An Elementary Approach to Functions

Second Edition

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Preface to the Second Edition

In this second edition the authors maintain the tenets of the first edition. The basic structure and tone of the original text remains the same. Two sections were deleted that proved to be of little use, and certain new materials were added. Notable among these changes are:

1 A new chapter on basic trigonometry, Chapter 8, has been added. This gives the user the option of a full course in numerical trigonometry; however, this chapter is independent of the others and may be omitted.

2 Systems of Linear Equations (in Chapter 1) has been expanded to include 3×3 systems. Determinants are introduced along with Cramer's rule.

3 The entire set of exercises in Chapter 2 has been reconstructed offering the student more problems starting at a slow pace and gradually increasing in difficulty.

4 Table 6 of the Appendix was shortened making its use more convenient.

5 Over 400 new problems were added throughout the book.

This text has a wide range of topics making it very flexible for many different meaningful courses. We recommend that individual instructors select topics most beneficial for their students.

Offered here are four tried course suggestions:

1 One-semester (no review of algebra and geometry; no analytic geometry); Chapters: 2, 4, 5, 6, 7, 9, and 10.

2 One-semester (emphasis on graphing techniques and analytic geometry); Chapters: 1, 2, 3, 5, 6, 11, and 12.

3 One-semester (algebra and analytic geometry); Chapters: 1, 3, 4, 11, and 12; Sections 6.4 and 10.5.

4 One-semester (algebra and trigonometry); Chapters: 1, 2, 3, 4, 8, and 10.

We wish to take this opportunity to thank all those who have expressed an interest in our text.

HENRY R. KORN
ALBERT W. LIBERI

Preface to the First Edition

The purpose of this text is to prepare students for an introductory course in the calculus. The approach is a concrete development motivated through problem solving and relevant applications. The student is led into a classical approach to mathematics, where a minimum amount of notation is employed and the reliance on set theory is omitted.

It is the opinion of the authors that a student entering the calculus should have a strong working knowledge of algebraic techniques and should also be familiar with such concepts as functions, absolute value, analytic geometry, and inequalities. Graphing techniques are used extensively throughout the text. The student is encouraged to draw information from algebraic procedures and apply the results in obtaining an illustrative graph.

An ample prerequisite for this text should be two years of high school mathematics (algebra and geometry). However, it is evident to most teachers that students with even stronger backgrounds often lack the mathematical maturity to bridge the gap between high school mathematics and the calculus. For this reason, there is a need for a one- or two-semester period of adjustment.

The examples in the text have been carefully selected and completely analyzed. Students should be encouraged to work through the examples with pencil and paper. Much of the theory and many useful algebraic techniques are exploited in the examples. The comments which are included should prove quite useful to the student since they were generated from actual classroom discussions.

There are over 300 examples with step-by-step solutions. The text also contains nearly 300 illustrations and 2000 exercises. The exercises in each chapter are of a varying degree of difficulty, and were chosen to illustrate as many facets of the topic covered as possible. Repetition has been avoided and algebraic manipulation emphasized.

The text contains adequate material for a meaningful two-semester course. However courses of this type are usually one semester in length and it is our recommendation that individual instructors select the topics most beneficial for their student.

Some course suggestions are as follows:

1 one-semester (no review of algebra and geometry; no analytic geometry); Chapters: 2, 4, 5, 6, 7, 8, and 9.

2 one-semester (emphasis on graphing techniques and analytic geometry); Chapters: 1, 2, 3, 5, 6, 10, and 11.

3 one-semester (algebra and analytic geometry); Chapters: 1, 3, 4, Sections: 6.5, 9.5, and Chapters: 10, and 11.

The authors wish to thank their wives, Susan and Jackie, for their endless hours of typing and constructive criticism, without which the manuscript would have certainly been disorganized and unfinished. We would also like to express special appreciation to Ms. Shelly Levine Langman, McGraw-Hill editing supervisor, not only for her enthusiasm and her professional abilities, but also for the extra attention she gave to us and the detailed analyses which transformed our manuscript into this text. We owe a debt of gratitude to Mr. Jack Farnsworth, McGraw-Hill editor, who gave us the incentive for completing this project by demonstrating his faith in our unproven abilities.

Finally, we would also like to extend our thanks to our students and fellow faculty members at Westchester Community College, who struggled through the original version of the text for three semesters.

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Chapter 1

The Straight Line

1.1
A REVIEW OF THE RECTANGULAR COORDINATE SYSTEM

In order to locate a point in a plane, we follow the *rectangular coordinate method.* In this system a plane is divided into four regions, called *quadrants.* The system contains two straight lines, called *axes,* one of which is horizontal and the other vertical. The horizontal line, which extends indefinitely to the left and right, is called the *x axis.* The vertical line, which extends indefinitely up and down (and intersects the horizontal line at right angles), is called the *y axis.* The point of intersection of the two lines is called the *origin* (see Diagram 1).

Although it is not necessary that the two axes be mutually perpendicular, we will use this frame of reference throughout this development. The four quadrants into which the axes divide the plane are conventionally labeled I, II, III, and IV (counterclockwise convention). Along the x axis, positive and negative units are to the right and left of the origin, respectively. Likewise, along the y axis, positive and negative units are above and below the origin, respectively (see Diagram 2).

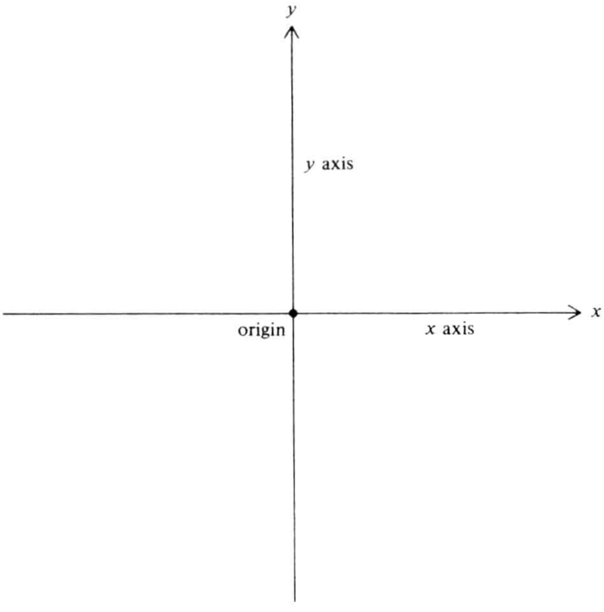

Diagram 1

To locate a point in this plane, we can choose two numbers. The first number will always indicate units to the right or left of the origin. The second number will always indicate units above or below the origin. The order in which we select these numbers is important in locating a point, and we write these numbers in parentheses. The first entry is called the *first coordinate,* and

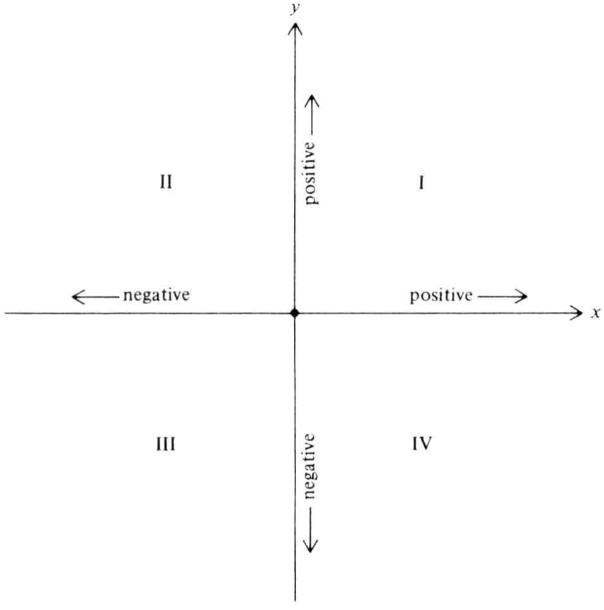

Diagram 2

the second entry is called the *second coordinate* of the point. The arrangement is called an *ordered pair.*

Example 1 (3, 1).
Here 3 is the first coordinate, and 1 is the second coordinate.

Example 2 (1, 3).
Here 1 is the first coordinate, and 3 is the second coordinate.

Let us locate these two points in Diagram 3. Note that each point in the plane has *two* coordinates and the order in which they are taken is important.

The first coordinate is referred to as the *abscissa* of the point, and the second coordinate is referred to as the *ordinate* of the point.

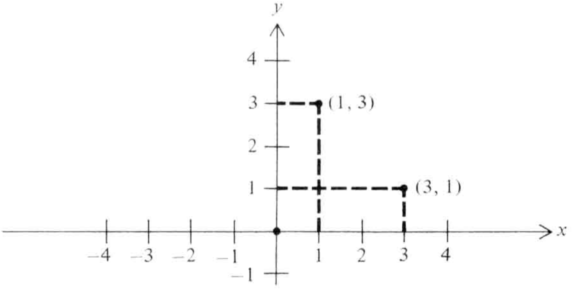

Diagram 3

DEFINITION 1 *Equality of ordered pairs* $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Example 3 Find a and b, given $(2, b) = (a + 1, 2)$.
From the definition for equality of ordered pairs $2 = a + 1$ and $b = 2$. Therefore, $a = 1$ and $b = 2$.

From Diagram 4, observe that the points P_1 through P_6 have associated with them the ordered pairs $P_1(2, 0)$, $P_2(0, -4)$, $P_3(4, 3)$, $P_4(-5, 4)$, $P_5(-3, -1)$, and $P_6(3, -2)$.

COMMENT It will be helpful to verify that the coordinates of points located in the different quadrants have the following signs:

Quadrant I:	(+, +)
Quadrant II:	(−, +)
Quadrant III:	(−, −)
Quadrant IV:	(+, −)

Points that lie on the x axis have the second coordinate zero, $(a, 0)$. See P_1 in Diagram 4. Points that lie on the y axis have the first coordinate zero, $(0, b)$. See P_2 in Diagram 4. There is no rule which demands using the same scale for both the x and y axes. A scale should be chosen which will be most adaptable.

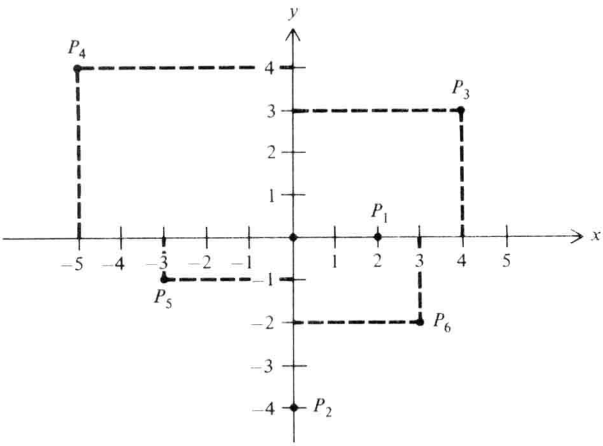

Diagram 4

Suppose that the point $P(-100, 1)$ is to be plotted. Note, in Diagram 5, that the scale of the x axis has been compressed to save space. In such cases the diagram should always be accurately labeled.

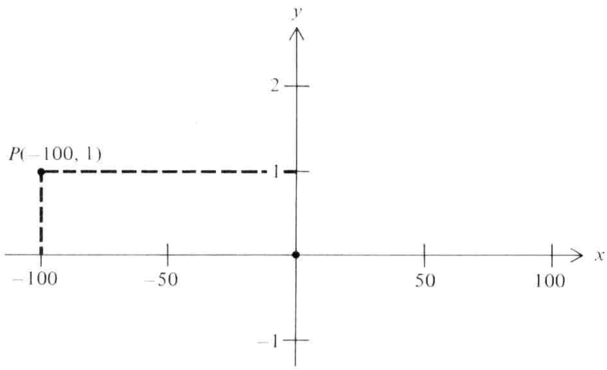

Diagram 5

1.2
THE SYMBOL Δ AND ABSOLUTE VALUE

If we select an *initial point* $A(x_1, y_1)$ and travel to a new point $B(x_2, y_2)$, which we call the *terminal point,* the value of the abscissa will change from x_1 to x_2 and the value of the ordinate will change from y_1 to y_2.

A symbol is used to represent such changes: Δx (read "delta x," not delta times x) and Δy (read "delta y," not delta times y).

$$\Delta x = x \text{ of the terminal point} - x \text{ of the initial point}$$

and $$\Delta y = y \text{ of the terminal point} - y \text{ of initial point}$$

DEFINITION 1 If a particle moves from an initial point $A(x_1, y_1)$ to a terminal point $B(x_2, y_2)$, the *increments* Δx and Δy are given by

$$\Delta x = x_2 - x_1 \qquad \text{and} \qquad \Delta y = y_2 - y_1$$

COMMENT It follows from the above definition that both Δx and Δy can be positive, negative, or zero.

Example 1 Consider a particle moving from $A(-1, 4)$ to $B(2, -3)$ (see Diagram 6). Find Δx and Δy.

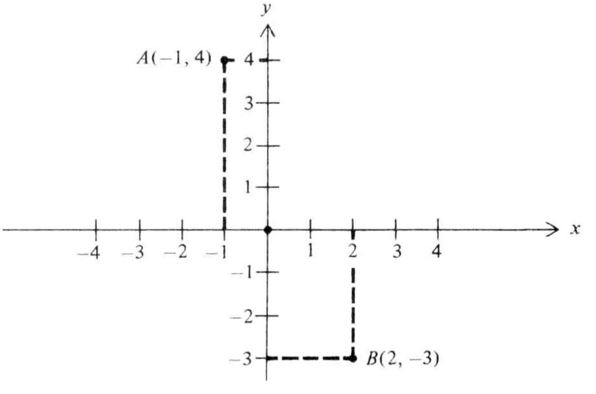

Diagram 6

Solution

Using Definition 1

$$\Delta x = 2 - (-1) = 3 \qquad \Delta y = -3 - (4) = -7$$

Note that x increases from A to B. Hence, $\Delta x > 0$. Since y decreases from A to B, $\Delta y < 0$.

Example 2 Consider a particle moving from $C(3, 2)$ to $D(-2, 2)$ (see Diagram 7). Find Δx and Δy.

Solution

$$\Delta x = (-2) - (3) = -5 \qquad \Delta y = (2) - (2) = 0$$

Example 3 Consider a particle moving from $E(-1, -2)$ to $F(-1, 0)$ (see Diagram 7). Find Δx and Δy.

Solution

$$\Delta x = (-1) - (-1) = 0 \qquad \Delta y = 0 - (-2) = 2$$

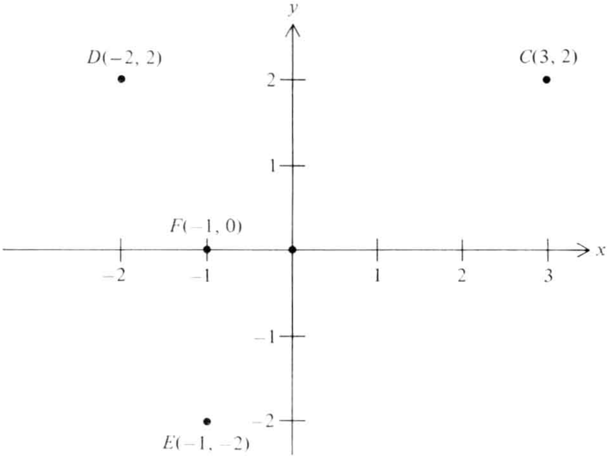

Diagram 7

We now discuss the concept of distance. In order to do this, we will define absolute value.

DEFINITION 2 The *absolute value* of x is denoted by $|x|$, where

$$|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example 4

$$|3| = 3$$
$$|0| = 0$$
$$|-3| = -(-3) = 3$$

Note that the absolute value of a negative number is never a negative number and, in general, $|x| = |-x|$.

This leads to the following theorem.

Theorem 1 $|x_2 - x_1| = |x_1 - x_2|$.

The proof of this theorem will be considered in Chapter 4. Let us look at a few examples to see what is implied by this theorem.

Example 5 If $x_1 = 4$ and $x_2 = 3$, then

$$|3 - 4| = |-1| = -(-1) = 1 \qquad |4 - 3| = |1| = 1$$