Lecture Notes in Computer Science

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Mathematical Foundations of Computer Science 1977

Proceedings, 6th Symposium, Tatranská Lomnica
September 5-9, 1977

Edited by J. Gruska

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MFCS'77

FOREWORD

This volume contains papers which were contributed for presentation at the 6th Symposium on Mathematical Foundations of Computer Science - MFCS´77, held at Tatranská Lomnica, Czechoslovakia, September 5-9, 1977.

The symposium was organized by the Computing Research Centre in Bratislava. The following institutions have cooperated in providing their support: The Faculty of Mathematics and Physics of the Charles University, Prague; the Faculty of Natural Sciences of the Šafárik University, Košice; the Faculty of Natural Sciences of the Komenský University, Bratislava; the Institute of Computing Technique of the Technical University, Prague; Institute of Technical Cybernetics of the Slovak Academy of Sciences; the Association of Slovak Mathematicians and Physicists and the Slovak Cybernetical Society.

The title of the symposium, "Mathematical Foundations of Computer Science" was chosen six years ago by the Polish organizers for the first meeting in the series and in 1974 it was used also for a seminar held at the International Banach Centre in Warsaw. In subsequent years it became a widely accepted designation for this new and important branch of science. It is understandable that this designation, or its close variants, will be used for other scientific events in the same areas, such as some of the recent symposia and seminars, held both in the United States and in Europe.

The present Proceedings include 15 invited papers and 46 short communications, the latter having been selected by the Program Committee from among the 117 submitted papers on the basis of originality and relevance to the following principal areas of interest: automata and formal languages, computability theory, analysis and complexity of algorithms, theoretical aspects of programming and of programming languages, theoretical aspects of operating systems and mathematical approaches to artificial intelligence.

The papers in these Proceedings were not formally refereed. It is anticipated that most of them will appear in a more polished and complete form in scientific journals.

The organizers of the symposium are much indebted to all of the contributors to this program, especially to the authors of the papers. Thanks are also due to all the above mentioned cooperating institutions for their valuable and all round assistance and to all people who helped in the organization of the Symposium.

Special thanks are due to Professor A. Klas, director of the Computing Research Centre in Bratislava for his generous support of not only MFCS´77 but of all MFCS symposia held in Czechoslovakia.

The Program Committee of MFCS´77 consisted of the following members: I.M. Havel /chairman/, J. Bečvář, J. Gruska, J. Hořejš, I. Korec, M. Novotný, B. Rovan and J. Šturc. A number of referees helped the Program Committee evaluate the submitted papers.

The organization was done mainly by the following members of the Organizing Committee: G. Andrejková, Z. Durayová, R. Filustek, J. Gruska /Symposium Chairman/, A. Guráňová, I.M. Havel, M. Chytil, A. Jelínková, M. Markusová, P. Mikulecký, I. Prívara, B. Rovan /Organizing Secretary/, and I. Šujan.

The help of Springer-Verlag, which has published these Proceedings, is also highly appreciated.

Bratislava, May 1977 Jozef Gruska

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ON THE STRUCTURE AND PROPERTIES OF NP-COMPLETE PROBLEMS AND THEIR ASSOCIATED OPTIMIZATION PROBLEMS

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SUMMARY

1. INTRODUCTION

Since the early work of Cook (1971) and Karp (1972) the research work on the properties of NP-complete problems has been intensive and widespread. The class of NP-complete problems contains all those problems which are in NP, that is which can be decided by a nondeterministic Turing machine in polynomial time, and to which all other problems in the class NP can be reduced in polynomial time. The characterization of the complexity of NP-complete problems leads to one of the most important (may be "the" most important) open questions in theoretical computer science: does there exist any Turing machine which decides any NP-complete problem in deterministic polynomial time? In that case, from the properties of the class NP, we would deduce that all NP-complete problems would be solvable within polynomial time and the two classes P and NP would coincide.

Even if it has been proved by Baker, Gill and Solovay (1975) that the question $P = NP$? can be positively answered in a relativized class of machines and negatively in another class of machines with different relativization and actually Hartmanis and Hopcroft (1976) have shown that there are relativized classes of machines for which the question $P = NP$? is independent from the axioms of set theory, the practical relevance of the issue and of its possible answers in the theory of algorithms is obvious and there is a wide belief that a solution will eventually be achieved.

Problems which have been recognized to be NP-complete problems belong to the widest variety of fields: among them we have combinatorial problems (such as the chromatic number or the node covering of a graph and the existence of hamiltonian circuits), scheduling problems, integer 0-1 programming, satisfiability of formulae of propositional calculus, salvability of quadratic Diophantine equations, inequivalence of simple programs with only one level of iteration. For any of these problems it would have important practical consequences to know whether the backtracking algorithms of polynomial depth that we should use for deterministically solving it could ever be replaced by an efficient deterministic polynomial time algorithm.

Beside the interest related to the solution of the question of whether P is equal to NP the research activities in this area of computational complexity have brought a new light on some of the most interesting properties of combinatorial problems. The results which have been obtained can very roughly be grouped into three classes. On one side (see Simon (1975), Hartmanis and Simon (1976), Hartmanis and Berman (1976,1977)),there has been a succesful attempt to strengthening the reductions among NP-complete problems in order to point out a deep similarity among these problems. The result of showing that all known NP-complete problems are isomorphic under a polynomial time mapping and the conjecture that this is the case with all infinite NP-complete problems seem to suggest that all these problems are very similar and that they actually are polynomial time permutations of the same problem.

From another point of view optimization problems related to NP-complete combinatorial problems have been considered. The literature in this field is overwhelmingly rich because this is the direction which is the most relevant for practical purposes and whose origin is certainly antecedent to the definition of NP-completeness . An extended annotated bibliography on approximate algorithms for combinatorial NP-complete problems is provided by Garey and Johnson (1976b). The general tendency in this area, analogously to what happens in concrete computational complexity , has been to consider one problem at a time and to look for the "best" approximate algorithm, that is the approximate algorithm with the best performance either from the point of view of efficiency or from the point of view of proximity to the exact solution or both.
Among those papers which have more particular interest there are also some papers with a deep methodological insight, such as those by Johnson (1974),Sahni and Gonzales (1976), Sahni (1976), Garey and Johnson (1976a); in these papers intrinsic differences among various optimization problems appear. While in some problems like knapsack and job sequencing

we can reach any desidered level of accuracy with polynomial time approximation algorithms, in some other problems, such as graph colouring, for example, any algorithm that comes too close to the optimal solution has the same complexity of the algorithms which give the optimal solution itself.

This type of results introduce a clear element of distinction among optimization problems and hence even if two NP-complete combinatorial problems are isomorphic, their isomorphism cannot be always extended to the associated optimization problems. The search for structural isomorphism among optimization problems and for classes of problems which are structurally isomorphic becomes, hence, an issue of great interest and this is the third kind of results that we want to consider in this introduction. Clearly when such a result can be proven, for those problems which are shown to be structurally isomorphic the same approximation algorithm can be used and good approximate solutions which have been found for an input in one problem can be mapped into good approximate solutions for the equivalent input in another problem. Beside these practical aspects, to be able of finding structural characterizations of classes of optimization problems and to relate their structural properties with the "degree of approximability" is certainly a relevant issue in the theory of computational complexity. Results in this direction have been achieved by Paz and Moran (1977) and Ausiello, D'Atri, Gaudiano and Protasi (1977a, 1977b).

2. ON THE ISOMORPHISM OF NP-COMPLETE COMBINATORIAL PROBLEMS

Let us first briefly review the basic terminology and notation.

Let Σ^* be the set of all words over a finite alphabet Σ. A language $L \subseteq \Sigma^*$ is said to be recognizable in time $t(n)$ by a Turing Machine (TM) M if for all $n \geq 0$, for every input x of length n M takes less then $t(n)$ steps either to accept or to reject x. If the TM is non deterministic we will consider the number of steps of the shortest accepting computation (if x is accepted) or the number of steps of the longest rejecting computation (if x is rejected).

DEFINITION 1. $NP = \{L \mid L$ is recognizable by a non-deterministic TM in time bounded by some polynomial $p\}$.

DEFINITION 2. A set $A \subseteq \Sigma^*$ is said to be p-*reducible to* a set $B \subseteq \Gamma^*$ (denoted $A \leq B$) if there is a mapping $f\colon \Sigma^* \rightarrow \Gamma^*$ which is computable in polynomial time on a deterministic TM and such that for every $x \in \Sigma^*$ $f(x) \in B$ if and only if $x \in A$.

DEFINITION 3. A set B is said to be *complete for* some class of sets C (denoted C-complete) if $B \in C$ and, for every $A \in C$, $A \le B$.

Well known examples of NP-complete sets (problems) are:

SATISFIABILITY = $\{w \mid w$ is a formula of propositional calculus in CNF and there exists a truth assignment that satisfies it$\}$.

CLIQUE = $\{\langle g, K\rangle \mid g$ is the encoding of a graph G,K is an integer and G has a complete subgraph of K nodes$\}$.

CHROMATIC-NUMBER = $\{\langle g, K\rangle \mid g$ is the encoding of a graph G,K is an integer and G can be coloured with K colours with no two adjacent nodes equally coloured$\}$.

DIOPH = $\{\langle a,b,c\rangle \mid a,b,c \ge 0$ are integers and the quadratic diophantine equation $ax^2 + by - c = 0$ can be solved with x,y positive integers$\}$.

SUBSET-SUM = $\{\langle a_1, \ldots, a_n, b\rangle \mid$ there is a subsequence $i_1, \ldots, i_m$ such that $\sum_{j=1}^{m} a_{i_j} = b\}$

JOB-SEQUENCING-WITH-DEADLINES = $\{\langle t_1, \ldots, t_n, d_1, \ldots, d_n, p_1, \ldots, p_n, k\rangle \mid$ there exists a permutation π such that $\sum_{j=1}^{n} (\textit{if } \sum_{i=1}^{j} t_{\pi(i)} \le d_j \textit{ then } p_j \textit{ else } 0) \ge k$.

DEFINITION 4. Two sets $A \subseteq \Sigma^*$ and $B \subseteq \Gamma^*$ are said to be p-*isomorphic* if there is a mapping $f\colon \Sigma^* \to \Gamma^*$ such that

- f is 1-1 and onto
- f is a p-reduction of A to B and f^{-1} is a p-reduction of B to A.

The following two theorems are both due to Hartmanis and Berman (1976) and are very important because they establish necessary conditions for two sets to be p-isomorphic; in the first case this fact can be derived from the properties of the p-reductions that hold among the two sets:

RESULT 1. Let p and q be length increasing invertible p-reductions of A to B and B to A respectively. Then A and B are p-isomorphic.

PROOF. Let us define

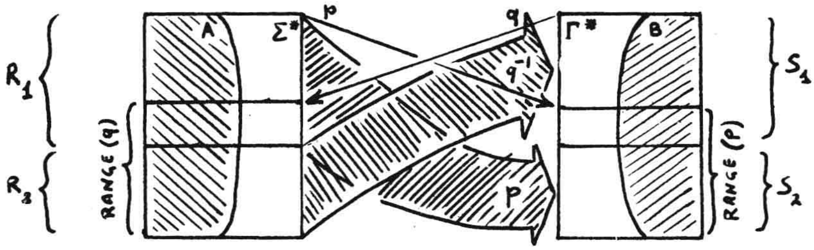
$$R_1 = \bigcup_{k=0}^{\infty} (q \circ p)^k \, \overline{\text{RANGE } (q)} \qquad R_2 = q(S_1)$$

$$S_1 = \bigcup_{k=0}^{\infty} (p \circ q)^k \, \overline{\text{RANGE } (p)} \qquad S_2 = p(R_1)$$

$$\varphi(z) = \begin{cases} p(z) & \text{if } z \in R_1 \\ q^{-1}(z) & \text{if } z \in R_2 \end{cases} \qquad \varphi^{-1}(z) = \begin{cases} p^{-1}(z) & \text{if } z \in S_2 \\ q(z) & \text{if } z \in S_1 \end{cases}$$

By the properties of p and q it follows that φ and φ^{-1} are inverses and:

- φ and φ^{-1} are computable in polynomial time;
- φ is 1-1 and onto and is a p-reduction of A to B. QED



The second theorem shows instead that the existence of a p-isomorphism to SATISFIABILITY is a consequence of intrinsic properties of some NP-complete sets:

RESULT 2. An NP-complete set B is p-isomorphic to SATISFIABILITY if and only if there exists a polynomial time computable length increasing padding function S_B for B such that

- for any x and y, $S_B(x,y) \in B$ iff $x \in B$
- the padding can be obtained back by a polynomial time computable function D_B and for every x,y $D_B(S_B(x,y)) = y$.

PROOF. Let us first prove that if there is any reduction p from SATISFIABILITY to B then there is also a reduction p' which is length increasing one-one and invertible in polynomial time.

For $x \in$ SATISFIABILITY let us define

$$p'(x) = S_B(p(x),x)$$

and $t(x) =$ *if* $x = S_B(p(D(x)),D(x))$ *then* $D(x)$ *else* undefined.

Clearly $p'(x) = p'(y)$ implies $x = y$ and besides $t = (p')^{-1}$ and $|p'(x)| > |x|$.

In order to prove that we can also find a length increasing one-one and invertible reduction q' from B to SATISFIABILITY we only have to prove (once and forever) that also SATISFIABILITY has a padding function S and a function D with the properties stated in the theorem. Once we have proved the existence of such reductions p' and q' which are length increasing, one-one and invertible, we are in the conditions of theorem 1 to prove the p-isomorphism between B and SATISFIABILITY. QED

The intuitive meaning of theorem 2 is shown in the figure.