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Attila Kuba  
László G. Nyúl  
Kálmán Palágyi (Eds.)

# Discrete Geometry for Computer Imagery

13th International Conference, DGCI 2006  
Szeged, Hungary, October 2006  
Proceedings



Springer

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# Discrete Geometry for Computer Imagery

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Proceedings



## Volume Editors

Attila Kuba

László G. Nyúl

Kálmán Palágyi

University of Szeged

Department of Image Processing and Computer Graphics

Árpád tér 2., 6720 Szeged, Hungary

E-mail: {kuba, nyul, palagyi}@inf.u-szeged.hu

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# Preface

DGCI 2006, the 13th in a series of international conferences on Discrete Geometry for Computer Imagery, was held in Szeged, Hungary, October 25-27, 2006. DGCI 2006 attracted a large number of research contributions from academic and research institutions in this field. In fact, 99 papers were submitted from all around the world. After review, 55 contributions were accepted from which 28 were selected for oral and 27 for poster presentation. All accepted contributions were scheduled in single-track sessions. The program was enriched by three invited lectures, presented by internationally well-known speakers: Jean-Marc Chassery (Domaine Universitaire Grenoble, France), T. Yung Kong (City University of New York, USA), and László Lovász (Eötvös Loránd University, Budapest, Hungary).

We were pleased that DGCI got the sponsorship of the International Association of Pattern Recognition (IAPR). DGCI 2006 is also a conference associated with the IAPR Technical Committee on Discrete Geometry (TC18). Hereby, we would like to thank all contributors, the invited speakers, all reviewers and members of the Steering and Program Committees, and all supporting personnel who made the conference happen. We are also grateful to the Institute of Informatics, University of Szeged, for the financial and infrastructural help, which was essential to the organization of a successful conference. Finally, we thank all the participants and hope that they found interest in the scientific program and also that they had a pleasant stay in Szeged.

October 2006

Attila Kuba  
László G. Nyúl  
Kálmán Palágyi

## DGCI meetings

<b>Edition</b>	<b>Venue</b>	<b>Date</b>	<b>Proc.</b>	<b>Editors / Organizers</b>
13 <sup>th</sup>	Szeged, Hungary	Oct. 25–27, 2006	LNCS 4245	A. Kuba L.G. Nyúl K. Palágyi
12 <sup>th</sup>	Poitiers, France	Apr. 13–15, 2005	LNCS 3429	E. Andres G. Damiani P. Lienhardt
11 <sup>th</sup>	Naples, Italy	Nov. 19–21, 2003	LNCS 2886	I. Nyström G. Sanniti di Baja S. Svensson
10 <sup>th</sup>	Bordeaux, France	Apr. 3–5, 2002	LNCS 2886	A. Braquelaire J.-O. Lauchaud A. Vialard
9 <sup>th</sup>	Uppsala, Sweden	Dec. 13–15, 2000	LNCS 1953	G. Borgefors I. Nyström G. Sanniti di Baja
8 <sup>th</sup>	Marne-la-Vallée, France	Mar. 17–19, 1999	LNCS 1568	G. Bertrand M. Couprie L. Perroton
7 <sup>th</sup>	Montpellier, France	Dec. 3–5, 1997	LNCS 1347	E. Ahronovitz C. Fiorio
6 <sup>th</sup>	Lyon, France	Nov. 13–15, 1996	LNCS 1176	S. Miguet A. Montanvert S. Ubeda
5 <sup>th</sup>	Clermont-Ferrand, France	Sep. 25–27, 1995	–	D. Richard
4 <sup>th</sup>	Grenoble, France	Sep. 19–20, 1994	–	J.-M. Chassery A. Montanvert
3 <sup>rd</sup>	Strasbourg, France	Sep. 20–21, 1993	–	J. Françon J.-P. Reveillès
2 <sup>nd</sup>	Grenoble, France	Sep. 17–18, 1992	–	J.-M. Chassery A. Montanvert
1 <sup>st</sup>	Strasbourg, France	Sep. 26–27, 1991	–	J. Françon J.-P. Reveillès

# Organization

DGCI 2006 was organized by the Department of Image Processing and Computer Graphics, University of Szeged, Hungary.

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# Duality and Geometry

## Straightness, Characterization and Envelope

Jean-Marc Chassery<sup>1</sup>, David Coeurjolly<sup>2</sup>, and Isabelle Sivignon<sup>2</sup>

<sup>1</sup> Laboratoire LIS

Domaine universitaire Grenoble - BP46

38402 St Martin d'Hères Cedex, France

`jean-marc.chassery@lis.inpg.fr`

<sup>2</sup> Laboratoire LIRIS - Université Claude Bernard Lyon 1

Bâtiment Nautibus - 8, boulevard Niels Bohr

69622 Villeurbanne cedex, France

`{david.coeurjolly, isabelle.sivignon}@liris.cnrs.fr`

**Abstract.** Duality applied to geometrical problems is widely used in many applications in computer vision or computational geometry. A classical example is the Hough Transform to detect linear structures in images. In this paper, we focus on two kinds of duality/polarity applied to geometrical problems: digital straightness detection and envelope computation.

## Introduction

In domain of geometry, notion of duality is often used to represent the same structure in different domains like spatial domain or parametric one. The objective is to facilitate transformations like characterization, detection, recognition or classical ones such as intersection or union. A first example is illustrated with Voronoi partition in which polygonal regions are not homogeneous in terms of number of vertices. Nevertheless, the corresponding dual mesh, called Delaunay mesh, is composed of triangles. According to applications the choice of the alternative representations can be used on optimality criteria (computational cost, database structure, ...).

Following this first example, we focus in this paper on dual transformations illustrated by problems of digital straightness and envelope.

## 1 Example of the Hough Transform

The Hough transform (HT for short) is a very classical tool in image analysis to detect geometric features in images. These features may be line segments, circles, ellipses or any other parameterized curve. The HT, introduced in 1962 by Hough [1], is a dual transformation that enables to find a set of global structures,

without any a priori knowledge on the number of structures to be found. Note also that this method is robust to noise and disconnected features.

### 1.1 Definition of Hough Transform

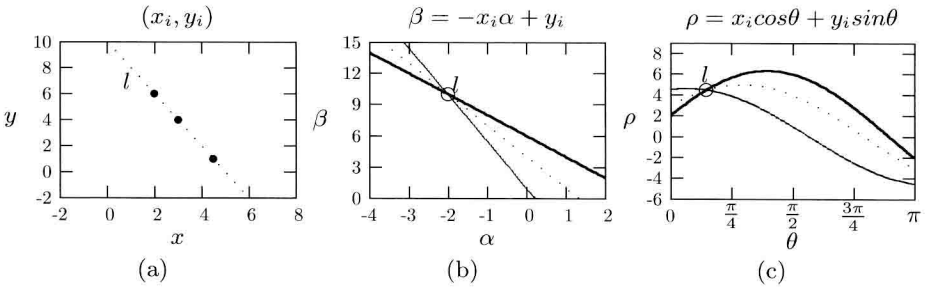
The general idea of this transform is that every point of the image contributes to the definition of the solution set for a given parameterized structure. Consider for instance a point  $p_0$  of coordinates  $(x_0, y_0)$  and the parameterization of lines  $y = \alpha x + \beta$ . Then the set of lines going through  $p_0$  are the ones of parameters  $(\alpha, \beta)$  fulfilling the equality  $y_0 = \alpha x_0 + \beta$ . This equality may be rewritten as  $\beta = -\alpha x_0 + y_0$ , and if a new geometrical space  $(\alpha\beta)$ , called *dual space*, or *parameter space*, is defined, this equation defines a line : in this dual space, each point of this line represents a line of the  $(xy)$  space going through the point  $p_0$ . An illustration of three points and the three corresponding lines in the dual space  $(\alpha\beta)$  are represented in Figure 1 (a)-(b): note that the three lines in  $(\alpha\beta)$  space are concurrent in one point, the coordinates of which defines a line going through the three points in  $(xy)$  space.

However, as noticed by Duda in [2], the linear parameterization of lines defined by  $y = \alpha x + \beta$  is not the handiest one since the two parameters  $\alpha$  and  $\beta$  are unbounded. Thus, another transform consists in using the polar parameterization of straight lines  $\rho = x \cos \theta + y \sin \theta$ . Any point in the  $(xy)$  space defines a sinusoidal curve in the  $(\theta\rho)$  space, where only the parameter  $\rho$  has unbounded values (see Figure 1(c) for an illustration).

General properties fulfilled by these two representations, and suitable for straight line detection in images were expressed by Duda [2]:

#### Property 1

- A point in the  $(xy)$  space matches up with one curve in the dual space;
- A point in the dual space matches up with a straight line in the  $(xy)$  space;
- Points lying on a same line in the  $(xy)$  space match up with concurrent curves in the dual space;
- Points on a same curve in the dual space match up with concurrent straight lines in the  $(xy)$  space.



**Fig. 1.** (a) Three points in the  $(xy)$  space; (b) Dual representation in the  $(\alpha\beta)$  space; (c) Dual representation in the  $(\theta\rho)$  space

## 1.2 Recognition of Parameterized Structures

Line segment detection in images does not consist in finding the pixels lined up according to the Euclidean straight line definition, but a relaxation of this definition has to be used. To do so, the method generally used consists of, first, decomposing, or quantifying the dual space along the two axis, and second, defining a counter for each cell of the dual space. Algorithm 1 describes the general algorithm for finding parameterized curves in an image using HT. The quantization step is a trade-off between precision on one part, and memory/computation cost on the other hand. Moreover, a good quantization should provide constant densities for equally probable line parameters. An illustration of Algorithm 1 is proposed in Figure 2.

---

**Algorithm 1.** Hough transform for parameterized curve detection

---

**Input:** Set of pixels  $P$

Quantify the dual space of the parameterized curve;

Set all the cell counters to zero;

**for** every pixel  $p$  of  $P$  **do**

    Compute  $HT(p)$  and digitize it according to the quantization grid;

    Add one to the counters of  $HT(p)$  digitization;

**end**

Look for local maxima among the cells counters: each maximum matches with the parameters of a curve found in  $P$ .

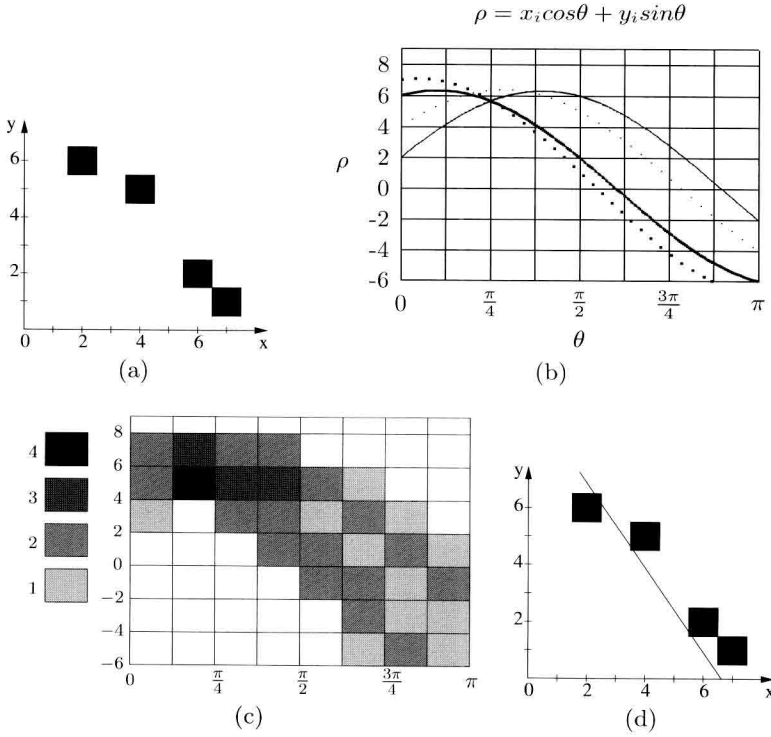
---

## 2 Duality in Discrete Geometry

During a HT, the discrete nature of the data processed is taken into account with a quantization of the dual space. On the contrary, we see in this section that the classical notion of dual space used in discrete geometry introduces the discrete nature of the data in the definition of the dual representation of a point.

### 2.1 Definition of the Dual Space

In digital geometry, pixels are said to be lined up if they belong to a digital straight line, which is the digitization of a straight line. In a general way, a digital straight line of parameters  $(a, b, \mu)$  and bounds  $\rho(a, b)$  and  $\omega(a, b)$  is the set of pixels  $(x, y)$  such that  $\rho(a, b) \leq ax - by + \mu \leq \omega(a, b)$ . Without loss of generality, we suppose that  $|b| > |a|$ , and  $b > 0$  in the following. With these conditions, the previous definition may be rewritten as  $\rho'(\alpha, \beta) \leq \alpha x - y + \beta \leq \omega'(\alpha, \beta)$ . Given a point  $p_0$  of coordinates  $(x_0, y_0)$ , the digital lines containing are the ones for which  $(x_0, y_0)$  fulfills the inequalities. Thus, we can once again define a dual space  $(\alpha\beta)$  to represent the space of line parameters, but contrary to HT, a given point  $p_0$  of coordinates  $(x_0, y_0)$  matches up with the intersection of two linear constraints defined by  $E^+ : \beta \geq -\alpha x_0 + y_0 + \rho'(\alpha, \beta)$  and  $E^- : \beta < -\alpha x_0 + y_0 + \omega'(\alpha, \beta)$ .

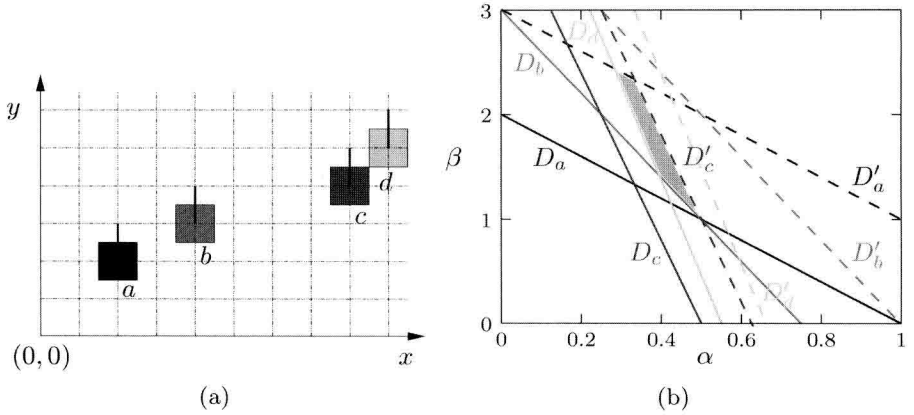


**Fig. 2.** Detection of a line segment with HT: (a) the four pixels of the set  $P$ ; (b) dual representation in the quantified dual space; (c) result of the digitization of the sinusoidal curves; (d) straight line computed from the local maximum found

**Definition 1.** Let  $P$  be a set of pixels. The preimage of  $P$  denoted by  $\mathcal{P}(P)$  is defined as follows:  $\mathcal{P}(P) = \{(\alpha, \beta), |\alpha| \leq 1 \mid \forall (x, y) \in P, \rho'(\alpha, \beta) \leq \alpha x - y + \beta < \omega'(\alpha, \beta)\}$ . (See Figure 3).

As we can see, in digital geometry, the linear parameterization of lines is used in order to define the dual space. Nevertheless, we pointed out that for the Hough transform, using a polar parameterization is more convenient in order to handle bounded parameters. Actually, the polar parameterization is not appropriate for preimage definition since intersection of sinusoidal curves would be involved. Thus, the handling of unbounded domains has to be tackled. First, the parameter  $\beta$  takes its values in an unbounded domain since it represents all the possible translation of a line. This problem is easy to solve, operating a translation of the set of pixels studied such that one particular pixel of the set is set to the origin. Next, the slope  $\alpha$  of the lines also have unbounded values. The idea here is to use two dual spaces instead of one :





**Fig. 3.** Illustration of the preimage of a set of pixels (digitization process fixed): each point matches up with two linear constraints, and the preimage is the intersection of these constraints

**Definition 2.** The dual space  $\mathcal{P}_y$  is defined as the space where one point  $(\alpha, \beta)$ ,  $|\alpha| < 1$  stands for the line  $\alpha x - y + \beta = 0$ . In the same way, a point  $(\alpha, \beta)$ ,  $|\alpha| < 1$  of the dual space denoted  $\mathcal{P}_x$  stands for the line  $\alpha y - x + \beta = 0$ .

## 2.2 Preimages of Digital Lines and Line Segments

The definition of preimage depends on the values of  $\rho'(\alpha, \beta)$  and  $\omega'(\alpha, \beta)$ , and in most applications, these values are defined according to the digitization process considered during the definition of the digital straight line. In this section, firstly we give some examples of preimages of digital straight lines in respect to the digitization process considered, and secondly, we emphasize on particular properties of the preimage of digital straight line segments (DSS for short) for one digitization process.

**Digitization and Preimage.** Let us first consider the *OBQ* (object boundary quantization) digitization scheme: given a straight line of equation  $ax - by + \mu = 0$ , its *OBQ* digitization is the set of pixels such that  $0 \leq ax - by + \mu < b$  (see conditions over  $a$  and  $b$  previously defined). Since the *OBQ* digitization is based on the definition of the inside and this outside of an object, this definition assumes that the line  $ax - by + \mu = 0$  is part of the boundary of an object the inside of which is given by the direction of the normal vector  $(a, -b)$ .

From this definition, we derive a characterization of the preimage of an infinite digital line according to the *OBQ* digitization process [3]:

*Property 2.* Let  $L$  be a digital straight line defined by  $0 \leq ax - by + \mu < b$ , with  $0 \leq a < b$ . Then the preimage of  $L$  according to the *OBQ* digitization process is the vertical segment  $[(\frac{a}{b}, \frac{\mu}{b}), (\frac{a}{b}, \frac{\mu+1}{b})]$ .