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# Pattern Analysis

## Lectures in Pattern Theory Volume II

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# PREFACE

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# INTRODUCTION

In this book we shall continue the mathematical study of regular structures begun in Volume I. With the help of the concepts of pattern synthesis introduced in the previous volume, we now turn to the inverse problem of pattern analysis.

Most of the results described below are due to the author and his co-workers, and have been presented in lectures or in the author's Research Seminar in Pattern Theory at Brown University. Some of them have appeared in the series of Reports on Pattern Analysis but only a few have been submitted for publication so that they appear here for the first time in print.

The progress of pattern theory has been more rapid than had been expected a few years ago when this series was planned. In particular, the study of tendencies toward regularity should be mentioned as a fast expanding research area. We shall need a third volume to report on this work, in addition to the two that were planned, before the more definitive and comprehensive fourth one appears. The third volume will present the science-theoretic background to pattern theory, regular structures as viewed from universal algebra, some topological considerations of image algebras, and analysis of patterns in addition to those examined in the present volume.

Our subject remains the same as in Volume 1 - the general study of combinatory regularity - but the emphasis is quite different. We shall be less concerned with the conceptual aspects and concentrate instead on the mathematical difficulties and to some extent, also on algorithmic questions.

It has been said that a mathematical theory is really a collection of special cases treated from a unifying perspective. In what follows, we shall indeed concentrate on certain special cases, some studied in depth while others receive only superficial attention. As the reader will notice, most of these special cases are motivated by possible applications. It would be misleading, however, to speak of them as actual applications. This has not been our purpose. As indicated by the title of this series, our main concern is of theoretical nature: explore the mathematical problems intrinsic to combinatory regularity.

This reservation should be borne in mind especially when reading the two last chapters. They are completely speculative in character, with little or no empirical support. From the standpoint of the extreme empiricism that dominates so much of applied mathematics, especially in the English speaking world, all mathematical models should be based on experimental data and observations. In the absence of such empirical information the scientist should abstain from speculation and mathematical constructions. In Chapter 6, we have done exactly the opposite, against the positivist's credo, and introduced assumptions that are not founded on hard data.

Once the assumptions have been made, their logical consequences are explored. To indicate that some of these results are quite preliminary, and will no doubt be reformulated and extended in the near future, we have designed some of the results as propositions rather than theorems.

Using the computer as the mathematician's laboratory we have repeatedly explored problems by performing computational experiments, often displaying the results through some form of computer graphics. In this way, we have been led to conjectures that were later proven by analytical arguments. This would not have been a practical research strategy without the flexible and mathematically congenial computer facility for interactive work that we were fortunate to have access to at Brown University.

The computing machine has not been used here to process large amounts of empirical data. Instead, we have used it as an extension of mathematical analysis, as a calculus ratiocinator in the sense of Leibnitz. It represents the continuation of analysis by other means.

As can be expected from lecture notes they are not as finished and complete as a regular monograph. The exposition is not yet polished and perfected and the reader will no doubt encounter obscurities and passages that are difficult to follow. In particular, section 2.4 and Chapter 6 are only preliminary in nature. The author would appreciate comments and advice for a more definitive version that is being planned.

# Chapter 1
# Ends and Means in Pattern Analysis

## 1. The Aims of Pattern Analysis.

The basic concepts of pattern theory consist of objects and relations. The pattern objects are generators, configurations, pure and deformed images, pattern classes. The pattern relations are given in terms of similarity transformations, combinatory relations, identification rules, and deformation mechanisms.

The objects are naturally arranged in levels (see 1.4.1) where generators form the lowest level, configurations the next and so on. In hierarchic pattern systems we can have a great number of levels.

In pattern synthesis one proceeds from low to high levels in the system. Pattern analysis represents the inverse process: starting from a high pattern level one tries to analyze the pattern object into lower level objects.

It will not be meaningful to speak of analysis in this sense unless the synthesis process has been specified. We have seen in Volume 1 that the theory is highly flexible as illustrated by a large number of special cases to which many more will be added in the present volume, and the third one. This implies, of course, that pattern analysis can appear in many forms depending upon the concrete nature of the generators, the connection type, etc. At this stage, it will suffice to mention explicitly only the most common types of analysis, and we will meet others as we go along.

In pattern analysis, we shall have to study mappings between the different spaces appearing in pattern theory: configuration space, the image algebra, the deformed image algebra, the pattern class structure. This is just as in classical algebraic systems, where we study, for example, mappings between groups. In particular, we study mappings that preserve algebraic properties, homomorphisms, or have other invariance - covariance properties.

<u>Case 1.1 (image restoration)</u>. This is the problem of finding a mapping from the deformed image algebra $\mathcal{I}^{\mathcal{D}}$ to the pure image algebra $\mathcal{I}$. The mapping $\mathcal{I}^{\mathcal{D}} \to \mathcal{I}$ is supposed to restore the pure image $I$ which was deformed by $\mathcal{D}$ into the observed object $I^{\mathcal{D}}$.

In what precise sense the image $I$ is restored will be discussed in Section 1.3.

As an example, we mention the problem of finding the set pattern that was deformed by a Poisson deformation mechanism (see 1, Case 4.5.2) into the point pattern observed.

In this case, as well as in the later ones, there is no reason to expect that the answer is unique. If it is, we have an unusual and degenerate situation. Otherwise, we will try to choose a mapping that is natural, reasonable, or even optimal in some precise sense.

Sometimes one wishes image restoration to be accompanied by a determination of the deformation that was applied. We then ask for a mapping $\mathscr{I}^{\mathscr{D}} \to \mathscr{I} \times \mathscr{D}$. This is motivated when the deformation itself is of interest to the analyst, or when he wishes to learn more about the structure of $\mathscr{D}$ than what is given to him a priori, for example, what is the probability measure operating on $\mathscr{D}$.

<u>Case 1.2 (image analysis)</u>. Given an image I find a configuration c that gives rise to I. This involves finding the generators and combinatory relations of c, so that the desired mapping is $\mathscr{I} \to \mathscr{C}(\mathscr{R})$.

In the case of temporal patterns of regime type, for example, (see 1, Case 3.4.3) one would have to solve a segmentation problem as well as finding the wave forms associated with the segments of the time axis.

<u>Case 1.3 (image approximation)</u>. With the usual set-up together with an additional image algebra $\mathscr{I}^*$, $\mathscr{I} \subset \mathscr{I}^*$, we want to find a "good" mapping $\mathscr{I}^{\mathscr{D}} \to \mathscr{I}^*$ such that the $I^*$ obtained is in some sense close to $I \in \mathscr{I}$.

In this case, one extends the pure image algebra to one $\mathscr{I}^*$ and tries to approximate I as well as possible employing elements from $\mathscr{I}^*$, using the information contained in the observed $I^{\mathscr{D}} = dI$.

If $\mathscr{I}$ is the image algebra of convex sets, for example, (see I, Case 3.5.9) and $\mathscr{I}^*$ is the set algebra of Borel sets, then a typical image approximation problem would consist of finding an element in $\mathscr{I}^*$ when one has observed a Poisson deformation of $I \in \mathscr{T}$.

<u>Case 1.4 (pattern recognition)</u>. Given $I^{\mathscr{D}}$ find the pattern class $\mathscr{P}_r$ to which $I$ belongs, $I^{\mathscr{D}} = dI$, $I \in \mathscr{P}_r$.

As an example, let $\mathscr{T}$ consist of motions as in I, Case 3.7.1, and the similarity group S is the direct product of Euclidean motions of $R^3$ and translations of the time axis. Let each $\mathscr{P}_r$ be formed from a prototype (see 1, 3.1) to which S is applied. Finding the prototype behind the observed motion image $I^{\mathscr{D}}$ is a pattern recognition problem.

In Case 1.3, we expanded the set $\mathscr{T}$ to $\mathscr{T}^*$. The idea behind this is, of course, that the original image algebra was too restricted and we had to enlarge it. Conversely, it can happen that $\mathscr{T}$ or $\mathscr{T}^{\mathscr{D}}$ is too large, carrying much information, some of which may be irrelevant or of small interest. We then introduce a partition $\mathscr{T}_*$ of $\mathscr{T}$ (or of $\mathscr{T}^{\mathscr{D}}$ as the case may be) and represent I by one of the elements in the partition $\mathscr{T}_*$, suitably chosen. We can look at it as a (partial) description of I (or of $I^{\mathscr{D}}$).

<u>Case 1.5 (image description)</u>. Given $\mathscr{T}$ and $\mathscr{T}_*$ find a mapping $f: \mathscr{T} \to \mathscr{T}_*$ such that the $I \in \mathscr{T}_*$ is a good representative of I.

For example, if $\mathscr{T}$ consists of contrast images with real contrast values and forms an infinite dimensional function space, we could try an $\mathscr{T}_*$ parametrized by a finite number of real valued parameters, a many to one mapping, and choose the parametrization and f such that little information is lost when replacing I by $I_*$.

For good image description one needs not only a good choice of the representatives within the partition but also the choice of $\mathcal{T}_*$ must be carefully made.

How to do this will depend upon the purpose of the image description. One possibility is for data storage when one aims for data compression. Another is for data transmission when the communication channel must also be taken into account when solving what is essentially a coding problem.

Case 1.6 (pattern inference and abduction). Given elements from $\mathcal{T}^{\mathcal{D}}$ make inferences concerning $\mathcal{T}$ and the underlying regularity structure. This may involve determining the generators, the connection type, the identification rule, etc.

When we process entire pattern structures, and not just an individual image, the pattern processor will sometimes be a complicated system which itself possesses pattern structures, as in Chapter 6. The relation of it to the structure theory being processed will then be crucial.

There are still other types of pattern analyses that ought to be mentioned and that will occur in later chapters. It may be that we know $\mathcal{T}$ except for some detail; we want to determine structural parameters in the image algebra, find attributes of generators, for example. Or, we may wish to determine the P-measures on $\mathcal{L}(\mathcal{R})$ and $\mathcal{T}$, see 1, Chapters 2-3, or even the probabilistic properties of the deformations.

Suppose anyway that we have been given a pattern system and are asked to carry out one of the above pattern analyses, how should one go about it? At this level of generality, the question is almost meaningless, and it is only when we get to more detailed and concrete pattern systems that we can start to offer specific methods of solution as will be done in the following chapters.

After we have examined some of them it will be seen that the major obstacle can be found in the presence of deformations that destroy information. If we only had to deal with a pure image algebra, some analysis problems would disappear, for example image restoration, and others would be simple (at least in principle!).

Take, for example, the case of recognizing handwritten text. This problem has received a lot of attention but the results have been less than impressive. If we only have to deal with stylized handwriting, say with segmented characters written in a specific alphabet, then recognition can indeed be organized in algorithmic form. The pioneering work of Eden (1961) should be mentioned here.

On the other hand, if the text is not segmented and if individual variations in writing style are allowed, then a feasible solution to the recognition problem still seems far away in spite of all the attempts that have been made.

Other examples could be cited. Automatic speech recognition is in a similar state, see e.g. Flanagan (1972). It has been demonstrated repeatedly that speech synthesis is possible with good quality. The inverse problem is so much harder that experts in the field have voiced pessimistic views about the possibility of real solutions in a flexible speech environment. Again, the reason seems to be the enormous variability and complicated interdependencies produced by the deformation mechanisms at work.

In both of these cases, we know that recognition <u>can</u> be obtained, since we, as humans, carry out such recognition all the time, although we do not know exactly how we do it.

In view of the disturbing effect of the deformations, it is tempting to organize pattern analysis in two stages. First, one would try to compensate for $\mathscr{D}$ by some preliminary processing: noise suppression, filtering, smoothing, or other cleaning procedures. Second, one would use the cleaned data, in a structured form, for the actual analysis, hoping that in this way the problem would have been reduced to amenable form.

The pattern recognition literature abounds with such attempts. The cleaning procedure is often borrowed from the disciplines of communication engineering and traditional statistical inference modified to suit the problem at hand. Sometimes the modifications try to incorporate global and qualitative properties of the patterns. Useful results have been attained in this way, especially when the deformations are of limited variability.

The success of such an attempt hinges on the choice of preprocessing. Ad hoc methods for selecting this procedure may work, but this author believes that general pattern analysis should exploit the whole pattern structure and be built directly on the underlying entire chain of pattern synthesis including $\mathscr{D}$.

At the same time, one has to remember the computational constraints, and aim for algorithms that can actually be executed with reasonable demands on the computational resources. The practical situation is a bit like the one in information theory. The general theorems in information theory do not always indicate optimal coding schemes that can be used in practice. They give, however, lower bounds that tell us what is possible theoretically, and this will