distinguished by different values of the conserved quantum numbers N_B (baryon number), N_ℓ (lepton number): gauge bosons and mesons, leptons and baryons. Only the lowest-mass states in each of these categories are stable: the photon, the electron and the neutrinos, and the proton. Quarks and gluons, being colored, cannot exist separately because of confinement. Certain particles decay only through electroweak interactions, and since these are relatively weak, the corresponding lifetimes are still sizable, and they are classified as *stable* particles by the Particle Data Group (1984).

Most hadrons, however, are much less stable because they decay strongly, i.e. they are not even stationary states under the action of the QCD Hamiltonian H_{QCD}. Such states have very short lifetimes, not directly observable, and constitute a very large part of the hadron spectrum. These hadron levels are characterized by their degeneracy with a continuum of multihadron states to which they can decay by QCD interactions. For example, $\Delta(1236)$ is degenerate with $N\pi$ states, and N(1520) with $N\pi$ and $N\pi\pi$ states. Certain states avoid decay because of the conservation of *flavor* quantum numbers by the QCD interaction. Indeed, each flavor or species of quark u, d, s, c, b or t can appear or disappear only by creation or annihilation with an antiquark of the same species, or, in terms of quark diagrams, any quark line has a definite species. This principle is only violated by the charged weak interactions. Let us then associate with each flavor an additive quantum number N_i, which is the number of quarks minus the number of antiquarks of flavor i. N_i must be conserved in a strong or electromagnetic process, and each hadron has a definite set of quantum numbers $\{N_i\}$. In each sector of hadrons having the same set of quantum numbers $\{N_i\}$, the lowest-mass state can decay only through the weak interaction if at least one $N_i \neq 0$; it can also decay electromagnetically if all $N_i = 0$. Examples of the first case are the decays of π^+ ($N_u = +1, N_d = -1$), n, Λ, D, ..., which decay weakly, or of the p, which cannot decay within the standard model $SU(2) \times U(1) \times SU(3)_c$. An example of the second case is the π^0 ($N_u = N_d = 0$), which decays electromagnetically through $\pi^0 \to 2\gamma$. The higher-mass hadrons in each flavor sector will in general decay strongly to the lowest-mass state plus a π. This may be still forbidden by energy–momentum conservation, as in the decay of Σ^0, which proceeds electromagnetically, $\Sigma^0 \to \Lambda\gamma$, or by additional conservation laws, P and C parity, and isospin conservation. Otherwise, the high-mass states are strongly unstable.

The above discussion on unstable hadrons has been rather imprecise. The concept of the mass of the unstable particle has been used, although such a state is, by definition, not an eigenstate of energy–momentum. The rigorous treatment of an unstable state is a standard question. The most direct way is to separate the Hamiltonian into a part H_0 for which the decaying state as well as the multihadron continuum are eigenstates, and a part H_I that is responsible for the transition to the continuum and which one hopes can be treated using perturbation theory. This perturbation H_I will also shift the energy levels. The separation $H = H_0 + H_I$ is trivial when the particle decays only through the electroweak interactions: $H_0 = H_{QCD}$, $H_I = H_{EW}$ (the electroweak Hamiltonian). On the other hand, if the particle is not stationary under H_{QCD}, i.e. if it decays strongly, there is no known systematic procedure for making such a separation. In fact, as we will see later, there are only phenomenological approaches based on the quark model, which try to define H_I through the creation of a $q\bar{q}$ pair for example.

It happens that some strongly unstable particles have a rather small width for an important reason. These are the hadrons that undergo only strong decays forbidden by the so-called Okubo–Zweig–Iizuka (OZI) rule (Okubo, 1963; Zweig, 1964; Iizuka, 1966), which applies when for example the quark–antiquark species of a meson annihilate and do not appear in the final state (see Chapter 2). This rule is reinforced in the case of heavy flavors like the $c\bar{c}$ mesons ψ below the $D\bar{D}$ threshold. In this case the decay amplitude itself is small and can be calculated by perturbation theory, because the strength of the QCD interaction decreases — a property of QCD called *Asymptotic Freedom.* However, even in this case, one cannot define such states as eigenstates of some part H_0 of H_{QCD}. A powerful method that circumvents the problem of an explicit separation of the Hamiltonian is to define the unstable state as a resonance in a scattering amplitude of stable particles. The unstable particle appears as a complex pole in the complex energy plane, or a bump in real energy. For instance, the $\Delta(1236)$ manifests itself as a bump in πN scattering. This is how strongly unstable particles are observed, as their lifetime is too short to observe a track. The definition of the unstable state as a complex pole in some energy variable applies not only to the scattering amplitude, but also to other quantities, which are easier to calculate — in particular to the correlation functions of local operators, a method that is used in lattice QCD calculations and QCD sum rules.

The idea of unstable hadrons that decay strongly but are nevertheless somewhat comparable to the stable ones, is fundamental in the analysis of low- and medium-energy processes. Strong decays of excited hadron states form a large component of the available experimental data. The decay of resonances to multihadron states can often be viewed as a decay sequence through intermediate resonances. For example, $N(1520) \to N\pi\pi$ can be viewed as $N(1520) \to \Delta(1236)\pi$, $\Delta(1236) \to N\pi$. The strong decays can thereby be reduced to a chain of two-body decays. In the same way, electroweak decays where the final state includes a photon or a lepton pair and a number of stable hadrons can quite often be described by a transition between the initial hadron and a resonance, followed by the strong decay of the resonance; for example D $\rightarrow K\pi e\nu$ can be viewed as $D \rightarrow K^* e\nu$, $K^* \rightarrow K\pi$. By such methods, the structure of decays is greatly simplified and more easily related with theoretical schemes. This procedure, on the other hand, allows the measurement of new quantities. For example, the transition $N(1520) \rightarrow \Delta\pi$ would not be otherwise observable. Since strong decays are much more rapid than electroweak ones, it might be feared that electroweak transitions of unstable hadrons such as $\Delta \rightarrow N\gamma$ would be impossible to measure, because they would be completely masked by the strong ones. But they are indirectly observable, because the transition $\Delta \rightarrow N\gamma$ partial width is also measured in the process $\gamma N \rightarrow N\pi$ in the vicinity of the $\Delta(1236)$ resonance. In the same way, the weak semileptonic transition $\Delta^{++} \rightarrow p\mu^+\nu_\mu$ is measurable through the neutrino-production reaction $\nu_\mu p \rightarrow \mu^- p\pi^+$. Finally, certain quantities are very akin to transition amplitudes, although they do not correspond to actual decays allowed by phase space. For example, the magnetic moment of the proton μ_p has essentially the same structure as the $\Delta \rightarrow N\gamma$ amplitude. The same applies to the axial couplings of the nucleon that can be measured by the reaction $\nu_\mu n \rightarrow \mu^- p$ at low momentum transfer or to the pion–nucleon coupling. The set of experimental data for particles decaying by the electroweak interactions is thus greatly enlarged by the hadron levels that decay strongly and by the various couplings of the stable states.

1.1.3 Fundamental Theory Versus the Quark Model

It is relatively easy to treat the effects of electroweak interactions by perturbation theory. However, for any type of transition under study, we are confronted with the problem of dealing with QCD. Hadrons, either stable or unstable, involve QCD essentially and there is no known general solution for this complicated field theory. On the other hand, one cannot expect much from perturbation theory in a situation that is fundamentally not one of weak coupling.

The usual way to treat local interactions is through perturbation theory, by expanding various quantitites in powers of the coupling constant. More precisely, taking as an example bound states in QED, one must first sum the effect of multiple Coulomb interactions, which generate the bound states, by solving the Schrödinger equation with a Coulomb potential. Any number of Coulomb exchanges within a bound state contribute with the same order of magnitude, in spite of the additional powers of $\alpha = e^2/4\pi$ in each Coulomb interaction. Once the Schrödinger equation has been solved, any matrix element can be approximated by a matrix element between Schrödinger wave functions, to be corrected by terms of order α, α^2, ... These corrective terms correspond to relativistic kinetic energy effects, to transverse photon exchanges, etc.

This procedure is not possible for QCD. The difficulties are the well-known difficulties of strong interaction theory. Since the strong interactions are characterized by large coupling constants, perturbation theory is not useful. However, this old idea is made much more precise by QCD. First, all the strong interactions are expressed in terms of a dimensionless coupling constant $\alpha_s = g_s^2/4\pi$. However, owing to the asymptotic freedom of QCD, this coupling constant is not always large. Indeed, it depends on a renormalization scale μ connected with the typical momentum transfers or gluon momenta in the process. At large μ, $\alpha_s(\mu^2)$ becomes small, which allows perturbation theory to be used in the so-called *hard processes*. Hard processes occur for example in electron– or neutrino–nucleon deep inelastic scattering, and also in some decays. This is the case, as we have already indicated, of the OZI-forbidden strong decays of heavy quarkonia (bound states of a quark and an antiquark of the same flavor) and also of some weak decays of heavy flavors. On the other hand, for a small renormalization scale, α_s is large. This small scale $\mu < 1\ \mathrm{fm}^{-1}$ is the one relevant for the strong decays of hadrons composed of light quarks, and in general for the OZI-allowed strong decays, for electromagnetic transitions (except when connected with OZI-rule violation, as in $J/\psi \to \eta\gamma$), and for weak decays of light flavors like hyperon or kaon decays. In these latter cases, which are the main part of our subject, perturbation theory in α_s is hopeless. In any case, even in processes where one can use perturbative QCD, there are in general parts where the relevant α_s is not small. For instance, in hard scattering processes only the parton process (e.g. the scattering of a quark by a photon, a gluon or another quark) is perturbatively calculable. However, the relation between the partons (quarks and gluons) and the observed hadrons is described by structure and fragmentation functions, which are not calculable in perturbation theory. Analogously, in the annihilation of J/ψ into light hadrons, one calculates by perturbative QCD the annihilation of the $c\bar{c}$ pair into hard gluons. However, the result has to be multiplied by the wave function of the $c\bar{c}$ system at the origin, a quantity that is not calculable by perturbation theory, i.e. with a Coulomb potential.

The fact that α_s is large is not the only point preventing the use of perturbative QCD. There are in QCD two main phenomena that are essentially nonperturbative and cannot be obtained even by summing the whole perturbation series. The first one is *confinement*, described above. The second is *dynamical breaking of chiral symmetry*. Both are connected with nonzero vacuum expectation values, which would vanish at any order in perturbation theory: the so-called *gluon condensate* and the *quark condensate*. The first seems related, as we have already mentioned, to an interaction energy that increases with distance (Leutwyler, 1979; Baker, Ball and Zachariasen,