Data Analysis for the Behavioral Sciences Using SPSS

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Data Analysis

for the

Behavioral Sciences Using SPSS

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DATA ANALYSIS FOR THE BEHAVIORAL SCIENCES USING SPSS

This text is written from the perspective that statistics is an integrated set of tools used to uncover the story contained in numerical data. The presentation is informal, yet rigorous; it is based on a conceptual approach supported by an understanding of underlying mathematical foundations. Students learn that more than one method of analysis is typically needed and that an ample characterization of results is a critical component of any data analytic plan. The use of real data contained in an accompanying disk not only enables a greater emphasis on conceptual understanding and interpretation, but also allows students to study statistics in a way that reflects statistical practice.

Sharon Lawner Weinberg is Professor of Quantitative Methods and Psychology and Vice Provost for Faculty Affairs at New York University. She is widely published, with more than 50 publications in her field, including books, book chapters, journal articles, and major reports. She is the author with Kenneth P. Goldberg of *Statistics for the Behavioral Sciences* (Cambridge University Press, 1990).

Sarah Knapp Abramowitz is Assistant Professor of Mathematics and Computer Science at Drew University.

Preface

While based loosely on *Basic Statistics for Education and the Behavioral Sciences* by Sharon Lawner Weinberg and Kenneth P. Goldberg, published by Cambridge University Press, 1990, this text capitalizes on the widespread availability of menu-driven software packages to create a course of study that links good statistical practice to the analysis of real data. Several important guiding principles motivate our presentation.

First, and perhaps most importantly, we believe that a good data analytic plan must serve to uncover the story behind the numbers, what the data tell us about the phenomenon under study. To begin, a good data analyst must know his data well and have confidence in it. Accordingly, we emphasize the usefulness of diagnostics in both graphical and statistical form to expose anomalous cases, which might unduly influence results, and to help in the selection of appropriate assumption-satisfying transformations so that ultimately we may have confidence in our findings. We also emphasize the importance of using more than one method of analysis to answer fully the question posed. Seeing a three-dimensional sculpture in its entirety requires viewing that sculpture from many vantage points. Likewise, fully understanding the phenomenon under study often requires delving into data from more than one vantage point via the application of more than one method of analysis.

Second, because we believe that data are central to the study of good statistical practice, this text comes with an accompanying disk that contains several data sets used throughout the text. One is a large set of real data containing 48 variables and 500 cases that we make repeated use of in both worked-out examples and end-of-chapter exercises. By posing interesting questions about variables in this large, real data set (e.g., Is there a gender difference in expected income at age 30 of eighth graders?), we are able to employ a more meaningful and contextual approach to the introduction of statistical methods and to engage students more actively in the learning process. The repeated use of this data set also contributes to creating a more cohesive presentation of statistics; one that links different methods of analysis to each other and avoids the perception that statistics is an often-confusing array of so many separate and distinct methods of analysis, with no bearing or relationship to one another.

Third, we believe that the result of a null hypothesis test (to determine whether an effect is real or merely apparent) is only a means to an end (to determine whether the effect being studied is important or useful), rather than an end in itself. Accordingly, in our presentation of null hypothesis testing, we stress the importance of evaluating the magnitude of the effect if it is deemed to be real, and of drawing clear distinctions between statistically significant and substantively significant results. Toward this end, we introduce the computation of standardized measures of effect size as common practice following a

statistically significant result. While we provide guidelines for evaluating, in general, the magnitude of an effect, we encourage readers to think more subjectively about the magnitude of an effect, bringing into the evaluation their own knowledge and expertise in a particular area.

Fourth, a course in applied statistics should not only provide students with a sound statistical knowledge base but also with a set of data analytic skills. Accordingly, we have incorporated SPSS, a popularly used statistical software package, into the presentation of statistical material in a highly integrated manner. SPSS is used to provide students with a platform for actively engaging in the learning process associated with what it means to be a good data analyst by allowing them to apply their newly learned knowledge to the real world of applications. This approach serves also to enhance the conceptual understanding of material and the ability to interpret output and communicate findings.

Finally, we believe that a key ingredient of an introductory statistics text is a clear, conceptual, yet rigorous, approach. We emphasize conceptual understanding through an exploration of both the mathematical principles underlying statistical methods and real world applications. We use an easy-going, informal style of writing that, we have found, gives readers the impression that they are involved in a personal conversation with the authors. And, we sequence concepts with concern for student readiness, reintroducing topics in a spiraling manner to provide reinforcement and promote transfer of learning.

The book is intended for use in a one-year course in introductory statistics at either the graduate or undergraduate level, although it certainly can be used in a one-semester course as well. The book consists of 16 chapters. In addition to topics traditionally found in introductory statistics texts in the behavioral and social sciences, the book covers such topics as data transformations, diagnostic tools for the analysis of model fit, the logic of null hypothesis testing, assessing the magnitude of effects, interaction and its interpretation in two-way analysis of variance and multiple regression, and non-parametric statistics. This broad coverage of topics gives the instructor flexibility in curriculum planning and provides students with more advanced material for future work in statistics.

The book has been piloted in manuscript form and has benefited from the many helpful comments of our New York University and Drew University students. The book also has benefited from the many helpful comments and suggestions of several colleagues. Of these, we must extend our special thanks to Chris Apelian, John Daws, Linda Lesniak, Kathleen Madden, Robert Norman, and Eileen Rodriguez whose careful reading have contributed to an improved text. Of course, any errors in the final version remain the responsibility of the authors. Finally, and most importantly, we would like to thank our families, Steve, Allison, and Carolyn Weinberg, Jason Barro, and Dave, Michelle, and Scott Abramowitz for their enduring love, patience, and support.

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Introduction

Welcome to the study of statistics! It has been our experience that many students face the prospect of taking a course in statistics with a great deal of anxiety, apprehension, and even dread. They fear not having the extensive mathematical background that they assume is required, and they fear that the contents of such a course will be irrelevant to their work in their fields of concentration.

Although it is true that an extensive mathematical background is required at more advanced levels of statistical study, it is not required for the introductory level of statistics presented in this book. Greater reliance is placed on the use of the computer for calculation and graphical display so that we may focus on issues of conceptual understanding and interpretation. While hand computation is de-emphasized, we believe, nonetheless, that a basic mathematical background – including the understanding of fractions, decimals, percentages, signed (positive and negative) numbers, exponents, linear equations and graphs – is essential for an enhanced conceptual understanding of the material.

As for the issue of relevance, we have found that students better comprehend the power and purpose of statistics when it is presented in the context of a substantive problem with real data. In this information age, data are available on a myriad of topics. Whether our interests are in health, education, psychology, business, the environment, and so on, numerical data may be accessed readily to address our questions of interest. The purpose of statistics is to allow us to analyze these data to extract the information that they contain in a meaningful way and to write a story about the numbers that is both compelling and accurate.

Throughout this course of study we make use of a series of real data sets that are contained in the data disk attached to this book. We will pose relevant questions and learn the appropriate methods of analysis for answering such questions. You will learn that more than one statistic or method of analysis typically is needed to address a question fully. You will learn also that a detailed description of the data, including possible anomalies, and an ample characterization of results, are critical components of any data analytic plan. Through this process we hope that you will come to view statistics as an integrated set of data analytic tools that when used together in a meaningful way will serve to uncover the story contained in the numbers.

THE ROLE OF THE COMPUTER IN DATA ANALYSIS

From our own experience, we have found that the use of a computer statistics package to perform computations and create graphs not only enables a greater emphasis on conceptual understanding and interpretation, but also allows students to study statistics in a way

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that reflects statistical practice. We have selected the latest version of SPSS for Windows available to us at the time of writing, version 10.0, for use with this text. We have selected SPSS because it is a well-established package that is widely used by Behavioral and Social Scientists. Also, because it is menu-driven, it is easily learned. SPSS is a computational package like MINITAB, JMP, Data Desk, Systat, Stata, and SPlus, which is powerful enough to handle the analysis of large data sets quickly. By the end of the course, you will have obtained a conceptual understanding of statistics as well as an applied, practical skill in how to carry out statistical operations. SPSS version 10.0 operates in a Windows environment.

STATISTICS: DESCRIPTIVE AND INFERENTIAL

The subject of statistics may be divided into two general branches: descriptive and inferential. *Descriptive statistics* are used when the purpose of an investigation is to *describe* the data that have been (or will be) collected. Suppose, for example, that a third-grade elementary school teacher is interested in determining the proportion of children who are firstborn in her class of 30 children. In this case, the focus of the teacher's question is her own class of 30 children and she will be able to collect data on all of the students about whom she would like to draw a conclusion. The data collection operation will involve noting whether each child in the classroom is firstborn or not; the statistical operations will involve counting the number who are, and dividing that number by 30, the total number of students in the class, to obtain the proportion sought. Because the teacher is using statistical methods merely to describe the data she collected, this is an example of descriptive statistics.

Suppose, on the other hand, that the teacher is interested in determining the proportion of children who are firstborn in all third-grade classes in the city where she teaches. It is highly unlikely that she will be able to (or even want to) collect the relevant data on all individuals about whom she would like to draw a conclusion. She will probably have to limit the data collection to some randomly selected smaller group and use inferential statistics to generalize to the larger group the conclusions obtained from the smaller group. Inferential statistics are used when the purpose of the research is not to describe the data that have been collected, but to generalize or make inferences based on it. The smaller group on which she collects data is called the *sample*, whereas the larger group to whom conclusions are generalized (or inferred), is called the population. In general, two major factors influence the teacher's confidence that what holds true for the sample also holds true for the population at large. These two factors are the method of sample selection and the size of the sample. Only when data are collected on all individuals about whom a conclusion is to be drawn (when the sample is the population and we are therefore in the realm of descriptive statistics), can the conclusion be drawn with 100 percent certainty. Thus, one of the major goals of inferential statistics is to assess the degree of certainty of inferences when such inferences are drawn from sample data. Although this text is divided roughly into two parts, the first on descriptive statistics and the second on inferential statistics, the second part draws heavily on the first.

INTRODUCTION

3

VARIABLES AND CONSTANTS

In the previous section, we discussed a teacher's interest in determining the proportion of students who are firstborn in the third grade of the city where she teaches. What made this question worthy of pursuit was the fact that she did not expect everyone in the third grade to be firstborn. Rather, she quite naturally expected that in the population under study, birth order would vary, or differ, from individual to individual and that only in certain individuals would it be first.

Characteristics of persons or objects that vary from person to person or object to object are called *variables*, whereas characteristics that remain constant from person to person or object to object are called *constants*. Whether a characteristic is designated as a variable or as a constant depends, of course, on the study in question. In the study of birth order, birth order is a variable; it can be expected to vary from person to person in the given population. In that same study, grade level is a constant; all persons in the population under study are in the third grade.

EXAMPLE 1.1. Identify some of the variables and constants in a study comparing the math achievement of tenth-grade boys and girls in the southern United States.

Solution. *Constants:* Grade level; Region *Variables:* Math achievement; Sex

EXAMPLE 1.2. Identify some of the variables and constants in a study of math achievement of secondary-school boys in the southern United States.

Solution. Constants: Sex; Region

Variables: Math achievement; Grade level

Note that grade level is a constant in Example 1.1 and a variable in Example 1.2. Because constants are characteristics that do not vary in a given population, the study of constants is neither interesting nor informative. The major focus of any statistical study is therefore on the variables rather than the constants.

THE MEASUREMENT OF VARIABLES

Measurement involves the observation of characteristics on persons or objects, and the assignment of numbers to such persons or objects so that the numbers assigned represent the amounts of the characteristics possessed. The specific characteristics observed and the particular rules used for making the number assignments determine the type of arithmetic operations and comparisons that can meaningfully be made.

Nominal Level. The simplest form of observation is to perceive that two objects are similar or dissimilar; for example, short versus non-short, heavy versus non-heavy, or college student versus non-college student. Those objects that are perceived to be the same are assigned to the same class, and within any one class, subclasses may be defined. That is, Male College Student and Female College Student may be defined as subclasses within the general class College Student. Within this most simple form of observation, however, comparisons between classes or subclasses cannot be made. One class may be different

from another in a particular characteristic, but it is neither better nor worse. The classes or subclasses are only named or enumerated in this level of measurement, called appropriately the *nominal level of measurement*; they are not compared.

If numbers were assigned to the classes in this nominal level of measurement (e.g., if a 1 were assigned to Male College Student, a 2 to Female College Student, and a 3 to Person Who Is Not a College Student), they would be assigned in the most arbitrary of ways. The fact that, for example, 2 is larger numerically than 1 would be irrelevant. The 2 could just as easily be assigned to the class Male College Student or Person Who Is Not a College Student and the 1 to the class Female College Student or Person Who Is Not a College Student. The only rule followed in this level of measurement is that different numbers are assigned to different classes or subclasses. All other possible comparisons between the numbers are disregarded.

Variables that have only two categories (often labeled as either yes or no) are called *dichotomous* variables. Sex is an example of a dichotomous variable. We may think of the categories, in this case, as either male or female, or as either yes or no if they are in response to the question, Is the respondent male? Dichotomous variables can be considered to be nominal-leveled.

Ordinal Level. If the college students in the foregoing example were rated according to great success, average success, and below-average success in college, then an ordinal (or ordered) comparison among the college students in terms of the property success could be made. Such ordered observations characterize the next level of measurement, the ordinal level of measurement.

Suppose you were asked to rank the 10 best college teachers you have had by giving the best teacher a 1, the second-best teacher a 2, the third-best teacher a 3, and so on. The rule for assigning the numbers in this case is simply "the better the teacher, the lower the number assigned." But we could just as well use any other increasing set of numbers, like $1, 3, 5, \ldots, 19$, or $5, 10, 15, \ldots, 50$ to label the teachers from best to worst. The two rules followed in this level of measurement are that (1) different numbers are assigned to different amounts of the property under observation and (2) the higher the number assigned to a person or object, the less (or more) of the property the person or object is observed to have. It is *not* true in the ordinal level of measurement that equal numerical differences along the numerical scale correspond to equal increments in the property being measured. Using the 1-to-10 ranking, for example, it would not necessarily be true that the difference in teaching ability between teacher 9 and teacher 7 is the same as the difference in teaching ability between teacher 5 and teacher 3, even though the numerical difference for each pair is 2(9-7=5-3=2).

While dichotomous variables can be considered to be nominal-leveled, in some situations, they are considered to be ordinal-leveled. In the case of gender, for example, if 1 = male and 2 = female, we may say that individuals with a score of 2 possess more femaleness than individuals with a score of 1. With this interpretation, the dichotomous variable gender is ordinal-leveled.

Interval Level. An ordinal level of measurement can be developed into a more meaningful scale if it is possible to assess how near to each other the persons or objects are in the property being observed. If numbers can be assigned in such a way that equal numerical differences correspond to equal increments in the property, we have, what is called an *interval level of measurement*. As an example of an interval level of measurement, consider